IMO 2011 – A student's report

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Abstract

Inspired by the success of Richard Freeland's and Martin Chan's student reports for the *Romanian Masters of Mathematics* and *Balkan Mathematical Olympiad*, respectively, I have composed this report detailing the 52nd *International Mathematical Olympiad* from a student's perspective. In a sense, it complements James Cranch's report, as well as describing the events at the pre-IMO camp. The IMO was hosted in Amsterdam, Netherlands.

1   Participants

As usual, the team comprised six contestants, a leader, deputy and two observers. In ascending order of 'UNK' identifiers, they were:

Code	Name	Position
UNK1	James *'Triple-A Ronson'* Aaronson	Contestant
UNK2	Andrew *'Carl'* Carlotti	Contestant
UNK3	Ben *'Belliott'* Elliott	Contestant
UNK4	Adam P. Goucher	Contestant
UNK5	Josh *'Laminator'* Lam	Contestant
UNK6	Jordan Millar	Contestant
UNK7	James Cranch	Leader
UNK8	Jack Shotton	Deputy Leader
UNK9	Geoff Smith	Observer A
UNK10	Sally Anne Huk	Observer C

The 'UNK' identifiers made a welcome change from the 'GBR' counterparts in the *Romanian Masters of Mathematics*. Not only does 'UNK' allow Jordan Millar (Northern Ireland) and James Cranch (Guernsey) to be included, it also forms the basis of a song and dance conceived by James and our guide, Vicky, whilst inebriated in the 'mosh-pit' of the NEMO science centre in Amsterdam.

The only roles that may require explanation are the Observers. Observer A stays with the leaders and has the power to coordinate the marking of scripts. By comparison, Observer C remains with the contestants, and has the opportunity to enjoy the various excursions.

2   Problems

The problems themselves can be found both on the IMO website and in James' report. Problem 1 was an easy number theory problem by Mexico, which admits two distinct solutions up to

multiplication by a scalar.

The second problem was a very interesting themed combinatorial geometry problem by Geoff involving windmills. Whilst being a very difficult problem, it possesses a deceptively simple proof; as such, it was classified as 'C3' when 'C8' would be a more appropriate indication of its comparative difficulty. By masquerading as an easy problem, and being suitably Amsterdam-themed, it outcompeted a medium geometry problem (G4) by 47 votes to 46, securing a place.

Problem 3 was a hybrid problem, straddling the border between functional equations and inequalities. Problem 4 was an enumeration problem involving weighing scales, where the solution involves double-factorials. Problem 5 was another problem involving functions, this time a divisibility relation. Finally, the hardest question on the paper was a geometry problem, sadistically included to balance out the absence of a Euclidean geometry problem in the first paper.

3   Results

The United Kingdom performed very strongly this year, winning two gold, one silver and two bronze medals. Optimistically, we hoped to accomplish the elusive 'double traffic lights' of two gold, silver and bronze medals. Indeed, our combined score of 132 would have been sufficient to attain this, if the marks were distributed differently amongst us. Individual results are summarised below:

Code	Name	Actual Score	Cranch expected score	Award
UNK1	James Aaronson	29	0	Gold Medal
UNK2	Andrew Carlotti	28	0	Gold Medal
UNK3	Ben Elliott	23	0	Silver Medal
UNK4	Adam P. Goucher	21	0	Bronze Medal
UNK5	Josh Lam	13	0	Honourable Mention
UNK6	Jordan Millar	18	0	Bronze Medal

We all managed to transcend our Cranch expected scores by at least 13 marks – a remarkable accomplishment! Moreover, we surpassed Hungary for the third time in James Cranch's lifetime, which shows that this year's team was indeed particularly strong. Carlotti, Ben and I imitated each others' RMM performances in a cyclic way: Carlotti landed directly on the gold medal boundary, as I did in the RMM; Ben was one mark above the silver boundary, as Carlotti was in the RMM; I was one mark below, imitating Ben's RMM performance.

Carlotti and I both had no partial marks whatsoever: we had four and three complete solutions, respectively, and no marks on any of the other questions. The other partial marks mainly resulted from the Windmill Problem: anyone proving a trivial fact about the cyclic nature of the windmill problem earned a single 'charity mark' to account for the dearth of complete solutions for that question.

