Junior Mathematical Challenge 2008

1. Which of these calculations produces a multiple of 5?

A $1\times2+3+4$ B $1+2\times3+4$ C $1\times2+3\times4$ D $1+2\times3\times4$ E $1\times2\times3\times4$

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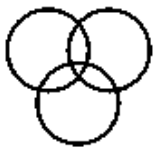
1. D The results of the five calculations are 9, 11, 14, 25, 24 respectively.

2. Which of these diagrams could be drawn without taking the pen off the page and without drawing along a line already drawn?

A  B  C  D  E 

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2. **E** For it to be possible to draw a figure without taking the pen off the paper and without drawing along an existing line, there must be at most two points in the figure at which an odd number of lines meet. Only E satisfies this condition.

3. All of the Forty Thieves were light-fingered, but only two of them were caught red-handed. What percentage is that?

A 2 B 5 C 10 D 20 E 50

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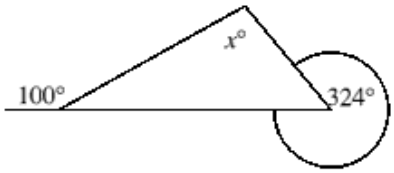
3. **B** $\frac{2}{40} = \frac{1}{20} = \frac{5}{100} = 5\%$.

4. In this diagram, what is the value of x?

A 16 B 36 C 64 D 100 E 144



4. **C** The unmarked interior angle on the right of the triangle = $(360 - 324)° = 36°$.
So, by the exterior angle theorem, $x = 100 - 36 = 64$.

5. At Spuds-R-Us, a 2.5kg bag of potatoes costs £1.25. How much would one tonne of potatoes cost?

A £5 B £20 C £50 D £200 E £500

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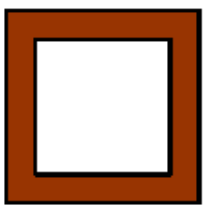
5. **E** The cost of 1 kg of potatoes is £1.25 ÷ 2.5 = 50 p. So the cost of 1 tonne, that is 1000 kg, is 1000 × 50p = £500.

6. The diagram shows a single floor tile in which the outer square has side 8cm and the inner square has side 6cm. If Adam Ant walks once around the perimeter of the inner square and Annabel Ant walks once around the perimeter of the outer square, how much further does Annabel walk than Adam?



A 2 cm B 4 cm C 6 cm D 8 cm E 16 cm

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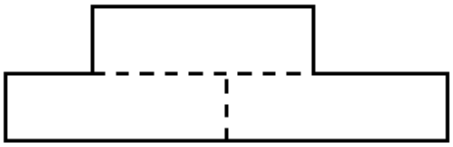
6. **D** Adam Ant walks 24 cm, whilst Annabel Ant walks 32 cm.

7. King Harry's arm is twice as long as his forearm, which is twice as long as his hand, which is twice as long as his middle finger, which is twice as long as his thumb. His new bed is as long as four arms. How many thumbs length is that?

A 16 B 32 C 64 D 128 E 256

7. **C** In terms of length, 1 arm = 2 forearms = 4 hands = 8 middle fingers = 16 thumbs. So 4 arms have the same total length as 64 thumbs.

8. The shape on the right is made up of three rectangles, each measuring 3cm by 1cm. What is the perimeter of the shape?



A 16 cm B 18 cm C 20 cm D 24 cm E More information needed

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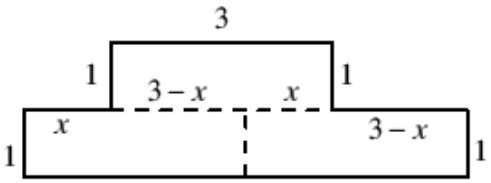
8. **A** From the diagram, in which all lengths are in cm, it can be seen that the perimeter
$= [4 \times 1 + 3 \times 3 + x + (3 - x)]$ cm
$= 16$ cm .



9. Which of the following has the smallest value?

A $\frac{1}{2} - \frac{1}{3}$ B $\frac{1}{3} - \frac{1}{4}$ C $\frac{1}{4} - \frac{1}{5}$ D $\frac{1}{5} - \frac{1}{6}$ E $\frac{1}{6} - \frac{1}{7}$

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9. **E** The values of the five expressions are $\frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}, \frac{1}{42}$ respectively.

10. The faces of a cube are painted so that any two faces which have an edge in common are painted different colours. What is the smallest number of colours required?

