United Kingdom Mathematics Trust

British Mathematical Olympiad

Round 2 : Thursday, 28 January 2016

Time allowed *Three and a half hours.*

Each question is worth 10 marks.

Instructions

- *Full written solutions – not just answers – are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt.*

  Rough work should *be handed in, but should be clearly marked.*
- *One or two* complete *solutions will gain far more credit than partial attempts at all four problems.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Staple all the pages neatly together in the top* left *hand corner, with questions 1, 2, 3, 4 in order, and the cover sheet at the front.*
- To accommodate candidates sitting in other time zones, please do not discuss any aspect of the paper on the internet until 8am GMT on Friday 29 January.

In early March, twenty students eligible to represent the UK at the International Mathematical Olympiad will be invited to attend the training session to be held at Trinity College, Cambridge (31 March-4 April 2016). At the training session, students sit a pair of IMO-style papers and eight students will be selected for further training and selection examinations. The UK Team of six for this summer's IMO (to be held in Hong Kong, China 6–16 July 2016) will then be chosen.

Do not turn over until **told to do so.**

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2015/16 British Mathematical Olympiad
Round 2

1. Circles of radius r_1, r_2 and r_3 touch each other externally, and they touch a common tangent at points A, B and C respectively, where B lies between A and C. Prove that $16(r_1+r_2+r_3) \geq 9(AB+BC+CA)$.

2. Alison has compiled a list of 20 hockey teams, ordered by how good she thinks they are, but refuses to share it. Benjamin may mention three teams to her, and she will then choose either to tell him which she thinks is the weakest team of the three, or which she thinks is the strongest team of the three. Benjamin may do this as many times as he likes. Determine the largest N such that Benjamin can guarantee to be able to find a sequence $T_1, T_2, \ldots, T_N$ of teams with the property that he knows that Alison thinks that T_i is better than T_{i+1} for each $1 \leq i < N$.

3. Let $ABCD$ be a cyclic quadrilateral. The diagonals AC and BD meet at P, and DA and CB produced meet at Q. The midpoint of AB is E. Prove that if PQ is perpendicular to AC, then PE is perpendicular to BC.

4. Suppose that p is a prime number and that there are different positive integers u and v such that p^2 is the mean of u^2 and v^2. Prove that $2p - u - v$ is a square or twice a square.