Level 3 Technical Level
IT: Programming

Y/507/6469-Unit 5 Mathematics for programmers
Mark scheme

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

The following annotation is used in the mark scheme:

; - means a single mark

// - means alternative response

/ - means an alternative word or sub-phrase

A - means acceptable creditworthy answer

R - means reject answer as not creditworthy

NE - means not enough

I - means ignore

DPT - in some questions a specific error made by a candidate, if repeated, could result in the candidate failing to gain more than one mark. The DPT label indicates that this mistake should only result in a candidate losing one mark on the first occasion that the error is made. Provided that the answer remains understandable, subsequent marks should be awarded as if the error was not being repeated.

Level of response marking instructions

Level of response mark schemes are broken down into levels, each of which has a descriptor. The descriptor for the level shows the average performance for the level. There are marks in each level.

Before you apply the mark scheme to a student's answer read through the answer and annotate it (as instructed) to show the qualities that are being looked for. You can then apply the mark scheme.

Step 1 Determine a level

Start at the lowest level of the mark scheme and use it as a ladder to see whether the answer meets the descriptor for that level. The descriptor for the level indicates the different qualities that might be seen in the student's answer for that level. If it meets the lowest level then go to the next one and decide if it meets this level, and so on, until you have a match between the level descriptor and the answer. With practice and familiarity you will find that for better answers you will be able to quickly skip through the lower levels of the mark scheme.

When assigning a level you should look at the overall quality of the answer and not look to pick holes in small and specific parts of the answer where the student has not performed quite as well as the rest. If the answer covers different aspects of different levels of the mark scheme you should use a best fit approach for defining the level and then use the variability of the response to help decide the mark within the level, ie if the response is predominantly level 3 with a small amount of level 4 material it would be placed in level 3 but be awarded a mark near the top of the level because of the level 4 content.

Step 2 Determine a mark

Once you have assigned a level you need to decide on the mark. The descriptors on how to allocate marks can help with this. The exemplar materials used during standardisation will help. There will be an answer in the standardising materials which will correspond with each level of the mark scheme. This answer will have been awarded a mark by the Lead Examiner. You can compare the student's answer with the example to determine if it is the same standard, better or worse than the example. You can then use this to allocate a mark for the answer based on the Lead Examiner's mark on the example.

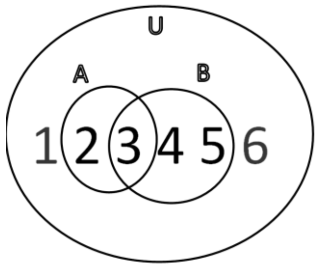
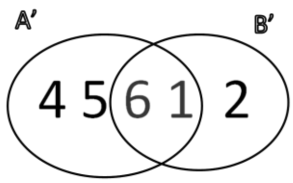
You may well need to read back through the answer as you apply the mark scheme to clarify points and assure yourself that the level and the mark are appropriate.

Indicative content in the mark scheme is provided as a guide for examiners. It is not intended to be exhaustive and you must credit other valid points. Students do not have to cover all of the points mentioned in the Indicative content to reach the highest level of the mark scheme.

An answer which contains nothing of relevance to the question must be awarded no marks.

