UNIVERSITY OF ABERDEEN **SESSION 2005–06**

DEGREE EXAMINATION

EG1006 Engineering Mathematics 1

Monday 16 January 2006 (3pm to 5pm)

NOTE: *(i)* *Candidates are permitted to use approved calculators*

(ii) *Candidates are permitted to use the Engineering Mathematics Handbook, which will be made available to them.*

(iii) *Marks may be deducted for answers that do not show clearly how the solution is reached.*

Attempt ALL FIVE of the questions in SECTION A and THREE of the questions in SECTION B.

SECTION A

1. Differentiate the following functions:

$$3x^5 - \frac{1}{x^3}, \qquad e^{2x}\ln(3x), \qquad \frac{\tan(x^2-1)}{x^2}.$$ **[3,3,3 marks]**

2. **(a)** A curve is given, parametrically, by the equations $x = t^2$, $y = t^5$. Show that the point $(1, -1)$ lies on this curve and find the equation of the tangent to the curve at this point. **[4 marks]**

(b) A curve is given by the implicit relation

$$x^2y^2 - y\cos x + x\sin y = 0.$$

Find an expression for $\dfrac{dy}{dx}$ in terms of x and y. **[5 marks]**

3. Evaluate the following integrals:

$$\int \left(x^2 - \frac{1}{x^4}\right) dx, \qquad \int x\cos 2x\, dx, \qquad \int x\sin(x^2)dx.$$ **[3,3,3 marks]**

4. **(a)** Let $z = -6 + j$ and $w = 4 + 5j$. Express each of the following complex numbers in the form $x + jy$ where x and y are real numbers.

$$5z + \bar{w}, \qquad zw, \qquad \bar{z}/w.$$ **[5 marks]**

(b) Express the complex number $z = 2 - 2j$ in modulus-argument form. Hence find the modulus and *principal* argument of z^5. **[4 marks]**

5. **(a)** Define matrices A, B and C by

$$A = \begin{pmatrix} 3 & 0 \\ 1 & -5 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 3 \\ -5 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 & -2 \\ 3 & 4 & 1 \end{pmatrix}$$

Calculate those of the following expressions that can be evaluated and explain why the others cannot be:

$$AB, \quad A^2, \quad CB, \quad 2BC, \quad A+C, \quad C^{-1}.$$ **[5 marks]**

(b) Find the inverse of the matrix

$$\begin{pmatrix} 2 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 3 \end{pmatrix}.$$ **[4 marks]**

SECTION B

6. **(a)** In the plane with usual Cartesian coordinates x and y and at a certain time $t = 0$, A is at the point with coordinates $(0, 1)$ and B is at the point with coordinates $(1, 0)$. The point O is the origin. In the units in which the x and y axes are calibrated, the point A moves at 3 units/sec along the y-axis in the direction of increasing y and B moves at the rate of 4 units/sec along the x-axis in the direction of increasing x. Find the rates at which the distance AB and the area of the triangle AOB are increasing after 3 seconds. **[6 marks]**

(b) The population $P(t)$ of a certain town at time t satisfies the condition that its rate of change at time t is proportional to $P(t)$. If this population was 20,000 in 1950 and 30,000 in 2000 what was the population in 1970 and in 1900 ? **[7 marks]**

7. **(a)** Find the local and global maximum and minimum of the function

$$f(x) = (x+2)^2(x-2)$$

on the range $-4 \leq x \leq 4$. **[6 marks]**

(b) Evaluate the definite integral

$$\int_1^3 \frac{(t-1)dt}{(t+3)(t+1)}.$$ **[7 marks]**

8. **(a)** Find the area enclosed by the graphs of the functions $y = x^2$ and $y = x + 2$. **[6 marks]**

(b) Find the volume enclosed when that part of the graph of the function $y = 1 + \sqrt{x}$ lying between $x = 0$ and $x = 2$ is rotated about the x-axis. **[7 marks]**

9. **(a)** Let A and B be the matrices given by

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 3 \\ -2 & 2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 3 \\ 3 & -4 & 0 \\ -2 & 0 & -6 \end{pmatrix}$$

Calculate the matrix AB and show it is invertible. **[5 marks]**

(b) For which value of λ is the following system of linear equations inconsistent ?

$$\begin{array}{rcrcrcr} 2x & - & y & + & \lambda z & = & 1 \\ x & + & y & + & (1-\lambda)z & = & -1 \\ x & + & 2y & - & \lambda z & = & 0 \end{array}$$

Solve the system for $\lambda = 1$.

[8 marks]

10. **(a)** Solve the polynomial equation

$$z^4 - 4z^3 + 12z^2 + 4z - 13 = 0$$

given that $z = 2 - 3j$ is a root. **[4 marks]**

(b) Show that the function $y(x) = x \cos x$ satisfies the differential equation

$$\frac{dy}{dx} + y \tan x = \cos x$$

[4 marks]

(c) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0.$$

Find a solution $y(x)$ of this equation which satisfies $y(0) = 2$, $y(\frac{\pi}{4}) = 1$. **[5 marks]**