The only perfect score this year was attained by Lisa Sauermann, placing her at the top of the all-time IMO hall of fame, directly above a fellow German.

4 Occurrences approximately arranged chronologically

4.1 Saturday, 9th July

This year, the pre-IMO camp took place at Trinity College, Cambridge (a place described by one person as 'UKMT-mathsy-heaven' due to the large number of former IMO contestants studying there). Indeed, Joseph Myers' website, the UK IMO Register, confirms that half of all British IMO team members studied at Trinity. The other contestants mainly went to other colleges in Cambridge and Oxford – truly inspirational for anyone wanting to study maths at Oxbridge. I arrived later than the other UK team members (affectionately known as 'UNKs' by the 'Aussies' who arrived even earlier) to see them happily congregating in the Butler House Common Room. James Cranch and the Australian leader, Ivan, were reclining on a sofa which was, like all the furniture in the room, crimson.

We had a discussion about dietary requirements. Our deputy leader, Jack Shotton, is a vegetarian, and James Aaronson can only eat food in the intersection of 'kosher' and 'dairy-free' food. Additionally, I dislike food with a non-zero value on the Scoville Heat Scale, which severely restricted my choice in the Indian Restaurant. Indeed, I had only recently emotionally recovered from a frightening encounter with a jalapeño several weeks earlier. Nevertheless, we were all perfectly versatile in our gastronomic preferences by comparison with a certain former *Balkan Mathematical Olympiad* contestant who could only eat four specific types of food.

4.2 Sunday, 10th July

The day began with the first *Extraneous Selection Test*, or XST1. This was a $4\frac{1}{2}$-hour paper in the format of an IMO designed to train our teams for the forthcoming competitions. The first problem had a nice perturbation argument, eloquently described by James in terms of siphoning milk and water between vessels.

Paul Russell gave a lecture on Hall's Marriage Problem. This involved a hypothetical situation where the hypothetical Mrs. Cranch leaves her husband in favour of Joseph Myers to allow their passionate love for graph minors to flourish. As a result, the equally hypothetical Mrs. Myers had to settle for James Cranch instead. The idea of doubly-generalised Hall's Marriage Theorem emerged from this, where the women are either spinsters or uniformly polygamous, and all the men are monogamous.

Later on, we exchanged games with the Australians. Their deputy leader, Graham, subjected us to the enjoyably addictive card game, *We Didn't Playtest This At All*. In return, we introduced them to the games of *Mao* and *Contact*. Whilst the latter is perfectly acceptable in the polite company of James Cranch, the former is strictly forbidden due to its hideously complicated and adaptable rules. The most mathematical game was *Set*, where the objective is to form lines in toroidal modulo-3 4-space, $\mathbb{Z}_3^4$. The players soon realised that one of Euclid's axioms of geometry meant that they only had to point to two cards to determine uniquely the third card in the set.

4.3 Monday, 11th July

The second *Extraneous Selection Test*, XST2, commenced as planned. This was highly memorable, involving a problem (Question 1) discussed by the great Paul Erdős and Joseph Myers. Joseph was sixteen years of age at the time, and received a letter beginning 'Dear Dr Myers...' Specifically, the problem asked for all values of natural numbers a and b such that every natural number can

be expressed as a sum of numbers of the form $a^x b^y$, none of which divides any other. The only solution up to permutation happens to be $a = 2, b = 3$.

Another problem was Question 2, a David-Monk-style functional equation, which I solved using an intuitive two-dimensional geometric visualisation of the problem as a space-filling fractal. Although James liked my solution, he criticised me for mentioning the theory of Lindenmayer systems (L-systems), as this terminology is not recognised by most IMO coordinators. Indeed, the only other person at the camp who knew of their existence (and indeed wrote his thesis on them) was Ross Atkins, an Australian with half of a beard. However, Ross had not arrived at Trinity at this point, and we shall learn much more about his antics later in this report.

Question 3 was a geometry shortlist problem considered too difficult for the IMO. The problem was invariant under inversion (the typically British approach to geometry) and involved such beasts as curvilinear quadrilaterals! No-one made non-trivial progress on this question, including me, whose attempts to find a conformal map failed miserably. The intended solution was based on the *power of a point* paradigm, and would have met with a response of 'How do *humans* solve it?' from Richard.

