UNIVERSITY OF ABERDEEN **SESSION 2004–05**

DEGREE EXAMINATION

EG1006 Engineering Mathematics 1

Monday 17 January 2005 (12noon to 2pm)

NOTE: (i) *Candidates are permitted to use approved calculators*
(ii) *Candidates are permitted to use the Engineering Mathematics Handbook, which will be made available to them.*
(iii) *Marks may be deducted for answers that do not show clearly how the solution is reached.*

Attempt ALL FIVE of the questions in SECTION A and THREE of the questions in SECTION B.

SECTION A

1. Differentiate the following functions:

$$2x^4 + \frac{1}{x^2}, \qquad x^2 \ln(x^3), \qquad \frac{e^{x^2}}{\tan x}.$$ **[3,3,3 marks]**

2. **(a)** A curve is given, parametrically, by the equations $x = 1 + t^3$, $y = t - 1$. Show that the point $(9, 1)$ lies on this curve and find the equation of the tangent to the curve at this point. **[4 marks]**

(b) A curve is given by the implicit relation

$$x^2y^2 + y \sin x - x \cos y = 0.$$

Find an expression for $\dfrac{dy}{dx}$ in terms of x and y. **[5 marks]**

3. Evaluate the following integrals:

$$\int \left(x^3 - \frac{1}{x^2}\right) dx, \qquad \int xe^{2x}\,dx, \qquad \int \frac{x\,dx}{x^2 - 4}.$$ **[3,3,3 marks]**

4. **(a)** Let $z = 3 - 2j$ and $w = 3 + 5j$. Express each of the following complex numbers in the form $x + jy$ where x and y are real numbers.

$$2z - 3w, \qquad z\bar{w}, \qquad \frac{z}{w}.$$ **[5 marks]**

(b) Express the complex number $z = -3 + 3j$ in modulus-argument form. Hence find the modulus and *principal* argument of z^2. **[4 marks]**

5. **(a)** Define matrices A, B and C by

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 4 & -1 \\ 2 & 3 \end{pmatrix}, \qquad C = (1, 2, -3).$$

Calculate those of the following expressions that can be evaluated and explain why the others cannot be:

$$AB, \quad BA, \quad A^{-1}, \quad B^2, \quad A + C.$$ **[5 marks]**

(b) Find the inverse of the matrix

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}.$$ **[4 marks]**

SECTION B

6. **(a)** Ice in the form of a cube melts in such a way that 3cc of ice is lost each second and the remaining ice is always in the form of a cube. At what rate is the length of the side of the ice cube changing when the side of the ice cube is 2cm long? **[6 marks]**

(b) A ladder of length 4m has one end A against a vertical wall and the other end B on horizontal ground. If B moves away from the wall at 2m/sec (with A remaining in contact with the wall), at what speed is A moving when B is 3m from the wall? **[7 marks]**

7. **(a)** Find the local and global maxima and minima of the function

$$f(x) = (x + 1)(x - 2)^2$$

for the range $-2 \le x \le 4$. **[6 marks]**

(b) Evaluate the definite integral

$$\int_0^3 \frac{dt}{(t+1)(t+2)}.$$ **[7 marks]**

8. **(a)** Find where the curve $y = x^2$ and the line $y = x + 2$ meet. Hence find the area enclosed by them. **[6 marks]**

(b) Find the volume enclosed when that part of the curve $y = x^3 + 2$ between $x = 1$ and $x = 2$ is rotated about the x-axis. **[7 marks]**

9. **(a)** Let A and B be the matrices given by

$$A = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 4 & 0 \\ 0 & 2 & -3 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 0 & 4 \\ 1 & 3 & -3 \end{pmatrix}.$$

Calculate the matrices $3A - 2B$ and AB. **[5 marks]**

(b) Solve each of the following two systems of linear equations, stating clearly in each case the solutions obtained.

$$\text{(i)} \quad \begin{aligned} x - y + z &= 6 \\ 2x + y + 2z &= 9 \\ x \quad\;\; - z &= -1 \end{aligned} \qquad\qquad \text{(ii)} \quad \begin{aligned} x + y + z &= 5 \\ 3x + 5y - z &= 9 \\ x + 2y - z &= 2 \end{aligned}$$

[8 marks]

10. **(a)** Solve the polynomial equation

$$z^3 - 2z^2 - 3z + 10 = 0$$

given that $z = 2 - j$ is a root. **[4 marks]**

(b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0.$$

Find a solution $y(x)$ of this equation which satisfies $y(0) = 2$, $y'(0) = 16$. **[5 marks]**

(c) Show that the function $y(x) = x^2 + e^x$ satisfies the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2 - 4x + x^2$$

[4 marks]