NATIONAL COMMITTEE FOR MATHEMATICAL CONTESTS

Further International Selection Test 1989

Wednesday, 1st March 1989

Time allowed: 3½ HOURS.

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

Write on one side of the paper only. Use a fresh sheet or sheets of paper for each question. At the top of each sheet write your name and initials and the number of the question. Arrange your answers in order. Complete the proforma provided and attach it at the front of your script.

1. Find the smallest positive integer a with the property:

There exist integers b,c such that the equation

$$ax^2 - bx + c = 0$$

has two distinct roots in the interval $0 < x < 1$.

2. Find the number of different arrangements in a row of the letters

A,A,A,A,A,B,B,B,B,B,C,C,C,C,C

such that each letter is adjacent to an identical letter.

Indicate if you can a generalization to the case of n letters each appearing five times.

3. Let $f(x)$ be a polynomial of degree n such that

$$f(k) = \frac{k}{k+1}, \quad k = 0,1,2,\ldots,n .$$

Find $f(n+1)$, expressing your result as simply as possible.

4. M is a point on the side AC of triangle ABC such that triangles BAM, BMC have inscribed circles of equal radius. Find the length of BM in terms of the lengths a,b,c of the sides of triangle ABC.