The relaxing intermission of the day was punting on the River Cam. We were rather concerned about what would happen if our boat capsized, as it contained half of the British IMO team. Fortunately, this did not happen, although we did collide with a few weeping willows (*Salix x*

chrysocoma). After first getting rather lost, we had a race against Team Carlotti, winning by the narrowest of margins. This brought back memories of a previous maths camp in Oxford, where I caught gastroenteritis as a result of consuming an ice-cream after punting in the River Cherwell. Fortunately, this was not repeated in Cambridge, and none of us were ill for the IMO.

Monday's lecture was by the Fields medallist, Tim Gowers, entitled *Multiplication and Division*. Despite its innocuous-sounding title, it was actually involved with algorithmic complexity, irrespective of what James Cranch's parallel report leads you to believe... A brief introduction to Fourier transforms featured in the session, although we had all familiarised ourselves with them during Ben Green's lecture at Oundle. The importance of integer factorisation in cryptography was mentioned, whilst we were blissfully unaware that RSA cryptography was being taught in a much more in-depth way at the Birmingham maths camps held simultaneously with the IMO.

4.4 Tuesday, 12th July

In the morning, the most important mathematical competition, the Ashes, was held. The competition parallels the cricketing Ashes, and relates to an urn containing the remnants of the solutions to the 2008 competition. Australia won the first competition, and we won the other two. This year's result was highly anticipated, as it would determine whether or not the Australians would equalise us, and thereby win the Ashes and Trophy. Rather than inform you of the result now, I shall allow the tension to manifest.

James Cranch departed for the IMO via Gatwick airport, leaving us in the responsible hands of Jack Shotton and Graham White. After the Ashes exam, we were pleased to relax and have lunch in the Hall. James Aaronson queried as to what animal the meat originated from, as it would determine whether or not he would be able to eat it. The chef gave a most sincere and helpful response: 'Hippopotamus.' Possibly assisted by the outbreak of laughter amongst us, James successfully detected the sarcasm. He repeated the question, and managed to ascertain that the meat was either beef or pork. He decided not to risk it.

The results for the Ashes were displayed in ascending order, alternating between the British and Australian teams. It transpired that we had equal scores overall (specifically, 66), and we observed that neither of our sequences majorised the other. Geoff remarked that this was the best possible outcome, as we retain the Ashes whilst we are both content with the result. This may explain why both our teams performed well at the IMO.

As a tie-breaker, we played a game of Ultimate Frisbee in the Fellows' Garden at Trinity. We were losing initially, but then managed to make a miraculous recovery and seize a victory from the clutches of defeat. A factor affecting this was the low light intensity – the Australians are acclimatised to a greater exposure to sunlight, so the poor visibility impeded them to a greater extent. Indeed, I commented that 'my vision has become monochromatic due to my cones deactivating and my rods taking over'; the potential for innuendo was noticed firstly by Nancy Fu, shortly followed by the rest of us. James did not participate, as Ultimate Frisbee falls into the category of sport. The game terminated at approximately 22:00[1], when we decided that it would not be safe to wander the Fellows' Garden in case we were attacked by vampires.

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[1]Martin Chan's report for the Balkan Mathematical Olympiad involves decimal time, for the sake of simplicity. For obfuscatory purposes, I have used an ancient sexagesimal system for measuring time.

4.5   Wednesday, 13th July

It made a pleasant change from the Ashes and XSTs to have a day with no exams! After breakfast, we boarded a train from Cambridge to London for our excursion. On the train journey, we were busily selecting questions for each other in a competition known as the Cambridge Mathematical Olympiad, or CMO. Our team quickly rejected the first question (linear algebra) on the basis of its simplicity: I declared that it was equivalent to finding the tangent to an ellipse, and produced a mindless algebra bash capable of solving the question. Out of the remaining five, we chose the more symmetrical geometry question, a Diophantine equation, and a combinatorics problem involving a teacher and students. Unsatisfied with the boring setting of the final problem, Carlotti rephrased it to involve animals at Cambridge Zoo. This problem caused the most difficulty, as it appeared to be trivialised by the doubly-generalised version of Hall's Marriage Theorem. However, I remarked that noticing its equivalence to Hall's Marriage Theorem was non-trivial, and that any Australian who did so deserved to solve the problem. Also, we were not informed as to the statement of doubly-generalised Hall's Marriage Theorem, so we couldn't verify as to whether or not it applies. Finally, it was noted that neither intuitive guess at doubly-generalised Hall's Marriage Theorem is actually correct, prompting us to include the problem anyway.

