

# Junior Mathematical Challenge 2004



1. How many letters of the word **MATHEMATICS** do not have any lines of symmetry?
- A 0                      B 1                      C 2                      D 3                      E 4

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1. **B** Of the letters in question, only **S** does not have at least one line of symmetry.



2. Which of the following numbers is exactly divisible by 7?

A 104

B 106

C 108

D 110

E 112

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2. E  $112 \div 7 = 16$ . As none of the other options differs from 112 by a multiple of 7, 112 is the only one of these numbers which is exactly divisible by 7.



3. The year 2004 has the units digit equal to twice the thousands digit. How many years will it be before this next happens?

A 10

B 36

C 220

D 1002

E 2004

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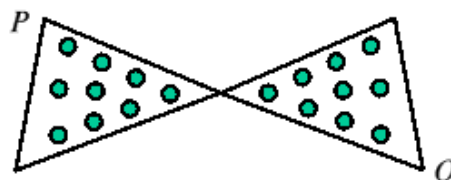


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3. A The required condition will next be met in 2014.



4. A ladybird has landed at point  $P$  on Sam's bow-tie. If it travels only along the edges of the bow-tie, but cannot travel along any edge more than once, how many different ways are there for it to get from  $P$  to  $Q$ ?



- A 1    B 2    C 3    D 4    E 5

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4. D Every allowable route must pass through the centre point of the bow tie. There are two routes from  $P$  to the centre point, and for each of these there are two routes from the centre point to  $Q$ . So the total number of different routes  $= 2 \times 2 = 4$ .



5. The word 'thirty' contains 6 letters and  $6 = 30 \div 5$ . Similarly, the word 'forty' contains 5 letters and  $5 = 40 \div 8$ . Which of the following is not a multiple of the number of letters it contains?
- A six                      B twelve                      C eighteen                      D seventy                      E ninety

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5. C Eighteen has 8 letters and 18 is not a multiple of 8. Of the other options, 6 is a multiple of 3, 12 is a multiple of 6, 70 is a multiple of 7 and 90 is a multiple of 6.



6. Which of these fractions is nearest to 1?
- A  $\frac{12}{23}$                       B  $\frac{23}{34}$                       C  $\frac{34}{45}$                       D  $\frac{45}{56}$                       E  $\frac{56}{67}$

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6. E The differences between the given fractions and 1 are, respectively,  $\frac{11}{23}$ ,  $\frac{11}{34}$ ,  $\frac{11}{45}$ ,  $\frac{11}{56}$  and  $\frac{11}{67}$ . The smallest of these is  $\frac{11}{67}$ , so  $\frac{56}{67}$  is nearest to 1.



7. In music, a demisemiquaver is half of half of half a crotchet, and there are four crotchets in a semibreve. How many demisemiquavers are there in a semibreve?

A 8                      B 16                      C 32                      D 64                      E 128

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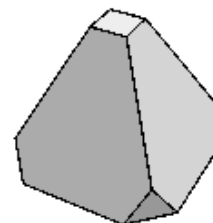
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7. C There are 8 demisemiquavers in a crotchet and 4 crotchets in a semibreve, so there are 32 demisemiquavers in a semibreve.



8. A solid square-based pyramid has all of its corners cut off, as shown. How many edges does the resulting shape have?

- A 8    B 13    C 15    D 20    E 24



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8. E The original pyramid had 8 edges. Cutting off the top corner adds 4 edges, whilst cutting off the other 4 corners adds 3 extra edges in each case. So the total number of edges is  $8 + 4 + 4 \times 3 = 24$ .



9. The Bean family are very particular about beans. At every meal all Beans eat some beans. Pa Bean always eats more beans than Ma Bean but never eats more than half the beans. Ma Bean always eats the same number of beans as both children together and the two children always eat the same number of beans as each other. At their last meal they ate 23 beans altogether. How many beans did Pa Bean eat?

- A 7                    B 9                    C 11                    D 13                    E 15

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9. **C** Pa Bean does not eat more than half the beans, so he eats at most 11 beans. Ma Bean eats the same number of beans as both children together, so she eats an even number of beans which is at least one quarter of the total number of beans eaten. Therefore she eats at least 6 beans. If she does eat 6 beans, then Pa Bean eats 11 beans, which is consistent with the information given. However, if Ma Bean eats 8 or more beans, then Pa Bean eats at most 7 beans and this is impossible as we are told that Pa Bean eats more beans than Ma Bean. So Pa Bean eats 11 beans.