A 2 B 3 C 4 D 5 E 6

10. **B** Consider one corner of the cube. There are three faces which meet there, and each pair of them has an edge in common. So three different colours are needed. No other colours will be needed provided that opposite faces are painted in the same colour since opposite faces have no edges in common.

11. In 1833 a ship arrived in Calcutta with 120 tons remaining of its cargo of ice. One third of the original cargo was lost because it had melted on the voyage. How many tons of ice was the ship carrying when it set sail?

A 40 B 80 C 120 D 150 E 180

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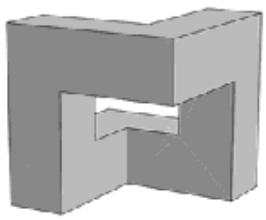
11. E The 120 tons of ice which remain represent two-thirds of the original cargo. So one-third of the original cargo was 60 tons.

12. The sculpture 'Cubo Vazado' [Emptied Cube] by the Brazilian artist Franz Weissmann is formed by removing cubical blocks from a solid cube to leave the symmetrical shape shown.

If all the edges have length 1, 2 or 3, what is the volume of the sculpture?



A 9 B 11 C 12 D 14 E 18

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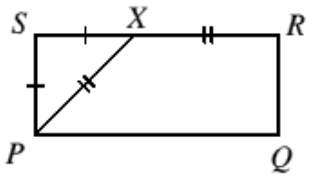
12. **C** Consider the sculpture to consist of three layers, each of height 1. Then the volumes of the bottom, middle and top layers are 5, 2, 5 respectively. So the volume of the sculpture is 12.
(*Alternatively: the sculpture consists of a* $3 \times 3 \times 3$ *cube from which two* $2 \times 2 \times 2$ *cubes have been removed. The* $2 \times 2 \times 2$ *cubes have exactly one* $1 \times 1 \times 1$ *cube* (*the cube at the centre of the* $3 \times 3 \times 3$ *cube*) *in common. So the volume of the sculpture* $= 27 - (2 \times 8 - 1) = 12$.)

13. A rectangle $PQRS$ is cut into two pieces along PX, where $PX = XR$ and $PS = SX$ as shown. The two pieces are reassembled without turning either piece over, by matching two edges of equal length. Not counting the original rectangle, how many different shapes are possible?



A 1 B 2 C 3 D 4 E 5

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13. **C** New shapes may be formed by joining PX to XR (quadrilateral) or SP to RQ (parallelogram) or XS to RQ (trapezium). Triangle SPX shows that PX and SX have different lengths; and PX and PQ have different lengths because XR is shorter than SR. So there are no other places to position the triangle.

14. A solid wooden cube is painted blue on the outside. The cube is then cut into eight smaller cubes of equal size. What fraction of the total surface area of these new cubes is blue?

A $\frac{1}{8}$ B $\frac{1}{3}$ C $\frac{3}{8}$ D $\frac{1}{2}$ E $\frac{3}{4}$

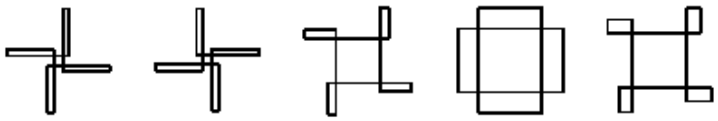
14. D As the original cube was divided into eight cubes of equal size, these smaller cubes have side equal to half the side of the original cube. So each of the new cubes originally occupied one corner of the large cube and hence has three faces painted blue and three faces unpainted. So the fraction of the total surface area of the new cubes which is blue equals one half.

15. An active sphagnum bog deposits a depth of about 1 metre of peat per 1000 years. Roughly how many millimetres is that per day?

A 0.0003 B 0.003 C 0.03 D 0.3 E 3

15. **B** A rate of 1 metre per 1000 years is equivalent to 1 mm per year, that is just under three thousandths of 1 mm per day.

16. The figures below are all drawn to scale. Which figure would result from repeatedly following the instructions in the box on the right?



A B C D E

Move forward 2 units.
Turn right.
Move forward 15 units.
Turn right.
Move forward 20 units.
Turn right.

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16. **A** Of the five alternatives, only A and B have straight lines in the ratio 2:15:20. However, B would be formed by repeatedly moving forward 2 units, turning right, moving forward 20 units, turning right, moving forward 15 units, turning right.