The main highlights of the day included a visit to the Science Museum, where the maths section had been conveniently closed off, possibly to prevent an unhealthy over-exposure. This was mainly for the benefit of our guests, the Australians, as most (if not all) of the UNKs have already explored the museum. Indeed, it was my fourth time there, so it became rather repetitive. Ben and I discussed predictions for the 2012 IMO team, as it is traditional to do so a year beforehand. Apparently, the previous prediction was imprecise, and our current one does not account for surprises.

This was followed almost immediately with a guided tour around Westminster, where we were informed of the history of the Houses of Parliament, Westminster Abbey and other famous buildings in the area. Apparently, Westminster Abbey and Westminster School (renowned for producing a large quantity of maths camp attendees) are actually abbreviations of much longer names ending '. . . of St. Peter in Westminster.' We learnt that the idiom 'robbing Peter to pay Paul' originates from the erection of St. Paul's Cathedral nearby. As such, there is a resulting long-standing rivalry between Westminster School and St. Paul's School (*alma mater* of Imre Leader, and James Aaronson's current school).

The excursion was punctuated with meals at *Nando's* and *the Delhi Brassiere*. At the former, most of the IMO teams scoffed burgers and chips and guzzled Coke, whilst I enjoyed my Caesar salad and freshly-squeezed orange juice. The other restaurant kindly provided us with $85mm \times 54mm$ metallic gold discount cards, which come in very handy when one wishes to recall the name of the restaurant for the purposes of writing an IMO report. A discussion of Conway's Game of Life emerged spontaneously, with Joseph Myers explaining the precise rules to Jack Shotton.

4.6   Thursday, 14th July

After breakfast, we had our penultimate exam at the pre-IMO camp, namely XST3. The afternoon was occupied by the Sisyphean task of finding somewhere to play Ultimate Frisbee without the Trinity College porters dispersing us. Eventually, we settled on a grassy area near a tennis court and cricket pitch, and enjoyed obscure games such as 'non-planar frisbee' and 'tennisbee'. Joseph photographed a gratuitous struggle between Carlotti and me over the frisbee, which lasted several minutes!

Ross Atkins revealed his favourite inequality: "Jensen's Inequality $\geq$ all other inequalities."

Bryn Garrod's immersive lecture on Infinite Random Graphs soon followed. An unexpected result was that all (countably) infinite random graphs are isomorphic to each other, and a very non-random graph known as the Rado graph. I asked whether the graph was planar – probably the stupidest question in the entire pre-IMO camp.

4.7   Friday, 15th July

The final day of the pre-IMO camp included the Cambridge Mathematical Olympiad, where we selected questions for the Australians and vice-versa.

4.8 Saturday, 16th July

A fresh and early start to the day commenced with breakfast in the common room. This served mainly to use up any remaining croissants and several litres of fruit juice. At this point, we had already hauled our luggage from the 'D', 'E' and 'F' sections of Burrell's Field, including James, who was having some slight difficulties with the retractable handle on his suitcase. Nevertheless, after a coach journey with disco lights and intervening lavatory stop, we reached Gatwick Airport.

The flight was supposed to last a mere 45 minutes. However, the presence of several other aeroplanes on the runway delayed our flight – obviously, representing Britain in the IMO, we were obliged to queue. The airborne part of the flight was indeed rather rapid, and the cabin was not de-pressurised to the same extent as en route to Romania (my only previous experience of flying).

At Schipol Airport, Vicky Simon-Akerboom welcomed us to the Netherlands. She was brandishing a large sign marked 'United Kingdom.' Several other guides were seen with similar signs, to identify themselves. It was apparent at once that IMO 2011 was very well-organised. Ben was excited about being able to travel on a double-decker train to central Amsterdam. . .

