Tuesday 14th June 2016 Junior Kangaroo Solutions

1. **D** At all the times given, the minute hand is pointing to 12. When the minute hand is pointing to 12 and the angle between the hands is 150°, the hour hand has turned $\frac{150}{360} = \frac{5}{12}$ of a complete turn. Therefore the hour hand will point at 5 and the time will be 5 pm. (There are other times when the angle between the hands is 150° but, of these, only at 7 pm does the minute hand point to 12 and 7 pm is not one of the times given.)

2. **B** The number of people who can sit on each side of the square table is $12 \div 4 = 3$. When eight of these tables are arranged to make a long rectangular table, there will be room for $8 \times 3 = 24$ people on each long side and for three extra people at each end. Hence, the number of people that can sit round the long table is $2 \times 24 + 2 \times 3 = 48 + 6 = 54$.

3. **E** Since one ball and one bat cost £90, two balls and two bats cost $2 \times$ £90 = £180. Now, since three balls and two bats cost £210, one ball costs £210 − £180 = £30. Therefore a bat costs £90 − £30 = £60.

4. **A** The surface areas of the two solids are the same. Hence the same amount of paint is required to cover them. Therefore it would take 9 litres of paint to cover the surface of the second solid.

5. **B** The calculation is equivalent to $\dfrac{1}{10} \times \dfrac{3}{10} \times \dfrac{5}{10} \times 7000 = 1 \times 3 \times 5 \times 7 = 105$.

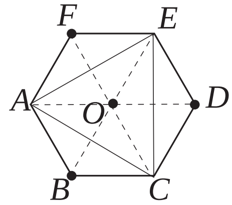
6. **E** Let the number of pupils in the class be x. The information in the question tells us that $3x + 31 = 4x + 8$, which has solution $x = 23$. Hence the number of sheets of paper Miss Spelling has is $3 \times 23 + 31 = 100$.

7. **C** Nets A and D would produce cubes with holes on two edges of the same face. Net E would produce a cube with a hole in the centre of two opposite faces while net B would produce a cube with one hole on an edge and two small holes. The given partial cube has holes on two opposite edges and therefore its net will have a hole on the edge of four different faces.

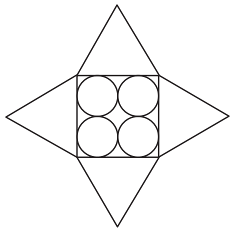
Hence only net C can be used to build the required shape.

8. **E** Since one angle of the isosceles triangle is 30°, there are two possibilities. Either the other two angles are equal, in which case the difference between them is 0°, or one of the other angles is 30°. In this case, since angles in a triangle add to 180°, the second missing angle is 120° and hence the difference between the two missing angles is $120° - 30° = 90°$.

9. **E** Label vertices A, B, C, D, E, E and F as shown. Since the hexagon is regular, it can be divided into six equilateral triangles as shown. Therefore quadrilateral $OABC$ is a rhombus and hence its diagonal AC is a line of symmetry. Therefore, if vertex B is folded onto O, the fold will be along AC. Similarly, if vertices D and F are folded onto O, the folds will be along CE and EA respectively. Hence the figure that is formed will be a triangle and, since all three of the rhombuses $OABC$, $OCDE$ and $OEFA$ are made out of two congruent equilateral triangles, the lengths of their diagonals AC, CE and EA will be equal. Hence the shape ACE that is formed is an equilateral triangle.

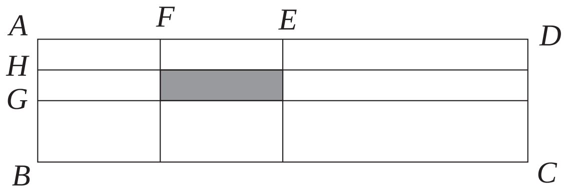


10. D The radius of each of the circles is 5 cm and hence the diameter of each is 10 cm. The length of the side of the square is equal to the sum of the diameters of two circles and hence is equal to 20 cm. The length of each side of the equilateral triangle is equal to the length of the side of the square. Hence the perimeter of the star, which is made up of eight sides of congruent equilateral triangles, is $8 \times 20 \text{ cm} = 160 \text{ cm}$.



11. B Joey's two numbers are 99 and 12 and hence his sum is 111. Zoë's two numbers are 98 and 10 and hence her sum is 108. Therefore the difference between their answers is $111 - 108 = 3$.

