UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 1st MAY 2008

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

http://www.ukmt.org.uk

The Actuarial Profession
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SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

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1. **D** The results of the five calculations are 9, 11, 14, 25, 24 respectively.

2. **E** For it to be possible to draw a figure without taking the pen off the paper and without drawing along an existing line, there must be at most two points in the figure at which an odd number of lines meet. Only E satisfies this condition.

3. **B** $\frac{2}{40} = \frac{1}{20} = \frac{5}{100} = 5\%$.

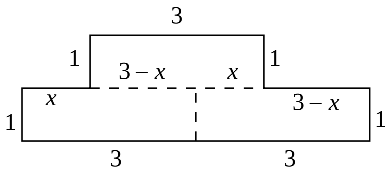
4. **C** The unmarked interior angle on the right of the triangle = (360 – 324)° = 36°. So, by the exterior angle theorem, $x = 100 - 36 = 64$.

5. **E** The cost of 1 kg of potatoes is £1.25 $\div$ 2.5 = 50 p. So the cost of 1 tonne, that is 1000 kg, is 1000 $\times$ 50 p = £500.

6. **D** Adam Ant walks 24 cm, whilst Annabel Ant walks 32 cm.

7. **C** In terms of length, 1 arm = 2 forearms = 4 hands = 8 middle fingers = 16 thumbs. So 4 arms have the same total length as 64 thumbs.

8. **A** From the diagram, in which all lengths are in cm, it can be seen that the perimeter $= [4 \times 1 + 3 \times 3 + x + (3 - x)]$ cm $= 16$ cm.



9. **E** The values of the five expressions are $\frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}, \frac{1}{42}$ respectively.

10. **B** Consider one corner of the cube. There are three faces which meet there, and each pair of them has an edge in common. So three different colours are needed. No other colours will be needed provided that opposite faces are painted in the same colour since opposite faces have no edges in common.

11. **E** The 120 tons of ice which remain represent two-thirds of the original cargo. So one-third of the original cargo was 60 tons.

12. **C** Consider the sculpture to consist of three layers, each of height 1. Then the volumes of the bottom, middle and top layers are 5, 2, 5 respectively. So the volume of the sculpture is 12.
(*Alternatively: the sculpture consists of a* $3 \times 3 \times 3$ *cube from which two* $2 \times 2 \times 2$ *cubes have been removed. The* $2 \times 2 \times 2$ *cubes have exactly one* $1 \times 1 \times 1$ *cube* (*the cube at the centre of the* $3 \times 3 \times 3$ *cube*) *in common. So the volume of the sculpture* = $27 - (2 \times 8 - 1) = 12$.)

13. **C** New shapes may be formed by joining *PX* to *XR* (quadrilateral) or *SP* to *RQ* (parallelogram) or *XS* to *RQ* (trapezium). Triangle *SPX* shows that *PX* and *SX* have different lengths; and *PX* and *PQ* have different lengths because *XR* is shorter than *SR*. So there are no other places to position the triangle.

14. **D** As the original cube was divided into eight cubes of equal size, these smaller cubes have side equal to half the side of the original cube. So each of the new cubes originally occupied one corner of the large cube and hence has three faces painted blue and three faces unpainted. So the fraction of the total surface area of the new cubes which is blue equals one half.

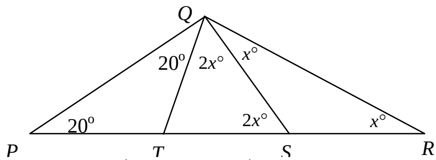
15. **B** A rate of 1 metre per 1000 years is equivalent to 1 mm per year, that is just under three thousandths of 1 mm per day.

16. **A** Of the five alternatives, only A and B have straight lines in the ratio 2:15:20. However, B would be formed by repeatedly moving forward 2 units, turning right, moving forward 20 units, turning right, moving forward 15 units, turning right.

17. **B** Consider the leading diagonal: $p \times 1 \times \frac{1}{8} = 1$ so $p = 8$.
Consider the bottom row: $u \times 4 \times \frac{1}{8} = 1$ so $u = 2$.
Consider the left-hand column: $p \times s \times u = 8 \times s \times 2 = 1$ so $s = \frac{1}{16}$.
Consider the non-leading diagonal: $r \times 1 \times u = r \times 1 \times 2 = 1$ so $r = \frac{1}{2}$.
Therefore $r + s = \dfrac{1}{2} + \dfrac{1}{16} = \dfrac{9}{16}$.

18. **B** Let my age now be x. So Granny's age is $4x$. Considering five years ago: $4x - 5 = 5(x - 5)$, giving $x = 20$. So Granny is 80 and I am 20.