17. In this *Multiplication Magic Square*, the **product** of the three numbers in each row, each column and each of the diagonals is 1. What is the value of $r + s$?

p	q	r
s	1	t
u	4	$\frac{1}{8}$

A $\frac{1}{2}$ B $\frac{9}{16}$ C $\frac{5}{4}$ D $\frac{33}{16}$ E 24

17. B Consider the leading diagonal: $p \times 1 \times \frac{1}{8} = 1$ so $p = 8$.
Consider the bottom row: $u \times 4 \times \frac{1}{8} = 1$ so $u = 2$.
Consider the left-hand column: $p \times s \times u = 8 \times s \times 2 = 1$ so $s = \frac{1}{16}$.
Consider the non-leading diagonal: $r \times 1 \times u = r \times 1 \times 2 = 1$ so $r = \frac{1}{2}$.
Therefore $r + s = \frac{1}{2} + \frac{1}{16} = \frac{9}{16}$.

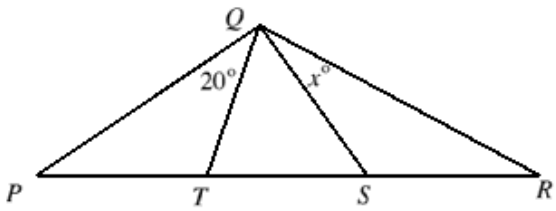
18. Granny swears that she is getting younger. She has calculated that she is four times as old as I am now, but remembers that 5 years ago she was five times as old as I was at that time. What is the sum of our ages now?

A 95 B 100 C 105 D 110 E 115

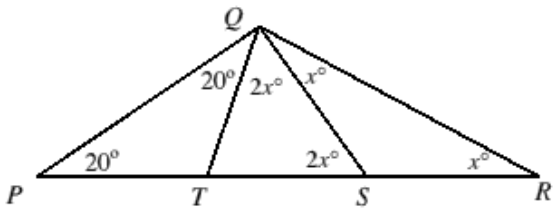
18. B Let my age now be x. So Granny's age is $4x$. Considering five years ago: $4x - 5 = 5(x - 5)$, giving $x = 20$. So Granny is 80 and I am 20.

19. In the diagram on the right, $PT = QT = TS$, $QS = SR$, $\angle PQT = 20°$. What is the value of x?

A 20 B 25 C 30 D 35 E 40



19. **D** As $QS = SR$, $\angle SRQ = \angle SQR = x°$.
So $\angle QST = 2x°$ (exterior angle theorem). Also $\angle TQS = 2x°$ since $QT = TS$.
As $PT = QT$, $\angle TPQ = \angle TQP = 20°$.



Consider the interior angles of triangle PQR: $20 + (20 + 2x + x) + x = 180$. So $4x + 40 = 180$, hence $x = 35$

20. If all the whole numbers from 1 to 1000 inclusive are written down, which digit appears the smallest number of times?

A 0 B 2 C 5 D 9 E none: no single digit appears fewer times than all the others

20. **A** Consider the nine numbers from 1 to 9 inclusive: each digit appears once, with the exception of zero. Now consider the 90 two-digit numbers from 10 to 99 inclusive: each of the 10 digits makes the same number of appearances (9) as the second digit of a number and the digits from 1 to 9 make an equal number of appearances (10) as the first digit of a number, but zero never appears as a first digit. There is a similar pattern in the 900 three-digit numbers from 100 to 999 inclusive with zero never appearing as a first digit, but making the same number of appearances as second or third digit as the other nine digits. This leaves only the number 1000 in which there are more zeros than any other digit, but not enough to make up for the fact that zero appears far fewer times than the other nine digits in the numbers less than 1000. (*It is left to the reader to check that* 0 *appears* 192 *times*, 1 *appears* 301 *times and each of* 2 *to* 9 *appears* 300 *times*.)

21. What is the value of ♥ if each row and each column has the total given?

			Total
♥	☼	♫	12
♫	♥	♥	11
☼	☼	♫	13
Total	12	11	13

A 3 B 4 C 5 D 6 E more information needed

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21. **A**

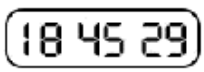
Consider the third column: $2♫ + ♥ = 13$ [1]

Consider the second row: $♫ + 2♥ = 11$ [2]

$2 \times [2] - [1]$: $3♥ = 9$, so $♥ = 3$.

(*Although their values are not requested, it is now straightforward to show that* $♫ = 5$, $☼ = 4$.)

22. On a digital clock displaying hours, minutes and seconds, how many times in each 24-hour period do all six digits change simultaneously?



