UK JUNIOR MATHEMATICAL CHALLENGE

TUESDAY 30th APRIL 2002

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **A** $2000 \times 5 = 10\,000$ and $2 \times 5 = 10$ so $2002 \times 5 = 10\,000 + 10$.

2. **D** $3 + 6 \times 4 = 3 + 24 = 27 \neq 36$.

3. **C** A is 12; B is 12; C is 15; D is 10 and E is 14.

4. **D** As $\triangle ABC$ is equilateral $\angle BCA = 60°$. Since $JKLM$ is a square, $\angle MJC = 90°$. So, since the angles of a triangle add up to 180°, $\angle JMC = 30°$.

5. **A** Of the numbers 1, 2, 3, 4, 5, the only one which is a multiple of 5 is 5 itself.

6. **A** The sum = $5432100 + 1 + 2 + 3 + 4 + 5 = 5432115$.

7. **C** The equal numbers cannot be 1 since $17 - 6 \times 1 = 11$, which is not a single digit number. Also they cannot be 3 or more as then the single digit would be negative. That leaves 2 which works since $17 - 6 \times 2 = 5$.

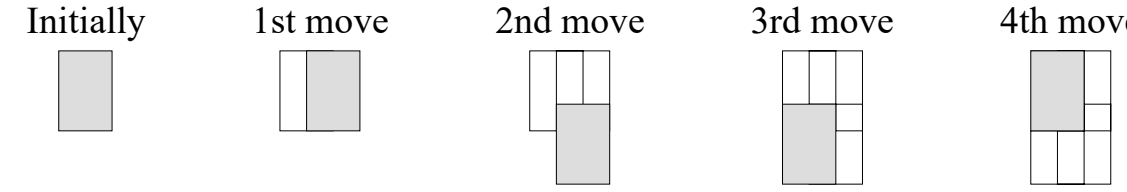
8. **E** Each centre is 2.5 cm from the nearer end. So the distance between them is $(9 - 2 \times 2.5)$ cm.

9. **C** The weight of the water poured out is $(21 - 12)$ kg. Thus the weight of the bucket is $(12 - 9)$ kg.

10. **E** In our notation, the subtraction becomes $927 - 54$ which is 873. Su Erasmus would write this as 378.

11. **E** The number of students in just the band is $60 - 12 = 48$. The number in just the orchestra is $20 - 12 = 8$. So the number in either band or orchestra (or both) is $(48 + 12 + 8)$.

12. **C**



So the area is that of a rectangle measuring 5m $\times$ 3m.

13. **A** To be in this century, the number in the hundreds place must be zero so the same must be true for the number in the tens place. The last digit must equal the first. (The next palindromic year will be 2112.)

14. **B** Inverting three glasses at a time means leaving one alone. In the sequence below, the first, the second, the third and then the fourth are left alone.

So four moves will suffice.

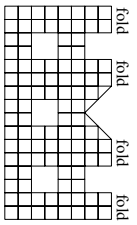
One move is clearly not enough. Two moves either return to the start or leave 2 up and 2 down. Three moves leave 1 up or 3 up so the least number is 4 moves.

15. **D** Since $\frac{3}{5} = 0.6 > 0.5$.

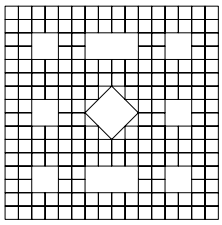
16. **C**

17. **B** With no 5p coins, there could be 0, 1, 2, 3, 4 or 5 2p coins, i.e. 6 possible ways. With one 5p coin, there could be 0, 1, 2 or 3 2p coins, i.e. 4 possible ways. With two 5p coins, there can be no 2p coins, i.e. 1 possible way.

18. **D** Unfold about the top edge



Now unfold about the right-hand edge



19. **D**

Number of sides	3	4	5	6	7	8	…
Number of diagonals	0	2	5	9	14	20	…

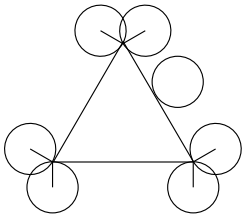
Or :− The number of corners to which a diagonal from a given corner can be drawn is $n - 3$. There are thus $n(n - 3)/2$ diagonals, the ½ because each diagonal has two ends. So $n(n-3)/2 = 2n$. Cancelling one n from each side (since $n \neq 0$) we get $n - 3 = 4$ and so $n = 7$.

20. **B** The sum of the length and breadth of the base is 8 cm and therefore we need to find two factors of 72 whose sum is 8 and whose product is also a factor of 72. We need consider only 1, 2, 3, 4 and 6 since these are the only factors of 72 less than 8. Of these, only 2 and 6 satisfy the above conditions. So the base of the cuboid is 2 cm × 6 cm and its height is 6 cm.

21. **A** Since ΔABC is isosceles, $\angle ABC = \angle BCA = y°$. As the exterior angle of a triangle is equal to the sum of the interior opposite angles, $x° = y° + z°$. Thus $z = x - y$.

22. **D** Since when N is divided into 26, the remainder is 2, N is a factor of 24 but not of 26. The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24. Both 1 and 2 are factors of 26. The sum of the other factors is $3 + 4 + 6 + 8 + 12 + 24 = 57$.

23. **C** Since its circumference and each side of the triangle are both of length 1, the circle completes 1 revolution as it rolls along each of the three sides. In addition, at each vertex, the circle makes a one third turn and so, since there are three vertices, there are four revolutions in total.



24. **E** Because AB ≠ DE, C ≠ 1. So C is 2 or more. Therefore A cannot be 3, 4 or 5 as D would be at least 6. So A = 1 or 2. If A = 2, then C is at least 3 and again D is at least 6. So A = 1.

$$\begin{array}{r} AB \\ \times\ C \\ \hline DE \end{array}$$

Neither B nor C can be 5 as, if they were, E would have to be 0 or 5 but this also means that E is not 5. So D = 5. We can now see that, to get D = 5 we must have C = 4 and B × C must have two digits. So B = 3 and E = 2.

25. **B**

Questions answered	1	2	3	4	5	6	n
Time taken (seconds)	1	3	7	15	31	63	$2^n - 1$

One hour is 60 minutes which is 3600 seconds. Since $2^{11} = 2048$, $2^{11} - 1 < 3600 < 2^{12} - 1$. So Gill answers the first 11 questions and obtains 55 marks.