Our residence for the next week was the Novotel – a four-star hotel with pleasant surroundings, brilliant accommodation, excellent food and wet trays. Team UNK resided on the sixth floor, giving us the option of taking the stairs or the lift. Only Carlotti and I favoured the former; James was particularly opposed to it. Ceri Fiddes joined us for our initial meal; she was a guest at the IMO, promoting the European Girls' Mathematical Olympiad, affectionately known as EGMO. We received a plethora of free gifts, including:

- IMO 2011 stationary, namely a pad of partially plain paper, a pen and 30-centimetre ruler;

- A metallic blue flask, colour-coordinated to match our UKMT ties and branded with MathCat to symbolise IMO 2011;

- A bimetallic medal puzzle, invented by Oskar van Deventer – probably the second most famous inventor of mechanical puzzles;

- An IMO 2011 Rubik's cube keyring;

- A Netherlands-themed cylindrical container, housing eight circular caramelised waffles;

- Invitations to the opening and closing ceremonies (or, if you're a set theorist, the clopen ceremonies!);

- A peaked cap, which I didn't wear in favour of my Panama hat;

- An inflatable plane, for no apparent reason;

- An IMO 2011 rucksack, which functioned to contain the above.

MathCat, in case you don't know, is the feline mascot of IMO 2011 – a domesticated version of the Germans' *Mathematiger*. There were 100 posters of MathCat situated around Amsterdam, and a competition for photographing your team in the proximity of one. Team UNK was far too busy to engage in such pleasantries. . .

Later, in the games room, Teodor von Burg *et al* were busily constructing a Jenga tower of maximal height. The next task was to familiarise ourselves with the adversaries. We divided into two groups, either playing Set with the Israelis or Uno with the Guatemalans. My experience of playing Mao inhibited my success, as I hesitated between saying "last card" and "Mao", despite the fact that the obverse of every card was marked with the required exclamation. The Guatemalan team comprised only four people, namely Alejandra, Sofia, Andrea and Fernando. We invited them to dine with us on an elongated table, where Jordan won ten pounds from Ben by eating yoghurt with chopsticks – a feat comparable to scoring 43 on the IMO and beating Lisa Sauermann. Suffice it to say, Jordan only accomplished one of these epic challenges. Jordan was rather excited about meeting Lisa, whom he idolised as a celebrity. I wonder whether he had the same reaction when meeting Geoff, who has both an MBE and Wikipedia article?

4.9   Sunday, 17th July

The first complete day of IMO 2011 involved the opening ceremony. There were speeches from dignitaries such as Robbert Dijkgraaf (one of the few people with three adjacent unit quaternions in his surname). This also featured the presentation of the various countries participating, backed by a particular piece of music being played ad nauseam. The countries were presented in *approximately* alphabetical order: a counter-example was Turkmenistan before Turkey. Shortly before the end of 'Europe' (it was done per continent, to separate the process into manageable chunks), we were called up by the energetic dancers to proceed in an orderly procession out of the theatre and on to the stage.

Some countries threw free gifts into the audience, such as Arabian dates (the fruits, not days in the Islamic calendar!) and Belgian chocolates. We left UKMT Frisbee distribution until later in the week, when we had more time.

4.10   Monday, 18th July

One of the events of the day was the first IMO paper. Amongst the questions was Geoff's elegant windmill problem. It received a rather mixed reception on Facebook, ranging from "The best

10

combinatorics IMO problem, the best IMO problem 2, the best IMO style IMO problem, I've met" (Hojoo Lee) to "F***[2] the windmills" (Baptiste Louf). Geoff had the last laugh, though, by smiling and winking at Baptiste as he collected his bronze medal in the closing ceremony.

To relax from the hardcore mathematics of the IMO paper, we watched the eighth and final Harry Potter film. The Dutch subtitles provided light relief, as Professor Snape's name became 'Sneep' and the wand measurements were converted from imperial to metric!

4.11 Tuesday, 19th July

I performed better on the second IMO paper, which seems to be a recurring theme for me (including BMO, FST, RMM and NST!). Maybe it was good that I didn't compete in the Balkan Mathematical Olympiad, as there was only one paper...

