United Kingdom Mathematics Trust

British Mathematical Olympiad

Round 1 : Friday, 28 November 2014

Time allowed $3\frac{1}{2}$ hours.

Instructions

- *Full written solutions – not just answers – are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.*
- *One* complete *solution will gain more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all the problems.*
- *Each question carries 10 marks. However, earlier questions tend to be easier. In general you are advised to concentrate on these problems first.*
- *The use of rulers, set squares and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top* left *hand corner and your name, initials and school in the top* right *hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by your solutions in question number order.*
- *Staple all the pages neatly together in the top* left *hand corner.*
- *To accommodate candidates sitting in other time zones, please do not discuss the paper on the internet until 8am GMT on Saturday 29 November.*

Do not turn over until **told to do so.**

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1. Place the following numbers in increasing order of size, and justify your reasoning:
$$3^{3^4}, 3^{4^3}, 3^{4^4}, 4^{3^3} \text{ and } 4^{3^4}.$$
Note that a^{b^c} *means* $a^{(b^c)}$.

2. Positive integers p, a and b satisfy the equation $p^2 + a^2 = b^2$. Prove that if p is a prime greater than 3, then a is a multiple of 12 and $2(p + a + 1)$ is a perfect square.

3. A hotel has ten rooms along each side of a corridor. An olympiad team leader wishes to book seven rooms on the corridor so that no two reserved rooms on the same side of the corridor are adjacent. In how many ways can this be done?

4. Let x be a real number such that $t = x + x^{-1}$ is an integer greater than 2. Prove that $t_n = x^n + x^{-n}$ is an integer for all positive integers n. Determine the values of n for which t divides t_n.

5. Let $ABCD$ be a cyclic quadrilateral. Let F be the midpoint of the arc AB of its circumcircle which does not contain C or D. Let the lines DF and AC meet at P and the lines CF and BD meet at Q. Prove that the lines PQ and AB are parallel.

6. Determine all functions $f(n)$ from the positive integers to the positive integers which satisfy the following condition: whenever a, b and c are positive integers such that $1/a + 1/b = 1/c$, then
$$1/f(a) + 1/f(b) = 1/f(c).$$