UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 25th APRIL 2013

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

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Institute and Faculty of Actuaries

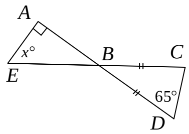
SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **E** All of the alternatives involve subtracting a number from 1. The largest result, therefore, will correspond to the smallest number to be subtracted, i.e. 0.00001.

2. **E** Their average height is $\dfrac{2.1 + 1.4}{2}$ m $= 1.75$ m.

3. **C** Triangle BCD is isosceles, so $\angle BCD = \angle BDC = 65°$.
The sum of the interior angles of a triangle is 180° so
$\angle CBD = (180 - 2 \times 65)° = 50°$.
Therefore $\angle ABE = 50°$ (vertically opposite angles). So
$\angle AEB = (180 - 90 - 50)° = 40°$.



4. **C** Distance travelled = average speed × time of travel, so Gill travelled between 15 km and 20 km. Of the alternatives given, only 19 km lies in this interval.

5. **D** The diagram shows the four lines of symmetry.

6. **A** $((1 - 1) - 1) - (1 - (1 - 1)) = (0 - 1) - (1 - 0) = -1 - 1 = -2$.

7. **D** Let the number of balls collected by Roger be x. Then Andy collects $2x$ balls and Maria collects $(2x - 5)$ balls. So $x + 2x + 2x - 5 = 35$, i.e. $5x = 40$, i.e. $x = 8$. So Andy collected 16 balls.

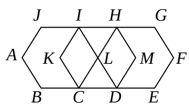
8. **A** The number 3 on the top ruler (which is 7cm from the left-hand end) aligns with the 4 on the bottom one (which is 6cm from the right-hand end). Thus $L = 7 + 6 = 13$.

9. **B** Let there be b boys and g girls in the family. Then Peter has g sisters and $(b - 1)$ brothers. So $g = 3(b - 1)$. Louise has $(g - 1)$ sisters and b brothers. So $g - 1 = 2b$. Therefore $2b + 1 = 3b - 3$, i.e. $b = 4$. So $g = 9$.
Therefore there are 4 boys and 9 girls in the family, i.e. 13 children in total.

10. **E** The top and bottom faces of the stack and the two touching faces form two pairs of opposite faces.
So the total number of pips on these four faces is $2 \times 7 = 14$. Therefore the total number of pips on the top and bottom faces of the stack is $14 - 5 = 9$.

11. **C** After Usain has run 100 m, his mum has run 50 m and Turbo has ‘run’ 10 m. So the distance between Usain's mum and Turbo is 40 m.

12. **E** Figure $ABEFGJ$ itself is a hexagon. There are three hexagons congruent to $ABCLIJ$; two hexagons congruent to $ABDMHJ$; four hexagons congruent to $ABCKIJ$; two hexagons congruent to $ABDLHJ$. So in total there are twelve hexagons.



13. **D** After the first coat, half of the paint is left. So after the second coat, the volume of paint remaining is one third of half of the capacity of the tin, i.e. one sixth of three litres = 500 ml.

14. **B** Let the two equal sides of the isosceles triangle have length a and the other side have length b. Then $2a + b = 20$. Since the sum of the lengths of any two sides of a triangle is greater than the length of the third, $2a > b$. Hence $4a > 2a + b$. So $4a > 20$, i.e. $a > 5$. Also $a < 10$ since $2a + b = 20$. So the possibilities are $a = 6, b = 8$; $a = 7, b = 6$; $a = 8, b = 4$; and $a = 9, b = 2$.

15. **A** When he starts to come down the hill, the Grand Old Duke of York has 90% of his men left. He loses 15% of these, so at the bottom of the hill he has 85% of 90% of the original number left. As $\frac{85}{100} \times 90 = 76\frac{1}{2}$, this means that $76\frac{1}{2}$% of his men were still there when they reached the bottom of the hill.

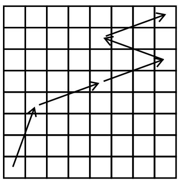
16. **B** The sum of the ages of the four children is $12 + 14 + 15 + 15 = 56$. Each year on their birthday, this sum increases by 4. So the number of years before the sum reaches 100 is $(100 - 56) \div 4 = 11$. Therefore their ages will first total 100 in 2024.