More importantly, we participated in some traditional Dutch games, including tug-of-war, the erection of structures built entirely from clipboards, and several races involving restricted motion. One example was where Team UNK sat on a long inflatable cylinder, and we had to oscillate vertically to propel it forwards. The Australians were beating us, until Carlotti had the epiphany that it would be more efficient to sit in reverse. Sure enough, our speed was two orders of magnitude faster in the Carlotti configuration. Team AUS positioned themselves to imitate us, but we had already gained far too much ground and stormed past the finish line. Success!

As is customary at all maths competitions, we went bowling. I ended up in Declan's $Diesel^{TM}$ trainers instead of the provided bowling shoes, due to an under-abundance of correctly-proportioned footwear. Nancy seemed to find the ordeal hilarious.

4.12 Wednesday, 20th July

Whilst the organisers were coordinating our scripts, we went sailing to a Dutch coastal village. We spent most of the sailing journey playing *Contact*, involving obscure words such as 'stichomythia' and 'meromorphic.' After disembarking, Vicky, Sally Anne and Team UNK purchased some ice-creams. Too lazy to decide for myself, I simply duplicated Josh's choice of cornet: an inventive concoction of caramel- and cherry-flavoured ice-cream. A suggested activity was to explore the town and identify stone carvings.

We wondered where our Observer A had disappeared to, especially since there were ambulances around the Novotel and a particular member of the French team had already threatened Geoff over his windmill problem. James Cranch reassured us as to his safety, explaining that "Geoff's in bed with the geometry at the moment."

4.13 Thursday, 21st July

The second day of organisation had another excursion, this time to The Hague. We visited the museum of Maurits Cornelis Escher, where the walls were adorned with his depictions of impossible objects, fantastical beasts and tilings of Riemann surfaces. Some exhibits were interactive, such as a perspective chamber where we positioned ourselves to give the illusion that our heights were in the inverse order to reality. We were photographed in this position, and rather disappointed

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[2]The expletive was considered sufficiently profane for me to censor it. If you're really bothered, it is 'SHPX' in ROT-13.

that we hadn't brought a MathCat poster with us. Surely this display of the optical illusions of projective geometry would have won the poster competition?

Geoff described how an IMO Foundation is being set up in Amsterdam, to ensure the continuing survival of the International Mathematical Olympiad. Until now, each IMO has been separate, with no overseeing organisation. All that is about to change. . .

4.14   Friday, 22nd July

After the coordination had finished, a relaxing visit to the Amsterdam city centre followed. James reported that we had abandoned Geoff in a 'Geoff Park' as we visited Dam Square, waiting for him to return for the cruise. A visit to the red light district was not part of the official IMO 2011 sequence of events, and the organisers were amazingly successful in ensuring this did not occur. Instead, we went to a far more exciting tourist attraction – the NEMO science centre – where Ross Atkins demonstrated a most bizarre solid capable of resisting anti-clockwise rotation. There were several exhibits based around the area-minimising property of soap bubbles.

4.15   Saturday, 23rd July

The closing ceremony was similar in nature to the opening ceremony. Graham White, the Australian deputy leader, stealthily attached furry koalas and kangaroos to our Panama hats, so that they would be displayed as we collected our medals. These mascots remained un-noticed; I suspect that Graham is secretly a ninja in his spare time.

To mark the end of the IMO, a barbecue was held somewhere within walking distance of the Novotel. Lisa Sauermann had managed to obtain a Mexican hat from somewhere, and commented on how her hat was larger than mine. Coupled with her having a score twice as large as mine, I felt rather inadequate! Geoff and I left early, walking back in the cold weather without informing Vicky. Oops! Along the way, a random car passenger ridiculed my Panama hat and a Vietnamese official ambushed us to provide IMO training materials. We gratefully accepted his generous donation. I ended the day with a hot, steamy shower.

5   Acknowledgements

I would like to acknowledge the following people, who helped immeasurably:

- Vicky Simon-Akerboom and Sally Anne Huk, for looking after us in the Netherlands;

- The UKMT, BMOS and IMO 2011, for running everything from behind the scenes;

- James Gazet and Gerry Leversha, amongst others, for training potential IMO 2012, 2013 and 2014 teams at the Birmingham camp;

- Finally, James, Andrew, Josh, Jordan and Ben, for an excellent performance in the IMO!