UNIVERSITY OF ABERDEEN **SESSION 2006–07**

DEGREE EXAMINATION

EG2510 Engineering Mathematics 3

Tuesday 29 May 2007 (9 am to 11 am)

NOTES *(i) Candidates are permitted to use approved calculators.*

(ii) Candidates are permitted to use the Engineering Mathematics Handbook which will be provided for them.

Candidates should attempt ALL FOUR questions in SECTION A and TWO of the questions in SECTION B.

SECTION A

1. Let

$$A = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 7 & 1 \end{pmatrix}.$$

The matrix A has 3 distinct real eigenvalues. The vectors

$$u = \begin{pmatrix} 1 \\ -4 \\ 9 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

are eigenvectors of A. Find, for each of these two eigenvectors, its corresponding eigenvalue.

It is given, and you are NOT required to check, that 2 is an eigenvalue of A. Find an Eigenvector corresponding to the eigenvalue 2.

Finally, write down an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

[10 marks]

2. **(a)** A box contains 20 resistors, of which 4 are faulty. A sample of 2 is chosen at random.

(i) What is the probability that none of the resistors in the sample are faulty?

(ii) What is the probability that at least one of the resistors in the sample is faulty?

[4 marks]

(b) A fair coin is tossed 7 times. The coin lands heads or tails.

(i) What is the probability of getting exactly 3 heads?

(ii) What is the probability of getting at most 3 heads?

(iii) What is the probability of getting a head on the first toss and two more heads in the other six tosses?

[6 marks]

3. **(a)** Values of a function $f(x)$ are found by experiment to be:

x :	1.0	1.1	1.2	1.3	1.4	1.5
$f(x)$:	156	204	276	371	477	603

Estimate the value of $f(1.27)$ by using (i) linear interpolation and (ii) Bessel's second order interpolation formula.

[7 marks]

(b) Use inverse linear interpolation to estimate the value of x (in the given range) for which $f(x) = 300$.

[3 marks]

4. **(a)** Find the least squares best fit straight line to the data points:

x :	0.3	0.4	0.5	0.6	0.7
$f(x)$:	21	23	29	37	43

[8 marks]

(b) A real-valued function $f(x)$ has the property that $|f'(x)| \leq 5$ for all values of x in the range considered. Given that $f(2) = 2.6$, what is the maximum possible error in estimating $f(2.01)$ as 2.60?

[2 marks]

SECTION B

5. **(a)** Consider the real symmetric matrix $A = \begin{pmatrix} 8 & 2 \\ 2 & 5 \end{pmatrix}$. Find an orthogonal matrix P and a diagonal matrix D such that $P^T AP = D$.

[10 marks]

(b) The matrix

$$A = \begin{pmatrix} 3 & 5 \\ -1 & 1 \end{pmatrix}$$

has complex eigenvalues. Determine the eigenvalues of A and find a real invertible matrix P such that

$$P^{-1}AP = \begin{pmatrix} \alpha & -\omega \\ \omega & \alpha \end{pmatrix}$$

where α and ω are real numbers with $\omega > 0$.

Variables x_1 and x_2 are functions of a variable t (time). Write down the general solution of the system:

$$\frac{dx_1}{dt} = -3x_1 + 5x_2$$

$$\frac{dx_2}{dt} = -1x_1 + 1x_2$$

[10 marks]

6. **(a)** A company owns two factories which manufacture the same product: Factory 1 and Factory 2. Factory 1 produces 60% of the total output and Factory 2 produces 40%. In Factory 1, 2% of the items manufactured are defective, and in Factory 2, 4% are defective.

An item is chosen at random from a day's output and tested. Let A denote the event that the item was manufactured in Factory 1 and B denote the event that the item is defective.

Write down the probability $\mathbb{P}(A)$ and the conditional probability $\mathbb{P}(B|A)$.

Calculate the probabilities:

$$\text{(i) } \mathbb{P}(A \cap B); \qquad \text{(ii) } \mathbb{P}(B); \qquad \text{(iii) } \mathbb{P}(A|B).$$

[7 marks]

(b) A continuous random variable X has probability density function

$$f_X(x) = \begin{cases} 3x^2 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

What is the probability $\mathbb{P}(X < 2/3)$ that X is less than 2/3?

Calculate the expectation $\mathbb{E}(X)$ and the variance $\text{var}(X)$.

[6 marks]

(c) The lifetime, in hours, of a certain type of light bulb is a continuous random variable T with exponential probability density function

$$f_T(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{1}{5000} e^{-t/5000} & \text{if } t \ge 0. \end{cases}$$

(i) Write down the value of $\mathbb{E}(T)$, the expectation of T.

(ii) Determine the probability that the light bulb will function for less than 2500 hours.

(iii) Determine the probability that the light bulb will function for more than $10,000$ hours.

[7 marks]

7. **(a)** Consider the initial value problem:

$$\ddot{x} + \dot{x} = 2tx, \qquad x = 1, \dot{x} = 2 \text{ when } t = 0.$$

Write down an equivalent first order system and use two steps of Euler's method (with step length 0.1) to estimate the values of x and $\dot{x}$ when $t = 0.2$.

[10 marks]

(b) Find values of constants A, B and C such that

$$\int_0^{2h} f(x)dx = h(Af(0) + Bf(h) + Cf(2h))$$

for any function $f(x)$ that is a polynomial of degree less than or equal to 3 (and any step length $h > 0$).

[10 marks]