17. **E** Let x cm be the length of the ﹀ shape. Although x is not given, it is clear that $x > 1$. The lengths, in cm, of the perimeters of pieces A, B, C, D, E are $4 + 6x$, $2 + 10x$, $7 + 5x$, $6 + 6x$, $1 + 11x$ respectively. As $4 + 6x < 6 + 6x$, the piece with the longest perimeter is B, C, D or E. As $x > 1$, it may be deduced that $7 + 5x < 6 + 6x < 2 + 10x < 1 + 11x$, so E has the longest perimeter.

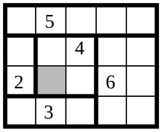
18. **A** Let the weights, in kg, of baby, nurse and me be x, y, z respectively. Then $x + z = 78$; $x + y = 69$; $y + z = 137$. Adding all three equations gives $2x + 2y + 2z = 284$, so $x + y + z = 284 \div 2 = 142$.
(*To find the combined weight, it is not necessary to find the individual weights, but baby weighs 5kg, nurse weighs 64 kg and I weigh 73 kg.*)

19. **D** For every 2 senior members in the swimming club there are 3 junior members. For every 5 senior members there are 2 veteran members. The lowest common multiple of 2 and 5 is 10, so it may be deduced that the number of senior members is a multiple of 10. For every 10 senior members in the swimming club there are 15 junior members and 4 veteran members. So the total number of members is a multiple of 29. Of the alternatives given, the only multiple of 29 is 58.

20. **B** The 'long knight' needs to move exactly seven squares to the right and exactly seven squares upwards. Although it is possible to move seven squares to the right in three moves (1, 3 and 3), in doing so it could move upwards by a maximum of five squares (3, 1 and 1). Similarly, it could move seven squares upwards in three moves, but could then move a maximum of five squares to the right. In four moves, the number of squares moved to the right must be even, since it is the sum of four odd numbers. So at least five moves are required and the diagram shows one way in which the task may be achieved in five moves.



21. **C** As 5 is a prime number, it must lie in a 5×1 rectangle. So the only possibility is the rectangle which covers the top row of the grid. Now consider 6: there is insufficient room for a 6×1 rectangle so it must lie in a 3×2 rectangle. There are only two such rectangles which include 6 but do not include either 4 or 3. If 6 comes in the middle of the top row of a 2×3 rectangle then there is space for a 3×1 rectangle including 3. But then there is not enough space for a rectangle including 4. So 6 must be placed in the rectangle shown. There is now insufficient room to place 4 in a 4×1 rectangle so it must lie in the 2×2 square shown, which includes the shaded square. This leaves the grid to be completed as shown.



22. **E** The diagram shows the totals of the rows and columns. The circled numbers are the total of the numbers in the two main diagonals. Note, by considering the average values of the rows and columns, that each should total 34. Row 2 and column 2 are both 2 short. So their common entry, 13, needs to increase by 2. So 13 must be interchanged with 15. (This change also reduces row 4 and column 3 by 2 and increases the main diagonal by 2, thus making all the sums equal 34 as desired.) So the sum of the numbers to be swapped is 28.

	9	6	3	16	34
	4	13	10	5	32
	14	1	8	11	34
	7	12	15	2	36
34	34	32	36	34	32

23. **D** Let the points awarded for a win and a draw be w and d respectively. Then $7w + 3d = 44$. The only positive integer solutions of this equation are $w = 2$, $d = 10$ and $w = 5$, $d = 3$. However, more points are awarded for a win than for a draw so we deduce that 5 points are awarded for a win and 3 points for a draw. So the number of points gained by my sister's team is $5 \times 5 + 2 \times 3 = 31$.

24. **B** Each of the overlapping areas contributes to the area of exactly two squares. So the total area of the three squares is equal to the area of the non-overlapping parts of the squares plus twice the total of the three overlapping areas i.e. $(117 + 2(2 + 5 + 8))\ \text{cm}^2 = (117 + 30)\ \text{cm}^2 = 147\ \text{cm}^2$.
So the area of each square is $(147 \div 3)\ \text{cm}^2 = 49\ \text{cm}^2$. Therefore the length of the side of each square is 7 cm.

25. **C** By arranging the tiles in suitable positions it is possible to place the 1×1 spotted square in any one of four corners of the steel sheet and then to place the grey square in any one of the other three corners. The other two corners will then be occupied by black squares. So, in total, there are $4 \times 3 = 12$ different looking installations.)