A 0 B 1 C 2 D 3 E 24

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22. **D** The only such occasions occur when the clock changes from 09 59 59 to 10 00 00; from 19 59 59 to 20 00 00 and from 23 59 59 to 00 00 00.

23. In a 7-digit numerical code each group of four adjacent digits adds to 16 and each group of five adjacent digits adds to 19. What is the sum of all seven digits?

A 21 B 25 C 28 D 32 E 35

23. **B** Let the 7-digit code be $abcdefg$. It may be deduced that $a = 3$ since $b + c + d + e = 16$ and $a + b + c + d + e = 19$. By using similar reasoning, it may be deduced that $b = c = e = f = g = 3$.
As $a + b + c + d = 16$, $d = 7$; so the code is 3337333.

24. The list 2, 1; 3, 2; 2, 3; 1, 4; describes itself, since there are two 1s, three 2s, two 3s and one 4. There is exactly one other list of eight numbers containing only the numbers 1, 2, 3, and 4 that, in the same way, describes the numbers of 1s, 2s, 3s and 4s in that order. What is the total number of 1s and 3s in this other list?

A 2 B 3 C 4 D 5 E 6

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24. E Let the other such list of numbers be a, 1; b, 2; c, 3; d, 4 and note that $a + b + c + d = 8$ since there are 8 numbers in the list.

If $d = 4$, then exactly two of a, b, c equal 4, but this would make $a + b + c + d > 8$, so $d \neq 4$.

Similar reasoning shows that $d \neq 3$, so $d = 1$ or $d = 2$.

If $d = 2$, then exactly one of a, b, c equals 4 and the remaining two both equal 1 since $a + b + c + d = 8$. So we have a, 1; b, 2; c, 3; 2, 4 and it is b which must equal 4 since we already have more than one 2. However, as a and c are now both equal to 1, we have 1, 1; 4, 2; 1, 3; 2, 4 and this is not correct.

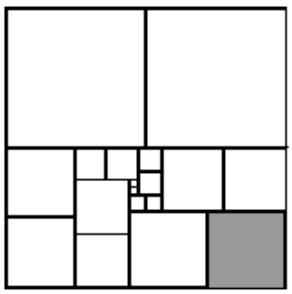
So $d = 1$ and we have $a + b + c = 7$ and $a, b, c \neq 4$. Clearly $a \neq 1$, since that would give at least two 1s so $a = 2$ or $a = 3$.

If $a = 2$, then we have 2, 1; b, 2; c, 3; 1, 4 with $b + c = 5$ and $b, c \neq 4$. So $b = 2, c = 3$ or vice versa. This gives either 2, 1; 2, 2; 3, 3; 1, 4 (incorrect), or 2, 1; 3, 2; 2, 3; 1, 4 (the example given in the question).

Finally, if $a = 3$, then we have 3, 1; b, 2; c, 3; 1, 4 with $b + c = 4$. The possibilities are 3, 1; 1, 2; 3, 3; 1, 4 or 3, 1; 2, 2; 2, 3; 1, 4 or 3,1; 3, 2; 1, 3; 1, 4 but only the first of these describes itself correctly. So the total number of 1s and 3s is 6.

25. A large square is divided into adjacent pairs of smaller squares with integer sides, as shown in the diagram (which is not drawn to scale). Each size of smaller square occurs only twice. The shaded square has sides of length 10. What is the area of the large square?

A 1024 B 1089 C 1156 D 1296 E 1444



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25. D Let the lengths of the sides of the squares, in increasing order, be $a, b, c, d, e, f, g, h, i$ respectively. So $h = 10$.

Note that $c = 2b - a$ and $d = 2c - 2a = 4b - 4a$. Also, $e = 2d - a = 8b - 9a$.

As $h = 2e - 2a - b = 15b - 20a$, we may deduce that $15b - 20a = 10$, that is $3b - 4a = 2$.

Since a and b are positive integers less than 10, the only possibilities are $a = 1$, $b = 2$ or $a = 4$, $b = 6$. However, $h = 10$ therefore b cannot be greater than 4.

So $a = 1$ and $b = 2$. It may now be deduced that $c = 4 - 1 = 3$; $d = 8 - 4 = 4$; $e = 16 - 9 = 7$. Also $2g = 2e + d$, so $g = 9$.

Now the length of the side of the larger square is $2h + e + g = 20 + 7 + 9 = 36$, so its area is $36^2 = 1296$.

(*Note that it was not necessary to find the values of f and i, but it is now quite simple to deduce that* $f = 8$ *and* $i = 18$.)