12. C

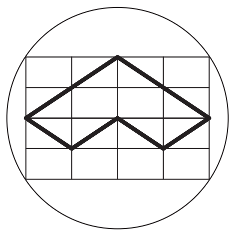


Since E is the midpoint of AD and F is the midpoint of AE, the length of FE is $\frac{1}{2} \times \frac{1}{2} \times 4 \text{ cm} = 1 \text{ cm}$. Similarly, since G is the midpoint of AB and H is the midpoint of AG, the length of HG is $\frac{1}{2} \times \frac{1}{2} \times 1 \text{ cm} = \frac{1}{4} \text{ cm}$. Therefore the area of the shaded rectangle is $\left(1 \times \frac{1}{4}\right) \text{cm}^2 = \frac{1}{4} \text{ cm}^2$.

13. B Let the units digit of the number be x. Hence the tens digit of the number is $x + 3$ and the sum of the digits of the number is $2x + 3$. The information in the question tells us that $10(x+3) + x = 7(2x+3) + 3$. Hence $11x + 30 = 14x + 24$ which has solution $x = 2$. Therefore the sum of the digits of the two-digit number is $2 \times 2 + 3 = 7$.

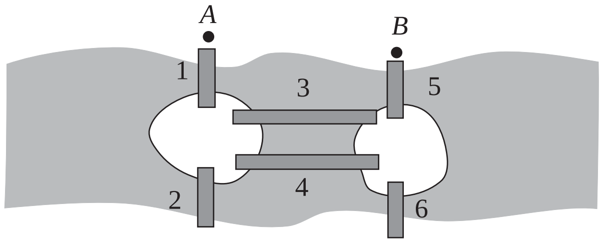
14. B Consider the case where two opposite faces are coloured red. Whichever of the four remaining faces is also coloured red, the resulting arrangement is equivalent under rotation to a cube with top, bottom and front faces coloured red. Hence, there is only one distinct colouring of a cube consisting of three red and three blue faces with two opposite faces coloured red. Now consider the case where no two opposite faces are coloured red. This is only possible when the three red faces share a common vertex and, however these faces are arranged, the resulting arrangement is equivalent under rotation to a cube with top, front and right-hand faces coloured red. Hence there is also only one distinct colouring of a cube consisting of three red and three blue faces in which no two opposite faces are coloured red. Therefore there are exactly two different colourings of the cube as described in the question.

15. C The diagonals of a rectangle bisect each other at the midpoint of the rectangle. Hence, the midpoint of a rectangle is equidistant from all four vertices and is the centre of a circle through its vertices.



In this case, the diameter of the circle is 10 cm. This is equal to the sum of the lengths of the diagonals of four of the smaller rectangles. Hence the diagonal of each small rectangle has length 2.5 cm. The perimeter of the marked shape is made up of eight diagonals of the small rectangles and hence has length $8 \times 2.5 \text{ cm} = 20 \text{ cm}$.

16. D Since Leonhard's walk always goes over bridge 1 first, it must conclude by going over bridge 5 to enable him to reach to B. Note also that bridges 2 and 6 must be crossed consecutively, in some order, as they are the only way to get to and from the opposite bank to the one from which he started and is to finish and so can be considered together.



Hence the number of days he can walk without repeating the order in which he crosses the bridges is the same as the number of ways of choosing ordered crossings of bridges 3, 4 and the pair 2 and 6. These can be chosen in six different ways (three choices for the first bridge, two for the second and then only one choice for the third). Hence Leonhard can walk for six days without repeating the order in which he crosses the bridges.

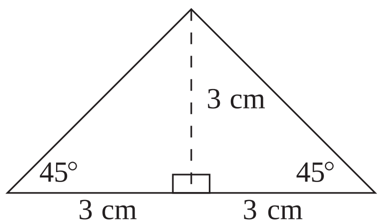
(The six orders are 126345, 126435, 134265, 136245, 143265 and 146235.)

17. A Let the length of each of the rectangles be x cm and the width be y cm. The perimeter of each of the rectangles is 40 cm and hence $2x + 2y = 40$. Therefore $x + y = 20$. From the diagram we can see that the length of each side of the square $ABCD$ is $(x + y)$ cm. Therefore the square $ABCD$ has side length 20 cm. Hence the area of $ABCD$ is $(20 \times 20)\ \text{cm}^2 = 400\ \text{cm}^2$.

