UNIVERSITY OF ABERDEEN **SESSION 2006–2007**

Degree Examination

EG2010 Engineering Mathematics 2

Wednesday 17 January 2007 (9 am to 11 am)

NOTES:

(i) Candidates are permitted to use approved calculators.

(ii) Candidates are permitted to use the Engineering Mathematics Handbook, which will be available to them.

(iii) You must not have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material must not be amended, annotated or modified in any way.

(iv) You must not have in your possession any material that could be determined as giving you an advantage in the examination.

(v) You must not attempt to communicate with any candidate during the exam, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

Failure to comply with (iii)-(v) will be regarded as cheating and may lead to disciplinary action as indicated in the Academic Quality Handbook, Section 4.14 and 5 (www.abdn.ac.uk/registry/quality/appendix7x1.pdf).

Candidates should attempt ALL questions from Section A and any TWO questions from section B.

SECTION A

1. **(a)** Find the general solution of the ordinary differential equation

$$\frac{dx}{dt} + 3x = 10\mathrm{e}^{2t},$$

using a particular integral of the form $x = p\mathrm{e}^{2t}$. [5 marks]

(b) Solve the initial value problem

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} - 6x = 0, \quad \text{with } x = 1 \text{ and } \frac{dx}{dt} = 8 \text{ when } t = 0.$$

[5 marks]

2. **(a)** A variable u is given as a function of two variables x, y by $u = (x - y)\sin(xy)$. Find the partial derivatives $\dfrac{\partial u}{\partial x}$, $\dfrac{\partial u}{\partial y}$ and show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = (x^2 - y^2)\cos(xy).$$

[5 marks]

(b) Let u be the function of three variables given by $u = \mathrm{e}^{-3x}\sin y \cos 2z$. Find the value of the constant c such that u satisfies the partial differential equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = cu.$$

[5 marks]

3. **(a)** Find the one critical point of the function

$$f(x, y) = 2x^2 - 2xy + y^2 + 6x - 4y + 5$$

and classify it as a local maximum, a local minimum or a saddle point. [5 marks]

(b) Show that $(2, 0)$ is a critical point of the function

$$f(x, y) = \mathrm{e}^{(x-2)y},$$

and classify that critical point as a local maximum, a local minimum or a saddle point.

[5 marks]

4. **(a)** Find the Laplace transforms of each of the following functions

(i) $f(t) = 4 - \mathrm{e}^{6t}$,

(ii) $f(t) = 3\sin 2t - 5\cos 2t$,

(iii) $f(t) = \mathrm{e}^{-t}\cos t$. [5 marks]

(b) Find the inverse Laplace transform of each of the following functions

(i) $F(s) = \dfrac{1}{s^3} + \dfrac{1}{(s+1)^2}$,

(ii) $F(s) = \dfrac{6}{(s-2)(s+1)}$. [5 marks]

SECTION B

5. **(a)** Find the general solution of the homogeneous linear differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 8x = 0.$$

Classify the system described by the differential equation as under-damped, critically damped or over-damped. Also give the value of the natural (angular) frequency and the damping parameter. [You are reminded of the damping term $2k\omega_n \frac{dx}{dt}$ in a standard second order linear differential equation.] [7 marks]

(b) Using your result from (a), find the general solution of the non-homogeneous differential equation:

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 8x = 20\sin 2t.$$

Describe the behaviour of the solution as $t \to \infty$. Write down the steady state response and find its period and amplitude. [13 marks]

6. **(a)** A variable u is given as a function of three variables x, y, z by $u = \dfrac{x^3y^4}{z^2}$. Find $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial u}{\partial z}$. Give a formula which estimates the change δu in u produced by small changes δx in x, δy in y and δz in z.

Use your formula to estimate the maximum possible percentage error in u produced by changes of $\pm 2\%$ in x, $\pm 1\%$ in y and $\pm 0.5\%$ in z. [8 marks]

(b) A variable u is a function of the two variables x and y and the variables x, y are given as functions of variables s, t by $x = 3s - 4t$, $y = 4s + 3t + 2$. Express the partial derivatives $\dfrac{\partial u}{\partial s}, \dfrac{\partial u}{\partial t}$ in terms of the partial derivatives $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}$.

Hence show that

$$\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right),$$

for a suitable constant k whose value should be found. [12 marks]

7. **(a)** Let f be an odd function which is periodic with period 2π. It is given in the range $0 \le x \le \pi$ by:

$$f(x) = \begin{cases} x & \text{if } 0 \le x < \pi, \\ 0 & \text{if } x = \pi. \end{cases}$$

Sketch the graph of $f(x)$ in the range $-\pi \le x \le 3\pi$.

Determine the Fourier sine coefficients b_n $(n \ge 1)$ for $f(x)$ and write down the Fourier sine series as far as the term in $\sin 3x$. [8 marks]

(b) Consider the initial value problem

$$\frac{dx}{dt} = 3x - y, \qquad \frac{dy}{dt} = 4x - 2y - 6, \quad \text{where } x = 0 \text{ and } y = 0 \text{ when } t = 0.$$

Show that the Laplace transform $X(s)$ of x is given by

$$X(s) = \frac{6}{(s+1)s(s-2)}.$$

Hence express x as a function of t. [12 marks]