Junior Mathematical Challenge 2007

1. What is the value of $0.1 + 0.2 + 0.3 \times 0.4$?

A 0.24 B 0.312 C 0.42 D 1.0 E 1.5

0741

1. **C** $0.1 + 0.2 + 0.3 \times 0.4 = 0.3 + 0.12 = 0.42$.

2. My train was scheduled to leave at 17:40 and to arrive at 18:20. However, it started five minutes late and the journey then took 42 minutes. At what time did I arrive?

A 18:21 B 18:23 C 18:25 D 18:27 E 18:29

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2. **D** The train arrived 5 + 42 = 47 minutes after 17:40, that is at 18:27.

3. What is the remainder when 354972 is divided by 7 ?

A 1 B 2 C 3 D 4 E 5

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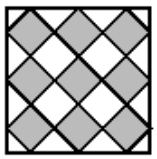
3. **B** Note that 7 divides 35, 49 and 7, so it divides 354970. So the remainder is 2.

4. Which of the following numbers is three less than a multiple of 5 and three more than a multiple of 6?

A 12 B 17 C 21 D 22 E 27

4. **E** Of the options given, only 27, which is three less than a multiple of 5, namely 30, and three more than a multiple of 6, namely 24, has both of the properties in the question.

5. In the diagram, the small squares are all the same size. What fraction of the large square is shaded?



A $\frac{9}{20}$ B $\frac{9}{16}$ C $\frac{3}{7}$ D $\frac{3}{5}$ E $\frac{1}{2}$

5. E The area of the large square may be considered to consist of thirteen equal squares (nine of which are shaded) plus eight 'half squares' and four 'quarter squares' (all of which are unshaded).
So the total unshaded area is $\left(4 + 8 \times \frac{1}{2} + 4 \times \frac{1}{4}\right)$ squares = 9 squares. Hence half of the large square is shaded.

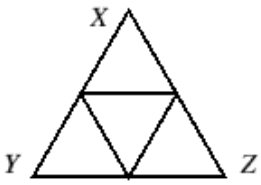
6. When the following fractions are put in their correct places on the number line, which fraction is in the middle?

A $-\frac{1}{7}$ B $\frac{1}{6}$ C $-\frac{1}{5}$ D $\frac{1}{4}$ E $-\frac{1}{3}$

6. **A** When put in their correct places on the number line, the order of the fractions is:
$-\frac{1}{3}, -\frac{1}{5}, -\frac{1}{7}, \frac{1}{6}, \frac{1}{4}$.

7. The equilateral triangle XYZ is fixed in position. Two of the four small triangles are to be painted black and the other two are to be painted white. In how many different ways can this be done?

A 3 B 4 C 5 D 6 E more than 6



7. **D** If the top triangle is painted black, then any one of the three remaining triangles may also be painted black. Similarly, if the top triangle is painted white, then any one of the three remaining triangles may also be painted white. So there are six different ways.

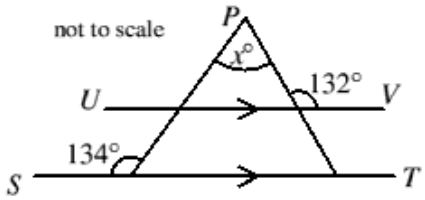
8. Amy, Ben and Chris are standing in a row. If Amy is to the left of Ben and Chris is to the right of Amy, which of these statements must be true?

A Ben is furthest to the left B Chris is furthest to the right C Amy is in the middle
D Amy is furthest to the left E None of statements A, B, C, D is true

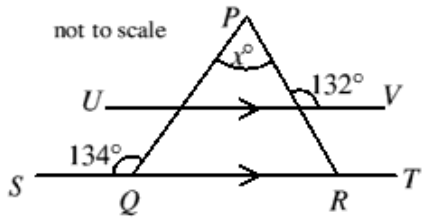
8. D From the information, we see that Amy is to the left of both Ben and Chris. So the three are in the order Amy, Ben, Chris or the order Amy, Chris, Ben. So D is certainly true and the others are all false either in one case or in both.

9. In the diagram on the right, ST is parallel to UV. What is the value of x ?

A 46 B 48 C 86 D 92 E 94



9. **C** As ST is parallel to UV, $\angle PRT = 132°$ (corresponding angles).
So $\angle PRQ = 48°$ (angles on a straight line).
From the exterior angle of a triangle theorem, $\angle SQP = \angle QPR + \angle PRQ$, so
$x = 134 - 48 = 86$.

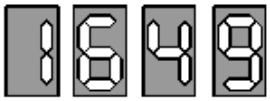


10. Which of the following has the largest value?

A $\frac{1}{2} + \frac{1}{4}$ B $\frac{1}{2} - \frac{1}{4}$ C $\frac{1}{2} \times \frac{1}{4}$ D $\frac{1}{2} \div \frac{1}{4}$ E $\frac{1}{4} \div \frac{1}{2}$

10. **D** The values of the five expressions are: A $\frac{3}{4}$; B $\frac{1}{4}$; C $\frac{1}{8}$; D 2; E $\frac{1}{2}$.

