UNIVERSITY OF ABERDEEN **SESSION 2005–2006**

DEGREE EXAMINATION
EG2010 Engineering Mathematics 2
Friday 20 January 2006 (9am to 11am)

NOTES:

(i) Candidates are permitted to use approved calculators.

(ii) Candidates are permitted to use the Engineering Mathematics Handbook, which will be available to them.

Candidates should attempt ALL questions from Section A and any TWO questions from section B.

SECTION A

1. **(a)** Find the general solution of the ordinary differential equation

$$\frac{dx}{dt} - 2x = 2t - 3,$$

using a particular integral of the form $x = pt + q$. [5 marks]

(b) Solve the initial value problem

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 6x = 0, \quad \text{with } x = 1 \text{ and } \frac{dx}{dt} = 7 \text{ when } t = 0.$$

[5 marks]

2. **(a)** A variable u is given as a function of two variables x, y by $u = xy^2e^{3y} + x^2 + y$. Find the partial derivatives $\frac{\partial u}{\partial x}$, $\frac{\partial^2 u}{\partial x^2}$, $\frac{\partial u}{\partial y}$, $\frac{\partial^2 u}{\partial x \partial y}$.

[5 marks]

(b) Let u be the function of three variables given by $u = xz^2 + yx^2 + zy^2$. Find the value of the constant k such that u satifies the partial differential equation:

$$x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + z^2\frac{\partial^2 u}{\partial z^2} = ku.$$

[5 marks]

3. **(a)** Find the two critical points of the function

$$f(x, y) = 1 + 3x^2 + 3y - 4y^3$$

and classify them as local maxima, local minima or saddle points. [5 marks]

(b) Find the one critical point of the function

$$f(x, y) = \mathrm{e}^{xy},$$

and classify that critical point as a local maximum, a local minimum or a saddle point.

[5 marks]

4. **(a)** Find the Laplace transforms of each of the following functions

(i) $f(t) = t^3 - 4t + 2$,

(ii) $f(t) = 2\cos(3t) + \mathrm{e}^{-4t}$,

(iii) $f(t) = \mathrm{e}^{3t}t^2$.

[5 marks]

(b) Find the inverse Laplace transform of each of the following functions

(i) $F(s) = \dfrac{4}{s^3} - \dfrac{5}{s}$,

(ii) $F(s) = \dfrac{1}{s^2 + 4}$.

(iii) $F(s) = \dfrac{12}{(s - 2)^4}$.

[5 marks]

SECTION B

5. **(a)** Find the general solution of the homogeneous linear ODE

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0.$$

Classify the system described by this differential equation as under damped, critically damped or over damped. Give the value of the natural (angular) frequency and the damping parameter. [You are reminded of the theoretical expression $2k\omega_n \frac{dx}{dt}$.] [7 marks]

(b) Using your result from (a), find the general solution of the non-homogeneous ordinary differential equation:

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = \cos t - 2\sin t.$$

Describe the behaviour of the solution as $t \to \infty$. Write down the steady state response and determine its amplitude.

[13 marks]

6. **(a)** Find the linear approximation to the function $f(x,y) = \cos(x+y) + \sin(2x-y)$ for (x,y) close to $(0,0)$. [6 marks]

(b) The radius of a cylinder can be manufactured to an accuracy of $\pm 1.5\%$ and the height of the cylinder can be manufactured to an accuracy of $\pm 3\%$. Estimate the maximum possible percentage error in the volume of the cylinder produced. [6 marks]

(c) A variable u is a function of the two variables x and y. New variables s and t are defined by $s = x + 2y$ and $t = 2x - y$.

Using the chain rule, express the partial derivatives $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}$ in terms of the partial derivatives $\dfrac{\partial u}{\partial s}, \dfrac{\partial u}{\partial t}$.

Hence rewrite the partial differential equation

$$2\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0 \qquad (1)$$

in terms of the new variables s, t and find the general solution. Determine the solution of (1) which satisfies the condition that $u(x,0) = \cos x$ for all x.

[8 marks]

7. **(a)** An odd function $f(x)$ is periodic with period 2π. It is given in the range $0 \le x \le \pi$ by:

$$f(x) = \begin{cases} 3 & \text{if } 0 < x < \pi, \\ 0 & \text{if } x = 0 \text{ or } x = \pi. \end{cases}$$

Sketch the graph of $f(x)$ in the range $-\pi \le x \le 3\pi$.

Determine the Fourier sine coefficients b_n $(n \ge 1)$ for $f(x)$ and write down the Fourier sine series as far as the term in $\sin(5x)$. [7 marks]

(b) Consider the initial value problem

$$\frac{dx}{dt} = 2x - 4y, \qquad \frac{dy}{dt} = x - 3y + 3, \quad \text{where } x = 0 \text{ and } y = 0 \text{ when } t = 0.$$

Show that the Laplace transform $X(s)$ of x is given by

$$X(s) = \frac{-12}{s(s+2)(s-1)}.$$

Hence express x as a function of t. [13 marks]