Supported by

The Actuarial Profession
making financial sense of the future

British Mathematical Olympiad

Round 2 : Tuesday, 30 January 2007

Time allowed *Three and a half hours.*

Each question is worth 10 marks.

Instructions

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt.*

  Rough work should *be handed in, but should be clearly marked.*
- *One or two* complete *solutions will gain far more credit than partial attempts at all four problems.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Staple all the pages neatly together in the top* left *hand corner, with questions 1,2,3,4 in order, and the cover sheet at the front.*

In early March, twenty students will be invited to attend the training session to be held at Trinity College, Cambridge (29th March - 2nd April). At the training session, students sit a pair of IMO-style papers and 8 students will be selected for further training. Those selected will be expected to participate in correspondence work and to attend further training. The UK Team of six for this summer's International Mathematical Olympiad (to be held in Hanoi, Vietnam 23-31 July) will then be chosen.

Do not turn over until **told to do so.**

Supported by

The Actuarial Profession
making financial sense of the future

2006/7 British Mathematical Olympiad
Round 2

1. Triangle ABC has integer-length sides, and $AC = 2007$. The internal bisector of $\angle BAC$ meets BC at D. Given that $AB = CD$, determine AB and BC.

2. Show that there are infinitely many pairs of positive integers (m, n) such that
$$\frac{m+1}{n} + \frac{n+1}{m}$$
is a positive integer.

3. Let ABC be an acute-angled triangle with $AB > AC$ and $\angle BAC = 60^o$. Denote the circumcentre by O and the orthocentre by H and let OH meet AB at P and AC at Q. Prove that $PO = HQ$.

Note: The circumcentre of triangle ABC is the centre of the circle which passes through the vertices A, B and C. The orthocentre is the point of intersection of the perpendiculars from each vertex to the opposite side.

4. In the land of Hexagonia, the six cities are connected by a rail network such that there is a direct rail line connecting each pair of cities. On Sundays, some lines may be closed for repair. The passengers' rail charter stipulates that any city must be accessible by rail from any other (not necessarily directly) at all times. In how many different ways can some of the lines be closed subject to this condition?