11. A station clock shows each digit by illuminating up to seven bars in a display. For example, the displays for 1, 6, 4 and 9 are shown.
When all the digits from 0 to 9 are shown in turn, which bar is used least?



A  B  C  D  E 

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11. A The number of times each bar is used is: A 4; B 6; C 8; D 7; E 7.

12. The six-member squad for the Ladybirds five-a-side team consists of a 2-spot ladybird, a 10-spot, a 14-spot, an 18-spot, a 24-spot and a pine ladybird (on the bench). The average number of spots for members of the squad is 12. How many spots has the pine ladybird?

A 4 B 5 C 6 D 7 E 8

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12. **A** The total number of spots which the six ladybirds have is $6 \times 12 = 72$. So the number of spots which the pine ladybird has is $72 - (2 + 10 + 14 + 18 + 24) = 4$.

13. Points P and Q have coordinates $(1, 4)$ and $(1, -2)$ respectively. For which of the following possible coordinates of point R would triangle PQR **not** be isosceles?

A $(-5, 4)$ B $(7, 1)$ C $(-6, 1)$ D $(-6, -2)$ E $(7, -2)$

13. **D** If R is $(-5, 4)$ then $PQ = PR = 6$. If R is $(7, 1)$ or if R is $(-6, 1)$ then R lies on the perpendicular bisector of PQ (the line $y = 1$), so in both cases $PR = QR$. If R is $(7, -2)$, then $QP = QR = 6$. However if R is $(-6, -2)$, then $PQ = 6$, $QR = 7$ and $PR > 7$, so triangle PQR is scalene.

14. If the line on the right were 0.2 mm thick, how many metres long would the line need to be to cover an area of one square metre?

A 0.5 B 5 C 50 D 500 E 5000

14. E The thickness of the line is 0.2 mm, that is 0.0002 m. So, in order to cover an area of one square metre, the length of the line would need to be $\frac{1}{0.0002}$ m, that is 5000 m.

15. I choose three numbers from this number square, including one number from each row and one number from each column. I then multiply the three numbers together. What is the largest possible product?

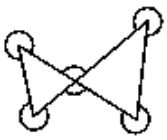
1	2	3
4	5	6
7	8	9

A 72 B 96 C 105 D 162 E 504

15. **C** We consider the different possible choices from the top row. If 1 is chosen, then the options are 1, 5, 9 and 1, 6, 8 giving products 45 and 48 respectively. If 2 is chosen, the options are 2, 4, 9 and 2, 6, 7 giving products 72 and 84 respectively. Finally, if 3 is chosen, the options are 3, 4, 8 and 3, 5, 7 giving products 96 and 105. So 105 is the maximum.

16. What is the sum of the six marked angles?

A 1080° B 1440° C 1620°
D 1800° E more information needed



16. **B** The six marked angles, together with the six interior angles of the two triangles, comprise all of the angles around five separate points. So the required sum is $(5 \times 360 - 2 \times 180)° = 1440°$.

17. Just William's cousin, Sweet William, has a rectangular block of fudge measuring 2 inches by 3 inches by 6 inches. He wants to cut the block up into cubes whose side lengths are whole numbers of inches. What is the smallest number of cubes he can obtain?

A 3 B 8 C 15 D 29 E 36

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17. C The only possible cubes have edge size 1 or 2. It takes 8 of the former to replace one of the latter, so William needs to cut as many cubes of edge size 2 as possible, namely 3. The number of one inch cubes, therefore, is $2 \times 3 \times 6 - 3 \times 8$, that is 12. So the smallest number of cubes is $3 + 12 = 15$.

18. The letters J, M, C represent three different non-zero digits. What is the value of $J + M + C$?

A 19 B 18 C 17 D 16 E 15

$$\begin{array}{r} J\ J \\ M\ M \\ C\ C \\ \hline J\ M\ C \end{array}$$

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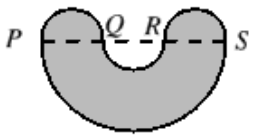
18. **B** The hundreds column shows us that $J = 1$ or 2. [We can't carry more than 2 from the units to the tens; and 2 plus the biggest feasible values 7, 8, 9 for the three letters is only 26.] The units column shows that $J + M$ is a multiple of 10 and it can't be 0 (or else $J + M = 0$); so $J + M = 10$ and $M = 9$ or 8 respectively. Also, the sum of the units column is $10 + C$, so there is exactly 1 to carry to the tens column. The tens column now tells us that $J + C + 1 = 10J$. So $J = 2$ is not possible and therefore $J = 1$, $C = 8$ and $M = 9$.

19. The points P, Q, R, S lie in order along a straight line, with $PQ = QR = RS = 2$ cm. Semicircles with diameters PQ, QR, RS and SP join to make the shape shown on the right. What, in cm^2, is the area of the shape?