18. D Let the cost of a can of cola be x pence and the cost of a croissant be y pence. The information in the question tells us that $6x + 7y = 8x + 4y$ and that both sides of the equation represent the total amount of money Ellen has. Hence $3y = 2x$. Therefore the total amount of money she has is $3 \times 3y + 7y$ pence $= 16y$ pence. Hence she could buy 16 croissants if she bought only croissants.

19. C If each person paid their fair share, each would have paid five times. Therefore Adam has paid on an extra three occasions and Bill has paid on an extra two occasions. Hence the £30 Chris owes should be divided in the ratio 3:2. Therefore Adam should get $\frac{3}{5} \times £30 = £18$ and Bill should get $\frac{2}{5} \times £30 = £12$.

20. D Consider one of the right-angled isosceles triangles as shown.



The longest side is $(30/5)$ cm $= 6$ cm. The triangle can be divided into two identical right-angled isosceles triangles with base 3 cm and hence with height 3 cm. Therefore the area of each of the original triangles is $\left(\frac{1}{2} \times 6 \times 3\right)\ \text{cm}^2 = 9\ \text{cm}^2$. Hence the total shaded area is $5 \times 9\ \text{cm}^2 = 45\ \text{cm}^2$.

21. C The information that in any group of four pencils, at least two have the same colour, tells us that there at most three different coloured pencils in Carl's pencil case. The information that in any group of five pencils, at most three have the same colour, tells us that there are at most three pencils of any single colour in the pencil case. Hence there are three pencils of each of the three different colours and so Carl's pencil case contains three blue pencils.

22. C Let the distance from London to Brighton be d miles. Since time = distance/speed, the times Lewis spent on the two parts of his journey are $\frac{d}{60}$ hours and $\frac{d}{40}$ hours. Hence the total time in hours that he travelled is

$$\frac{d}{60} + \frac{d}{40} = \frac{2d + 3d}{120} = \frac{5d}{120} = \frac{d}{24}.$$

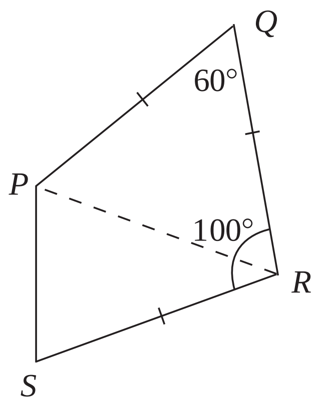
Therefore his average speed for the whole journey is $2d \div \left(\frac{d}{24}\right)$ mph $= 48$ mph.

23. E

$$\begin{array}{r} a\,b\,c \\ +\ \underline{a\,c\,b} \\ c\,4\,a. \end{array}$$

Since c is the digit in the hundreds column of the answer, we can deduce that $c > a$. Therefore, there must be a carry from the units column to the tens column and hence $a = 4 - 1 = 3$. Since there will also be a carry from the tens column to the hundreds column, we have $c = a + a + 1 = 7$. Therefore, $7 + b = 13$ and hence $b = 6$. Therefore the value of $a + b + c$ is $3 + 6 + 7 = 16$.

24. A Consider the quadrilateral $PQRS$ as shown with $PQ = QR = RS$, $\angle RQP = 60°$ and $\angle SRQ = 100°$.



Draw line PR. Since $PQ = QR$ and $\angle PQR = 60°$, triangle PQR is equilateral and hence $PR = PQ = QR = RS$ and $\angle PRQ = 60°$. Since $\angle SRQ = 100°$, $\angle SRP = 100° - 60° = 40°$. Since $PR = RS$, triangle PRS is isosceles and hence $\angle RPS = \angle PSR = \frac{1}{2}(180° - 40°) = 70°$. Therefore the largest angle of the quadrilateral is $\angle QPS = 70° + 60° = 130°$.

25. C There is no number that is both a multiple of three and a multiple of four without also being a multiple of two. Hence, the numbers underlined exactly twice are those that are a multiple of two and of three but not of four and those that are a multiple of two and four but not of three. The first set of numbers consists of the set of odd multiples of six. Since $2016 \div 6 = 336$, there are 336 multiples of 6 in the list of numbers and hence $336 \div 2 = 168$ odd multiples of six that would be underlined in red and blue but not green. The second set of numbers consists of two out of every three multiples of four and, since $2016 \div 4 = 504$, there are $\frac{2}{3} \times 504 = 336$ numbers that would be underlined in red and green but not blue. Hence there are $168 + 336 = 504$ numbers that Moritz would underline exactly twice.