10. When Harry bought his train ticket he received £2.50 in change. He noticed that for each coin in his change there was exactly one other coin of the same value. What was the coin of smallest value in Harry's change?

A 2p                      B 5p                      C 10p                      D 20p                      E 50p

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10. **B** The coins consisting of one of each type must add up to £1.25. The only way that this total may be made with coins of different denominations is by using a £1 coin, a 20p coin and a 5p coin.



11. The diagram shows a rod with five equally spaced points  $A, B, C, D$  and  $E$  marked on it.



The rod is rotated three times through  $180^\circ$ , first about  $A$ , then about  $B$  and finally about  $E$ . Which point finishes in the same position as it was at the start?

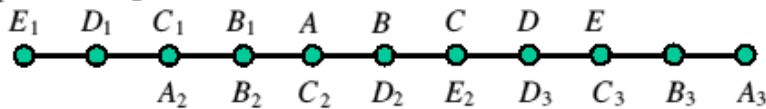
- A A                      B B                      C C                      D D                      E E

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11. D The diagram shows the positions of the points after the rotations. Note that  $A_1$ , which is not marked, is the same point as  $A$ . Similarly,  $E_3$ , which is not marked, is the same point as  $E_2$ .



12. The White Rabbit has an appointment to see the Red Queen at 4pm every day apart from weekends. On Monday, he arrives 16 minutes late. Each day after that he hurries more and more and so manages to halve the amount of time that he arrives late each day. On what day of the week does he arrive just 15 seconds late?

- A Monday              B Tuesday              C Wednesday              D Thursday              E Friday

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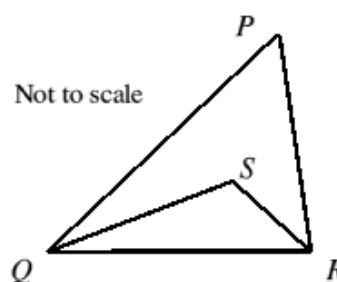


12. B On Tuesday the White Rabbit will be 8 minutes late, on Wednesday 4 minutes late, on Thursday 2 minutes late and on Friday 1 minute late. So on the following Monday the White Rabbit will be 30 seconds late and on the day after that he will be 15 seconds late.



13. In the triangle  $PQR$ , the angle  $QPR = 40^\circ$  and the internal bisectors of the angles at  $Q$  and  $R$  meet at  $S$ , as shown. What is the size of angle  $QSR$ ?

A  $110^\circ$  B  $120^\circ$  C  $130^\circ$  D  $135^\circ$  E  $140^\circ$



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13. A As  $\angle QPR = 40^\circ$ ,  $\angle PQR + \angle PRQ = 180^\circ - 40^\circ = 140^\circ$ .  
So  $\angle SQR + \angle SRQ = 140^\circ \div 2 = 70^\circ$ . Therefore  $\angle QSR = 180^\circ - 70^\circ = 110^\circ$ .



14. The Kings of Clubs, Diamonds, Hearts and Spades, and their respective Queens, are having an arm wrestling competition. Everyone must wrestle everyone else, except that no King will wrestle his own Queen. How many wrestling bouts are there?
- A 12                      B 16                      C 24                      D 28                      E 64

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14. C The 8 individuals will each wrestle 6 others. This suggests that the number of bouts is  $8 \times 6 = 48$ . However, each bout has been counted twice in this calculation, so the number of bouts is  $48 \div 2 = 24$ .



15. Granny spends one third of her weekly pension on Thursday night, and one quarter of what remains on Friday. What fraction of the original amount is left for her big night out on Saturday?
- A  $\frac{1}{12}$                       B  $\frac{2}{7}$                       C  $\frac{5}{12}$                       D  $\frac{1}{2}$                       E  $\frac{11}{12}$

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- 15. D** After Thursday, two thirds of Granny's pension is left. So after she has spent one quarter of this amount on Friday, the fraction of the original amount which remains is three quarters of two thirds, i.e. one half.



- 16.** A robot, which is initially facing North, is programmed to travel 5m then turn through  $10^\circ$ , travel 5m then turn through  $20^\circ$ , travel 5m then turn through  $30^\circ$ , and so on. Each move consists of moving 5m in a straight line and then turning clockwise through an angle which increases by  $10^\circ$  at each move.