19. **D** As $QS = SR$, $\angle SRQ = \angle SQR = x°$.
So $\angle QST = 2x°$ (exterior angle theorem). Also $\angle TQS = 2x°$ since $QT = TS$.
As $PT = QT$, $\angle TPQ = \angle TQP = 20°$.



Consider the interior angles of triangle PQR: $20 + (20 + 2x + x) + x = 180$. So $4x + 40 = 180$, hence $x = 35$.

20. **A** Consider the nine numbers from 1 to 9 inclusive: each digit appears once, with the exception of zero. Now consider the 90 two-digit numbers from 10 to 99 inclusive: each of the 10 digits makes the same number of appearances (9) as the second digit of a number and the digits from 1 to 9 make an equal number of appearances (10) as the first digit of a number, but zero never appears as a first digit. There is a similar pattern in the 900 three-digit numbers from 100 to 999 inclusive with zero never appearing as a first digit, but making the same number of appearances as second or third digit as the other nine digits. This leaves only the number 1000 in which there are more zeros than any other digit, but not enough to make up for the fact that zero appears far fewer times than the other nine digits in the numbers less than 1000. (*It is left to the reader to check that* 0 *appears* 192 *times,* 1 *appears* 301 *times and each of* 2 *to* 9 *appears* 300 *times*.)

21. **A** Consider the third column: $2♫ + ♥ = 13$ [1]

Consider the second row: $♫ + 2♥ = 11$ [2]

$2 \times [2] - [1]$ $3♥ = 9$, so $♥ = 3$.

(*Although their values are not requested, it is now straightforward to show that* ♫ = 5, ☼ = 4.)

22. **D** The only such occasions occur when the clock changes from 09 59 59 to 10 00 00; from 19 59 59 to 20 00 00 and from 23 59 59 to 00 00 00.

23. **B** Let the 7-digit code be $abcdefg$. It may be deduced that $a = 3$ since $b + c + d + e = 16$ and $a + b + c + d + e = 19$. By using similar reasoning, it may be deduced that $b = c = e = f = g = 3$.
As $a + b + c + d = 16$, $d = 7$; so the code is 3337333.

24. **E** Let the other such list of numbers be a, 1; b, 2; c, 3; d, 4 and note that $a + b + c + d = 8$ since there are 8 numbers in the list.
If $d = 4$, then exactly two of a, b, c equal 4, but this would make $a + b + c + d > 8$, so $d \neq 4$.
Similar reasoning shows that $d \neq 3$, so $d = 1$ or $d = 2$.
If $d = 2$, then exactly one of a, b, c equals 4 and the remaining two both equal 1 since $a + b + c + d = 8$. So we have a, 1; b, 2; c, 3; 2, 4 and it is b which must equal 4 since we already have more than one 2. However, as a and c are now both equal to 1, we have 1, 1; 4, 2; 1, 3; 2, 4 and this is not correct.
So $d = 1$ and we have $a + b + c = 7$ and a, b, $c \neq 4$. Clearly $a \neq 1$, since that would give at least two 1s so $a = 2$ or $a = 3$.
If $a = 2$, then we have 2, 1; b, 2; c, 3; 1, 4 with $b + c = 5$ and b, $c \neq 4$. So $b = 2$, $c = 3$ or vice versa. This gives either 2, 1; 2, 2; 3, 3; 1, 4 (incorrect), or 2, 1; 3, 2; 2, 3; 1, 4 (the example given in the question).
Finally, if $a = 3$, then we have 3, 1; b, 2; c, 3; 1, 4 with $b + c = 4$. The possibilities are 3, 1; 1, 2; 3, 3; 1, 4 or 3, 1; 2, 2; 2, 3; 1, 4 or 3,1; 3, 2; 1, 3; 1, 4 but only the first of these describes itself correctly. So the total number of 1s and 3s is 6.

25. **D** Let the lengths of the sides of the squares, in increasing order, be $a, b, c, d, e, f, g, h, i$ respectively. So $h = 10$.
Note that $c = 2b - a$ and $d = 2c - 2a = 4b - 4a$. Also, $e = 2d - a = 8b - 9a$.
As $h = 2e - 2a - b = 15b - 20a$, we may deduce that $15b - 20a = 10$, that is $3b - 4a = 2$.
Since a and b are positive integers less than 10, the only possibilities are $a = 1$, $b = 2$ or $a = 4$, $b = 6$. However, $h = 10$ therefore b cannot be greater than 4.
So $a = 1$ and $b = 2$. It may now be deduced that $c = 4 - 1 = 3$; $d = 8 - 4 = 4$; $e = 16 - 9 = 7$. Also $2g = 2e + d$, so $g = 9$.
Now the length of the side of the larger square is $2h + e + g = 20 + 7 + 9 = 36$, so its area is $36^2 = 1296$.
(*Note that it was not necessary to find the values of f and i, but it is now quite simple to deduce that $f = 8$ and $i = 18$.*)