UNIVERSITY OF ABERDEEN **SESSION 2006–07**

DEGREE EXAMINATION

EG1006 Engineering Mathematics 1

Tuesday 23 January 2007 (12 noon to 2 pm)

NOTE:
(i) Candidates are permitted to use approved calculators.
(ii) Candidates are permitted to use the Engineering Mathematics Handbook, which will be made available to them.
(iii) Marks may be deducted for answers that do not show clearly how the solution is reached.
(iv) You must not have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material must not be amended, annotated or modified in any way.
(v) You must not have in your possession any material that could be determined as giving you an advantage in the examination.
(vi) You must not attempt to communicate with any candidate during the exam, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

Attempt ALL FIVE of the questions in SECTION A and THREE of the questions in SECTION B.

SECTION A

1. Differentiate the following functions with respect to x:

$$x^3 + \frac{2}{x^2}, \qquad x\cos(1+x^3), \qquad \frac{x}{\ln x}, \qquad \arcsin(x^2).$$ **[2,2,2,3 marks]**

2. **(a)** A curve is given by the equation

$$y^3x + yx^2 = 2yx + 2\sin x.$$

Find an expression for $\dfrac{dy}{dx}$ in terms of x and y. **[5 marks]**

(b) A curve is given parametrically by

$$x = 1 + \cos t, \qquad y = 1 - \sin t.$$

Find the x and y coordinates of the point P on the curve that corresponds to the parameter $t = \dfrac{\pi}{4}$. Find also the equation of the tangent line to the curve at P. **[4 marks]**

3. Evaluate the following integrals:

$$\int \left(x^2 - \frac{1}{x^2}\right) dx, \qquad \int t^2 e^t \, dt, \qquad \int \frac{2u\,du}{1+u^2}.$$ **[3,3,3 marks]**

4. **(a)** Let $z = 3 + j$ and $w = 2 + 3j$. Express each of the following complex numbers in the form $x + jy$ where x and y are real numbers.

$$3z - 2\bar{w}, \qquad zw, \qquad \bar{z}/w.$$ **[5 marks]**

(b) Express the complex number $z = 3 - 3j$ in modulus-argument form. Hence find the modulus and *principal* argument of z^4. **[4 marks]**

5. **(a)** Define matrices A, B and C by

$$A = \begin{pmatrix} 4 & 0 \\ 2 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 2 \\ -7 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & 3 & -2 \\ 1 & 0 & 1 \end{pmatrix}$$

Calculate those of the following expressions that can be evaluated and explain why the others cannot be:

$$AB, \quad A^2, \quad 4CB, \quad BC, \quad 2A + C, \quad C^{-1}.$$ **[5 marks]**

(b) Find the inverse of the matrix

$$\begin{pmatrix} 4 & 0 & 2 \\ 1 & 0 & 0 \\ 0 & 2 & 1 \end{pmatrix}.$$ **[4 marks]**

SECTION B

6. An observer O and a rocket pod P are located on horizontal ground 100 m apart. A rocket R is launched from the pod and travels vertically upwards so that t seconds after launching its height is $100t$ m.

Calculate the rate of change of the distance OR and the rate of change of the angle $\widehat{POR}$ when $t = 2$. **[6,7 marks]**

7. **(a)** Find the local and global maximum and minimum of the function

$$f(x) = (x + 4)(x - 1)^2$$

on the range $-4 \leq x \leq 4$. **[7 marks]**

(b) Evaluate the definite integral

$$\int_0^2 \frac{(t - 1)dt}{(t + 1)(t + 2)}.$$ **[6 marks]**

8. **(a)** Find the area bounded by the lines $x = 1$, $x = 8$, the x-axis and the curve $y = x^{4/3}$. **[6 marks]**

(b) Find the volume generated when the area bounded by the curve $y = \sqrt{1 + x}$, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 2π radians about the x-axis. **[7 marks]**

9. **(a)** Let A and B be the matrices given by

$$A = \begin{pmatrix} 3 & 0 & 2 \\ 1 & 1 & 1 \\ -3 & 2 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 4 & 1 & 0 \\ 2 & -1 & 0 \\ 2 & 0 & -2 \end{pmatrix}$$

Calculate the matrix AB and show it is invertible. **[5 marks]**

(b) For which value of t is the following system of linear equations inconsistent ?

$$\begin{array}{rcrcrcr} 2x & - & y & + & tz & = & 1 \\ x & + & y & + & (1-t)z & = & -1 \\ x & + & 2y & - & tz & = & 0 \end{array}$$

Solve the system for $t = 1$.

[8 marks]

10. **(a)** Solve the polynomial equation

$$z^4 - 2z^3 + 3z^2 - 2z + 2 = 0$$

given that $z = 1 + j$ is a root. **[4 marks]**

(b) Write down the first eight derivatives of $\sin x$. Deduce the Taylor expansion of the function $\sin x$ about $x = 0$. **[4 marks]**

(c) Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0.$$

Hence find the solution of this equation which satisfies $x(0) = 0$, $x'(0) = \frac{\sqrt{3}}{2}$.
[5 marks]