How far has it travelled by the time it is first facing due East at the end of a move?

- A 9 m                      B 40 m                      C 45 m                      D 50 m                      E 90 m

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- 16. C** The total angle turned through after each of the first 4 moves is  $10^\circ$ ,  $30^\circ$ ,  $60^\circ$  and  $100^\circ$ . So the robot does not face due East at the end of a move in its first complete revolution. The total angle it has turned through after each of the next 5 moves is  $150^\circ$ ,  $210^\circ$ ,  $280^\circ$ ,  $360^\circ$  and  $450^\circ$ , so at the end of the 9th move the robot *does* face due East. As the robot moves 5m in each move, the distance it travels is 45m.

*Note that this solution assumes that the robot is not starting the process close to the North Pole!*



17. Exactly one of these statements is correct. Which one?

A  $44^2 + 77^2 = 4477$

B  $55^2 + 66^2 = 5566$

C  $66^2 + 55^2 = 6655$

D  $88^2 + 33^2 = 8833$

E  $99^2 + 22^2 = 9922$

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**17. D** Statements such as those in this question may sometimes be shown to be false by considering the units digit of the expressions on each side. The units digit of  $44^2 + 77^2$  is 5; the units digit of  $55^2 + 66^2$  and hence also of  $66^2 + 55^2$  is 1; the units digit of  $88^2 + 33^2$  is 3 and that of  $99^2 + 22^2$  is 5. So four of the statements are definitely false. It remains to check that  $88^2 + 33^2 = 7744 + 1089 = 8833$ .



18. A shape consisting of 2004 small squares is made by continuing the pattern shown in the diagram. The small squares have sides of length 1cm. What is the length, in cm, of the perimeter of the whole shape?

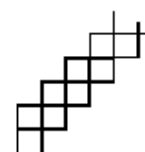
A 4008

B 4010

C 6012

D 6016

E 8016



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- 18. B** Each of the 2004 squares, apart from those at the top and bottom of the shape, contributes 2 cm to the perimeter of the figure.  
The other two squares contribute 3 cm each, so the perimeter is  $(2002 \times 2 + 2 \times 3)$  cm = 4010 cm.



19. If  $a \times b = 2$ ,  $b \times c = 24$ ,  $c \times a = 3$  and  $a$ ,  $b$  and  $c$  are all positive, what is the value of  $a + b + c$ ?
- A  $7\frac{1}{2}$                       B  $10\frac{1}{2}$                       C 12                      D 16                      E 19

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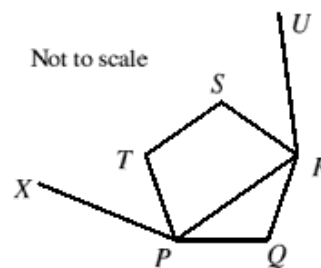
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- 19. B** From the three equations we see that  $(abc)^2 = 2 \times 24 \times 3 = 144$  and so, since  $abc$  is positive,  $abc = 12$ . Then the third equation tells us that  $a = \frac{1}{2}$ , the second that  $b = 4$  and the first that  $c = 6$ . Therefore  $a + b + c = 10\frac{1}{2}$ .



20. The figure shows a regular pentagon  $PQRST$  together with three sides  $XP, PR, RU$  of a regular hexagon with vertices  $PRUVWX$ . What is the size of angle  $SRU$ ?

- A  $48^\circ$    B  $54^\circ$    C  $60^\circ$    D  $63^\circ$    E  $72^\circ$



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20. A As  $PQRST$  is a regular pentagon, each of its internal angles is  $108^\circ$ . The internal angles of the quadrilateral  $PRST$  add up to  $360^\circ$  and so by symmetry  $\angle PRS = \angle RPT = \frac{1}{2}(360^\circ - 2 \times 108^\circ) = 72^\circ$ . Each interior angle of a regular hexagon is  $120^\circ$ , so  $\angle PRU = 120^\circ$ . Therefore  $\angle SRU = \angle PRU - \angle PRS = 120^\circ - 72^\circ = 48^\circ$ .



