**UNIVERSITY OF ABERDEEN** **SESSION 2004-2005**

Degree Examination

EG2510 Engineering Mathematics 3

Wednesday 10 August 2005 (3 pm—5 pm)

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**NOTES (i) Candidates are permitted to use approved calculators.**

**(ii) Candidates are permitted to use the Engineering Mathematics Handbook which will be provided for them.**

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*Candidates should attempt ALL FOUR questions in SECTION A and TWO of the questions in SECTION B.*

## SECTION A

**1.** Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & -2 \\ 0 & -2 & 3 \end{bmatrix}$. It is given (you are *not* required to show) that $-1$, 2 and 5 are eigenvalues of $\mathbf{A}$. Find an orthogonal matrix $\mathbf{Y}$ such that $\mathbf{Y}^T\mathbf{B}\mathbf{Y}$ is a diagonal matrix. Write down the diagonal matrix.

[10 marks]

**2.** **(a)** In a certain consignment of 15 thermostats, 4 are faulty. A sample of 2 thermostats is chosen at random from the consignment.

(i) By considering the number of ways of choosing the 2 thermostats, or otherwise, find the probability that both of the thermostats in the sample are faulty.

(ii) Find the probability that at most one of the two thermostats in the sample is faulty.

[4 marks]

**(b)** Sampling and testing over a long period has shown that 2% of the drill bits made on a production line are faulty. *Write down* the probability that a randomly chosen bit is satisfactory.

A sample of 8 bits is chosen at random. Assuming that the performance of each bit is independent of the others in the sample, find:

(i) the probability that all 8 of the bits in the sample are satisfactory;

(ii) the probability that exactly 6 of the bits in the sample are satisfactory.

[6 marks]

**3.** Values of a function $f(x)$ are found by experiment to be:

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ : | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| $f(x)$ : | 422 | 510 | 603 | 703 | 812 | 932 |

Estimate the value of $f(0.73)$ by using (i) linear interpolation and (ii) Bessel's second order interpolation formula.

Use inverse linear interpolation to estimate the value of $x$ (in the given range) for which $f(x) = 683$.

[10 marks]

**4.** **(a)** Find the least squares best fit straight line to the data points:

| | | | | | |
|---|---|---|---|---|---|
| $x$ : | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| $y$ : | 1534 | 1242 | 934 | 640 | 335 |

[5 marks]

**(b)** A real-valued function $f(x)$ has the property that $|f''(x)| \leq 2$ for all values of $x$ in the range considered. Given that $f(3) = 5$ and $f'(3) = 4$, estimate, as accurately as you can, the value of $f(3.01)$. What is the maximum possible error in your estimate?

[5 marks]

## SECTION B

**5.** Two wagons of mass $m$ are connected by a spring coupling with spring constant $2k$. Both wagons are connected by spring couplings with spring constants 2k and 5k to fixed points as indicated in Figure 1. The track is horizontal and frictional forces are negligible.

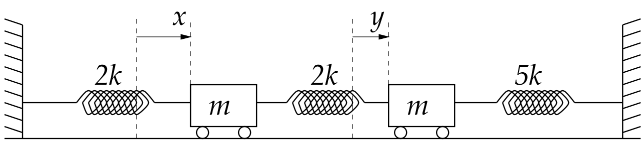


**Figure 1 (Question 5)**

Let $x$ and $y$ be the displacements of the wagons from their equilibrium positions as indicated in Figure 1 ($x$ and $y$ depend on time $t$). Show that

$$m\ddot{x} = -4kx + 2ky,$$
$$m\ddot{y} = 2kx - 7ky.$$

[8 marks]

By considering the matrix $\mathbf{B} = \begin{bmatrix} -4 & 2 \\ 2 & -7 \end{bmatrix}$, or otherwise, find the general solution of the system. Give the ratio of the frequencies of the normal modes of the system.

[12 marks]

**6.** **(a)** A manufacturer of cameras buys a certain kind of lens from two suppliers, obtaining 80% from the preferred supplier $X$ (who is unable to meet a higher target) and 20% from supplier $Y$. Records taken over several months show that 2% of the lenses supplied by $X$ and 4% of those supplied by $Y$ are faulty.

A lens is chosen at random from stock. Let $A$ be the event that the lens was supplied by $X$ and let $B$ be the event that the lens is faulty.

*Write down* the probability $P(A)$ and the conditional probability $P(B|A)$, and calculate $P(A \cap B)$.

By using a tree diagram, or otherwise, find the probabilities (i) $P(B)$, (ii) $P(A|B)$.

[7 marks]

**(b)** A continuous random variable $T$ has probability density function

$$f_T(t) = \begin{cases} 0 & \text{if } t < 0, \\ ct & \text{if } 0 \leq t \leq 2, \\ 0 & \text{if } t > 2, \end{cases}$$

where $c$ is a constant.

Explain why $c$ must equal $1/2$. Find the probability $P(T > 1)$ that $T$ is greater then 1.

Calculate the expectation $E(T)$ and the variance $\mathrm{var}\,(T)$ of $T$.

[7 marks]

**(c)** An engineering company manufactures metal rods with design length 20cm, but over a long period of time it is found that in practice the length of a rod is (to a good approximation) a normally distributed random variable $X$ with mean 20.01cm and standard deviation 0.05cm

A rod is chosen at random. What is the probability that its length is less than 19.98cm?

What percentage of the manufactured rods have length greater than 20.05cm?

[Values of the normal distribution $\Phi(x)$ are tabulated on the final page of the Examination Handbook.]

[6 marks]

**7.** **(a)** Consider the initial value problem:

$$\dot{x} = t^2 + x^2, \qquad x = 1 \text{ when } t = 0$$

Obtain two estimates for the value of $x$ when $t = 0.2$ by using (i) two steps of Euler's method (with step length 0.1) and (ii) one step of the Improved Euler method.

[10 marks]

**(b)** Find values of constants $A$, $B$ and $C$ such that

$$\int_{-h}^{h} f(x)\,dx = h(Af(-h) + Bf(0) + Cf(h))$$

for any function $f(x)$ that is a polynomial of degree less than or equal to 3 (and any step length $h > 0$).

[10 marks]