UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 29th APRIL 2010

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

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SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **B** The expression = 2010 + 2010 − 2010 − 2010 + 2010
= (2010 − 2010) + (2010 − 2010) + 2010 = 2010.

2. **E** In A, the letter T is incorrect; in B it is U which is incorrect; in C and D the incorrect letters are M and K respectively.

3. **A** 2010 mm = 2.01 m so, of the alternatives given, only a table could be expected to have a length of 2010 mm.

4. **D** Let X be on the top face of the cube. If the base is placed on a horizontal surface, then A, B, C, E will all be on vertical faces of the cube and D will be on the base, opposite X.

5. **D** Each of the five outer circles is divided into six regions, giving 30 regions in total. In addition, there is one region in the centre of the diagram and one region between the circles and the sides of the square. So, in all, there are 32 regions.

6. **C** The values of the expressions are A 12; B 15; C 16; D 15 and E 12.

7. **A** As 2, 5 and 10 are all factors of the correct product, this product is a multiple of 100. So the last digit and the last-but-one digit are both zero.

8. **D** If the mean of y and z is x, then $y + z = 2x$. So the sum of the interior angles of the triangle is $(x + y + z)^\circ = 3x^\circ$. So $3x = 180$, that is $x = 60$.

9. **A** One year is, at most, 366 days, so one-third of a year is less than 125 days. No month is longer than 31 days, so 4 months is also less than 125 days, as is 17.5 weeks which equals 122.5 days. However 3002 hours equals 125 days 2 hours, so this is the longest of the five periods of time.

10. **E** Third prize is worth one-sixth of the total prize money, so Mrs Keat received half of that amount, that is one-twelfth of the total.

11. **C** Divide the whole figure into horizontal strips of height 1 unit: its area is $(3 + 6 + 8 + 8 + 8 + 6 + 3)$ units^2 = 42 units^2. Similarly, the unshaded area is $(1 + 4 + 6 + 4 + 1)$ units^2 = 16 units^2. So the shaded area is 26 units^2.
Alternative solution: notice that if the inner polygon is moved a little, the answer remains the same – because it is just the difference between the areas of the two polygons. So, although we are not told it, we may assume that the inner one is so positioned that the outer shaded area can be split neatly into 1 by 1 squares – and there are 26 of these.

12. **C** There are 36 people to be seated so at least five tables will be required. The number of circular tables must be even. However, five rectangular tables will seat 40 people and three rectangular and two circular will seat 34. So at least six tables are needed. Two rectangular and four circular tables do seat 36 people: so six is the minimum number of tables.

13. **B** It is necessary to find a route for which the line is broken the first time it passes through any intersection and solid when it passes through that intersection for the second time. Only the route which starts at B and heads away from D satisfies this condition.

14. **D** The average number of vehicles per day $\approx \dfrac{300\,000\,000}{44 \times 365} \approx \dfrac{300\,000\,000}{40 \times 400}$
$= \dfrac{300\,000\,000}{16\,000} \approx \dfrac{300\,000\,000}{15\,000} = 20\,000.$

15. **C** The two shaded regions measure 3 by 7 and 1 by 6, so the total area outside the overlap is 27 units^2.

16. **E** As 108 marks represented 18% of the final total, 6 marks represented 1% of the final total. So this total was 600.

17. **D** As triangle *PQR* is equilateral, $x + 2y = 3x - y = 5y - x$. Equating any two of these expressions gives $2x = 3y$.
The only pair of given values which does not satisfy this equation is $x = 10$, $y = 6$.

18. **D** The other times that this has happened previously are when Sam's age in years went from 1 to 2; from 4 to 5; from 16 to 17 and from 36 to 37.
Note that since primes other than 2 are odd, the only squares which need to be checked, other than 1, are of even numbers.

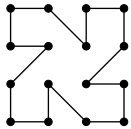
19. **C** Villages which have more than two roads leading to them (or from them) must all be visited more than once as a single visit will involve at most two roads. So Bentonville, Pencaster and Wytham must all be visited more than once. The route Home, Bentonville, Greendale, Wytham, Bentonville, Pencaster, Home, Wytham, Horndale, Pencaster, Home starts and finishes at Home and visits both Greendale and Horndale exactly once so the minimum number of villages is three.

20. **B** The seven numbers must total 49 if their mean is to be 7. The largest possible number will occur when the other six numbers are as small as possible, that is 1, 2, 3, 4, 5, 6. So the required number is $49 - 21 = 28$.

21. **C** The first and last hexagons both contribute 5 cm to the perimeter of the pattern. Every other hexagon in the pattern contributes 4 cm to the perimeter. The first and last thus contribute 10 cm, so we need another $2000 \div 4 = 500$ hexagons. Therefore the total number of hexagons required is 502.

22. **E** The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19. It is not possible for 2 to be one of the six numbers Kiran wrote down, since that would give one of the pairs an odd sum, whereas both of the other pairs would add up to an even number. The sum of the remaining 7 primes is 75 which is a multiple of 3. The sum of the six primes making up the three pairs must also be a multiple of 3 since each pair has the same total. So the odd prime not used in the six pairs must be a multiple of 3 too. Therefore 3 is the odd prime not used. So each pair totals $72 \div 3$, that is 24, and the pairs are $5 + 19$, $7 + 17$, $11 + 13$.

23. **E** The number of sides of the polygon is equal to the number of corners it has. As no dot is at more than one corner, the maximum number of corners is 16. So the maximum possible number of sides is 16, provided that a 16-sided figure may be drawn. The figure on the right shows one of several ways in which this can be achieved.



24. **B** In the 21st Century, to obtain a sequence of two years or more then either a 2 or a 0 must be repeated in each year, or the sequence include years such as 2011, 2033, 2044 etc. So the only sequence after that mentioned in the question will be from 2020 to 2030, but this is too short.
In the 22nd Century, either a 2 or a 1 must be repeated. The first such sequence is 2110 to 2129 which does include 20 years, one of which is 2120.

25. **A** The three-digit number *RRR* is equal to 111 multiplied by the single digit *R*. So $PQPQ \times R = 639027 \div 111 = 5757$. Now *PQPQ* equals the two-digit number *PQ* multiplied by 101. So $PQ \times R = 5757 \div 101 = 57$. The only ways in which 57 may be expressed as the product of a two-digit number and a single digit are 57×1 and 19×3. So $P = 5$, $Q = 7$, $R = 1$ or $P = 1$, $Q = 9$, $R = 3$. In both cases, $P + Q + R = 13$.