A 5π B $9\pi/2$ C 4π D $7\pi/2$ E 3π

19. **A** If the semicircle with diameter PQ is rotated through 180° about Q, the new shape formed has the same area as the original shape. It consists of a semicircle of diameter 6 cm and a semicircle of diameter 2 cm. So its area is $\left(\frac{1}{2} \times \pi \times 3^2 + \frac{1}{2} \times \pi \times 1^2\right)$ cm^2, that is 5π cm^2.

20. At halftime, Boarwarts Academy had scored all of the points so far in their annual match against Range Hill School. In the second half, each side scored three points. At the end of the match, Boarwarts Academy had scored 90% of the points. What fraction of the points in the match was scored in the second half?

A $\frac{3}{100}$ B $\frac{3}{50}$ C $\frac{1}{10}$ D $\frac{9}{50}$ E $\frac{1}{5}$

20. E Range Hill scored only three points in the match and these were scored in the second half. They represent 10% of the total points scored. As Boarwarts Academy also scored three points in the second half, the proportion of points scored after halftime was 20%, that is $\frac{1}{5}$.

21. A list of ten numbers contains two of each of the numbers 0, 1, 2, 3, 4. The two 0s are next to each other, the two 1s are separated by one number, the two 2s by two numbers, the two 3s by three numbers and the two 4s by four numbers. The list starts 3, 4, ... What is the last number?

A 0 B 1 C 2 D 3 E 4

21. **B** Let the list be $3, 4, a, b, c, d, e, f, g, h$. We can see that $c = 3$ and $e = 4$. So the list now reads $3, 4, a, b, 3, d, 4, f, g, h$. Now, the only pairs of letters two apart from each other are a, d and d, g. Therefore $d = 2$ and the list is $3, 4, a, b, 3, 2, 4, f, g, h$. The only pair now one apart are f, h. The list is $3, 4, a, b, 3, 2, 4, 1, g, 1$. Now a, b are the only pair zero apart. So $a = b = 0$ and $g = 2$.

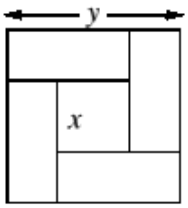
22. Only one choice of the digit d gives a prime number for each of the three-digit numbers read across and downwards in the diagram on the right. Which digit is d ?

	5	
1	d	3
	7	

A 4 B 5 C 6 D 7
E 8

22. **D** Four of the given values for d may be rejected since $143 = 11 \times 13$; $153 = 3 \times 51$; $567 = 3 \times 189$; $183 = 3 \times 61$. However, 173 and 577 are both prime, so $d = 7$.

23. The diagram shows a square with sides of length y divided into a square with sides of length x and four congruent rectangles. What is the length of the longer side of each rectangle?



A $\frac{y-x}{2}$ B $\frac{y+2x}{3}$ C $y-x$ D $\frac{2y}{3}$ E $\frac{y+x}{2}$

23. E Let the length of the longer side of each rectangle be l. Then the length of each shorter side is $l - x$. So $y = l + l - x$ and hence $l = \frac{1}{2}(y + x)$.

24. The pages of a book are numbered 1, 2, 3, … . In total, it takes 852 digits to number all the pages of the book. What is the number of the last page?

A 215 B 314 C 320 D 329 E 422

24. **C** Pages 1 to 9 inclusive require 9 digits; pages 10 to 99 inclusive require 180 digits. So, in total, 189 digits are required to number all of the pages before the three-digit page numbers commence with page number 100. This leaves 663 digits, so the last page in the book is the 221st page which has a three-digit number, namely page 320.

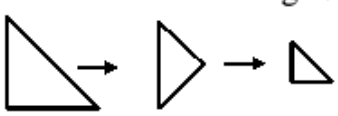
25. A piece of paper in the shape of a polygon is folded in half along a line of symmetry. The resulting shape is also folded in half, again along a line of symmetry. The final shape is a triangle. How many possibilities are there for the number of sides of the original polygon?

A 3 B 4 C 5 D 6 E 7

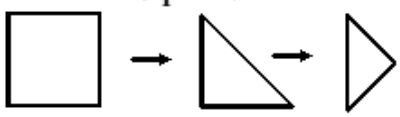
0765

25 **B** Imagine unfolding the final triangle once. Then one edge of the final triangle is inside the new shape obtained; and the other two triangle edges have 'mirror image' copies. So the new shape has at most 4 edges. After unfolding once more, one of these edges is now on the inside; and the remaining edges get mirror images again. So the shape obtained (the original shape) has no more than 6 edges. The diagrams below show that 3, 4, 5 and 6 sides are all possible.

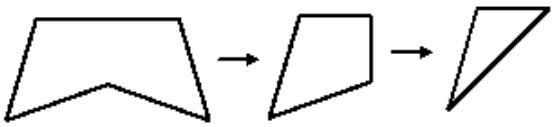
Triangle:



Square:



Pentagon:



Hexagon: