

UK JUNIOR MATHEMATICAL CHALLENGE

FRIDAY 6th MAY 2011

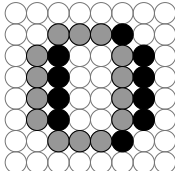
Organised by the **United Kingdom Mathematics Trust**
from the **School of Mathematics, University of Leeds**

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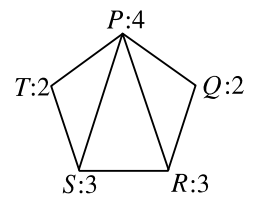
SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

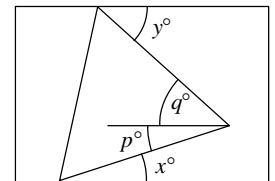
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1. **B** $2 \times 0 \times 1 + 1 = 0 \times 1 + 1 = 0 + 1 = 1$.
2. **E** If the sum of the digits is a multiple of 3 then the number is a multiple of 3. The sums of the digits of the given numbers are 6, 9, 12, 15, 18, so they are all multiples of 3.
(*Can you prove that all numbers consisting of three consecutive digits are multiples of 3? Hint: let the second digit be n .*)
3. **B** In the diagram, the extra cells which need to be lit are shown in black.
So, in total, 24 cells are lit in a bold 'o'.
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4. **C** $100 \text{ kg} = 100\,000 \text{ g}$. So the sum of money in £1 coins which would have the same mass as the world's largest coin is $\pounds(100\,000 \div 10) = \pounds10\,000$.
(*The coin was sold for \$4m (£2.6m) at an auction in Vienna in June 2010.*)
5. **B** One third is equal to four twelfths. Hence the children ate eight twelfths of the bar between them. Each child ate one twelfth of the bar, so old Mother Hubbard had eight children.

6. E The six marked angles are the interior angles of the two large triangles which make up the star shape in the diagram, so their sum is $2 \times 180^\circ = 360^\circ$.
7. D There are 9 bushels in a barrel. Each bushel is 4 pecks, so there are 36 pecks in a barrel. Therefore 35 more pecks are needed.
8. A Let the original square have side $3x$. Then its perimeter is $12x$.
The perimeter of the octagon is $2 \times 4x + 3 \times 3x + 3 \times x = 8x + 9x + 3x = 20x$.
So the required ratio is $12:20 = 3:5$.
9. A $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$ is the sum of the digits of each such number. As 45 is a multiple of 9, each such number is a multiple of 9 and so too is the difference between two of them. Thus the smallest feasible difference is 9. The two numbers 123 456 798 and 123 456 789 show that this can occur.

10. C The diagram shows the number of lines which meet at the vertices P, Q, R, S, T . When the path around the diagram passes through a vertex, it uses up two of the edges. So, apart from the first and last vertex used, each vertex must have an even number of edges meeting at it. So we are obliged to use R or S as the first vertex, and the other as the last. The path $RQPTSRPS$, together with its reverse, shows that either is a possible start. (It is a fact that such a path can be drawn through a connected graph precisely when either all, or all but 2, vertices have an even number of edges meeting there.)



11. C A line segment which is parallel to two sides of the rectangle has been added to the diagram, as shown. The angle marked p° is equal to the angle marked x° as these are alternate angles between parallel lines. So $x = p$. Similarly $y = q$. The angles marked p° and q° together form one interior angle of an equilateral triangle. Therefore $x + y = p + q = 60$.



12. E $\bullet = \blacksquare + \blacktriangle = \blacktriangle + \blacktriangle + \blacktriangle = 3\blacktriangle$. Therefore $\blacklozenge = \bullet + \blacksquare + \blacktriangle = 3\blacktriangle + 2\blacktriangle + \blacktriangle = 6\blacktriangle$.

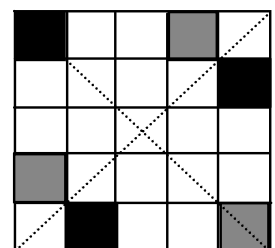
13. E The mean of $\frac{2}{3}$ and $\frac{4}{9}$ is $\left(\frac{2}{3} + \frac{4}{9}\right) \div 2 = \left(\frac{6}{9} + \frac{4}{9}\right) \div 2 = \frac{10}{9} \div 2 = \frac{5}{9}$.

(Note that the mean of two numbers lies midway between those two numbers.)

14. A Let the area of the shaded face be $x \text{ cm}^2$. Then the cuboid has two faces of area $x \text{ cm}^2$ and four faces of area $4x \text{ cm}^2$. So its total surface area is $18x \text{ cm}^2$.
Therefore $18x = 72$, that is $x = 4$.

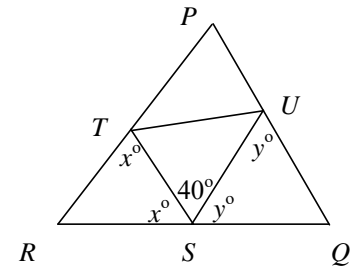
So the area of one of the visible unshaded faces is $4 \times 4 \text{ cm}^2 = 16 \text{ cm}^2$.

15. A In order that the figure has rotational symmetry of order 2, the three squares which appear in black must be shaded. When this has been done, we note that the broken lines shown are both lines of symmetry. So the minimum number of squares which must be shaded is 3.



- 16. B** The smallest possible number of votes the winner could receive corresponds to the situation in which the numbers of votes received by each of the candidates are as close together as possible.
As $83 \div 4 = 20.75$, at least one of the candidates receives 21 votes or more. However, it is not possible for the winner to receive 21 votes, since there are still 62 votes to be allocated which makes it impossible for each of the other three candidates to receive fewer than 21 votes. So the winner must receive more than 21 votes. If the numbers of votes received by the candidates are 22, 21, 20, 20 then there is a winner and, therefore, 22 is the smallest number of votes the winner could receive.
- 17. D** The lengths in minutes of the fifth set and the whole match are 491 and 665 respectively.
So the required fraction is $\frac{491}{665} = \frac{491 \times 3}{665 \times 3} \approx \frac{1500}{2000} = \frac{3}{4}$.
- 18. C** Until Peri reaches Granny's, he travels 9m in every 10 days. So he takes 90 days to travel the first 81m of his journey. There remains a distance of 9m to be covered and so, after a further 9 days, Peri is at Granny's. Therefore the length of Peri's journey is 99 days, that is 14 weeks 1 day. So Peri arrives at Granny's on Tuesday.
- 19. D** Of the given numbers, 2, 3 and 5 are all prime and therefore appear in the list. In addition, 1 appears in the list as it is the units digit of 11 and also of many other primes. However, all numbers with units digit 4 are even and therefore not prime, because the only even prime is 2. So only 1, 2, 3, 5 appear in the list.
- 20. A** Let the length of the side of each cube be x cm. Then the volume of the solid is $7x^3$ cm³. Therefore $7x^3 = 875$, that is $x^3 = 125$. So $x = 5$. The surface area of the solid comprises five of the faces of each of six cubes. Each face has area 25 cm² so the required area is $5 \times 6 \times 25$ cm² = 750 cm².
- 21. B** In total the train travels 27 km + 29 km = 56 km.
So the combined time for these two parts of the journey is $\frac{56}{96}$ hours = $\frac{7}{12}$ hours = 35 minutes.
The total journey time, therefore, is 38 minutes. So Gill arrives at 09:38.
- 22. D** Let the numbers of stamps bought by Evariste and Sophie be x and y respectively. Then $1.1x + 0.7y = 10$, that is $11x + 7y = 100$. As 100 has remainder 2 when divided by 7, we need a multiple of 11 which is two more than a multiple of 7. The multiples of 11 less than 100 are 11, 22, 33, 44, 55, 66, 77, 88, 99. Of these only 44 is two more than a multiple of 7. So the only positive integer solutions of the Diophantine equation $11x + 7y = 100$ are $x = 4$, $y = 8$. Therefore Evariste buys 4 stamps, costing £4.40, and Sophie buys 8 stamps, costing £5.60.

23. E Let $\angle RTS = x^\circ$. Then $\angle RST = x^\circ$ as $RS = RT$.
 Let $\angle QUS = y^\circ$. Then $\angle QSU = y^\circ$ as $QS = QU$.
 As RSQ is a straight line, $x + y + 40 = 180$; so $x + y = 140$.



Now $\angle TPU = 180^\circ - \angle TRS - \angle SQU$
 $= 180^\circ - (180 - 2x)^\circ - (180 - 2y)^\circ$
 $= 180^\circ - 180^\circ + 2x^\circ - 180^\circ + 2y^\circ$
 $= 2(x + y)^\circ - 180^\circ$
 $= 2 \times 140^\circ - 180^\circ$
 $= 100^\circ$.

24. D (We may assume that the party is initially on the near bank and wishes to cross to the far bank.)

If an adult crosses to the far bank then there has to be a child waiting there to bring the raft back (unless an adult immediately brings the raft back – but this represents a wasted journey). This is possible only if the first two crossings involve both children crossing to the far bank and one of them staying there whilst the other brings the raft back. The third crossing involves the first adult crossing to the far bank and on the fourth crossing the child waiting on the far bank brings the raft back to the near bank. So after four crossings, one of the adults is on the far bank and the remainder of the party is on the near bank. This procedure is repeated so that after eight crossings, both adults are on the far bank and both children are on the near bank. A ninth and final crossing then takes both children to the far bank.

25. C The three trapezia have 12 edges in total. Whenever two trapezia are joined together the total number of edges is reduced by at least 2. Therefore the maximum possible value of N is $12 - 2 \times 2 = 8$. As the shapes form a polygon, N cannot be less than 3. The diagrams below show that all values of N from 3 to 8 are indeed possible, so there are 6 different values of N .