21. Four of these jigsaw pieces fit together to form a rectangle. Which one is not used?

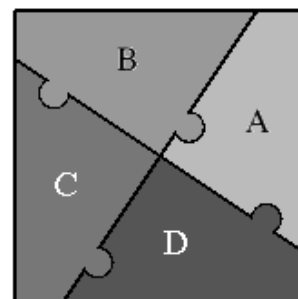


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- 21. E** In total, the five pieces have four “holes” and six “tabs”, so we can deduce that either B or E, both of which have two “tabs”, will not be used. Shape A, therefore, is used and, after a clockwise rotation of  $90^\circ$ , will fit together with shape B. This suggests that it is shape E which is not used and shapes C and D will indeed complete the jigsaw after rotations of  $90^\circ$  and  $180^\circ$  anticlockwise respectively.



- 22.** The digits in the product  $13 \times 2 = 26$  can be rearranged to give  $16 \times 2 = 32$  as well as  $31 \times 2 = 62$ . In which one of the following can the digits not be rearranged to give another correct product?
- A  $12 \times 3 = 36$     B  $12 \times 7 = 84$     C  $26 \times 3 = 78$     D  $16 \times 3 = 48$     E  $39 \times 2 = 78$

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- 22. D** In the four options other than D, the digits may be rearranged as follows:  
 A  $21 \times 3 = 63$     B  $18 \times 4 = 72$     C  $38 \times 2 = 76$     E  $29 \times 3 = 87$ .



23. In this addition each letter stands for a different digit, with  $S$  standing for 3. What is the value of  $Y \times O$ ?

- A 0    B 2    C 36    D 40    E 42

$$\begin{array}{r}
 \phantom{+} \phantom{M} \phantom{A} \phantom{N} \phantom{Y} \\
 + \phantom{M} \phantom{A} \phantom{N} \phantom{Y} \\
 \hline
 S \phantom{U} \phantom{M} \phantom{S}
 \end{array}$$

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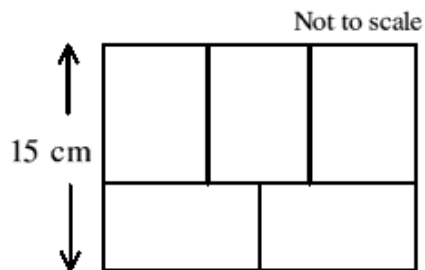
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23. E Consider the thousands column. The letters represent different digits so, as  $S$  is 3,  $M$  is 2 and there is a carry of 1 from the hundreds column. Therefore  $A$  is 9,  $U$  is 0 and there is also a carry of 1 from the tens column. In the units column,  $O + Y$  produces a units digit of 3, so  $O + Y = 3$  or  $O + Y = 13$ . However,  $O + Y = 3$  requires one of  $O, Y$  to equal zero (impossible as  $U = 0$ ) or 2 (also impossible as  $M = 2$ ). So  $O + Y = 13$ . We can also deduce that  $N$  is 8, since, in the tens column,  $1 + 3 + N = 12$ . The pairs of digits which produce a sum of 13 are 4 and 9, 5 and 8, 6 and 7. As  $A$  is 9 and  $N$  is 8, the only possible values for  $O$  and  $Y$  are 6 and 7. These are interchangeable, but in both cases  $Y \times O = 42$ .





24. Five identical rectangles fit together as shown.  
 What, in  $\text{cm}^2$ , is the total area which they cover?  
 A 270    B 300    C 330    D 360    E 450



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24. A Let the length and breadth of each of the rectangles be  $a$  cm and  $b$  cm respectively. Then  $2a = 3b$  and  $a + b = 15$ . So  $2a + 2b = 30$ . Therefore  $3b + 2b = 30$ , that is  $b = 6$ . So the total area covered by the five rectangles is that of a rectangle measuring 18 cm by 15 cm, i.e.  $270 \text{ cm}^2$ .



25. In a sequence of positive integers, every term after the first two terms is the sum of the two previous terms in the sequence. If the fifth term is 2004, what is the maximum possible value of the first term?  
 A 399            B 400            C 663            D 999            E 1001

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- 25. D** Let the first two terms of the sequence be  $a$  and  $b$  respectively. Then the next three terms are  $a + b, a + 2b, 2a + 3b$ . So  $2a + 3b = 2004$ . For  $a$  to be as large as possible, we need  $b$  to be as small as possible, consistent with their both being positive integers. If  $b = 1$  then  $2a = 2001$ , but  $a$  is an integer, so  $b \neq 1$ .  
However, if  $b = 2$  then  $2a = 1998$ , so the maximum possible value of  $a$  is 999.