Question	Guidance	Mark
01	C	**1**
02	A	**1**
03	B	**1**
04	B	**1**
05	A	**1**

Question	Guidance	Mark
06.1	**1 mark** for `11111101` **A.** -3 even if incorrect binary is given	**1**
06.2	**1 mark** for `01111000` **A.** 120	**1**
06.3	**1 mark** for the negative number –120 is converted to its positive form as 120 // it changes a negative number into its positive equivalent // the number has been made positive.	**1**
07.1	**1 mark** for $\frac{1}{3}$ **A.** equivalent values or description.	**1**
07.2	**2 marks** if answer is correct $\frac{1}{6}$ If answer given is not correct then awarding working marks as follows: **1 mark** for attempting to multiply by $\frac{1}{3}$ by $\frac{1}{2}$ **A.** 0.33 x 0.5 **R.** 0.3 x 0.5	**2**
08	**1 mark** for each correct step (max 3 marks), eg: Step 1: Start at middle position 14. Step 2: 47 > 14, so, start at middle position from 14 to 53 (1 mark) which is 45. Step 3: 47 > 45, so, start at middle position from 45 to 53 (1 mark) which is 47. **A.** Explanation as above but which discards 14. **A.** Steps as process or numbers so long as the logic is clear. **A.** Other reasonable sequences, eg smaller steps: ○ Step 1: Find middle number (14) ○ Step 2: Compare 14 with 47 ○ Step 3: Discard bottom half of numbers (and 14).	**3**

Question	Guidance	Mark
09	(A ∪ B) = {2, 3, 4, 5}, therefore (A ∪ B)' = {1, 6}. **2 marks** for correct diagram  **A.** U circle missing or labelled N. **A.** Labels/letters missing. If not fully correct then a maximum of 1 from: Identifying (A ∪ B)'; 2, 3, 4, 5 written once in the correct positions; 1 and 6 outside A and B even if labelled incorrectly (eg N); The numbers 1 and 6 linked in words or notation to (A U B)'; A' = {1, 4, 5, 6} and B' = {1, 2, 6}, therefore A' ∩ B' = {1, 6} **2 marks** for correct diagram  If not fully correct then a maximum of 1 from: Identifying A' ∩ B'; 4, 5, 2 written once in the correct positions; 1 and 6 linked in words or notation to A' ∩ B';	**4**

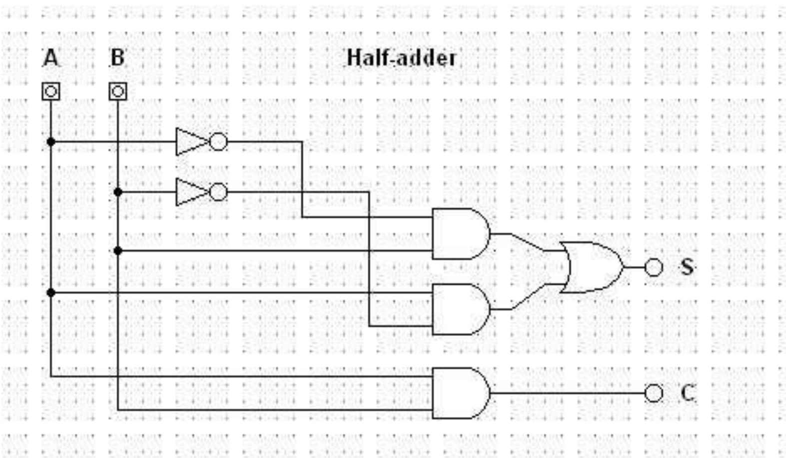
Question	Guidance	Mark

10.1	**1 mark** for
	
	`1 1 1`
	`1 0 1`
	`1 0 0`
		1

10.2	**1 mark** for 754	**1**

Question	Guidance	Mark
11.1	**1 mark** for Unit matrix I = $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ **A.** Unit matrix only. **A.** Description, eg ones on the main/first diagonal and zeroes elsewhere.	**1**
11.2	**4 marks** for the correct answer $A^{-1} = \begin{pmatrix}2 & -3\\-5 & 8\end{pmatrix}$ **A.** Without A^{-1} =. If not fully correct then a maximum of 3 from: **1 mark** for a pair of correct values, even in the wrong position/letter; **1 mark** for identifying the inverse matrix but with one diagonal incorrect; **1 mark** for the correct multiplication: If $A^{-1} = \begin{pmatrix}a & b\\c & d\end{pmatrix}$ then $\begin{pmatrix}a & b\\c & d\end{pmatrix} \times \begin{pmatrix}8 & 3\\5 & 2\end{pmatrix} = I = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ **A.** Answer which only shows the two matrices being multiplied. **1 mark** for the correct equation: therefore $\begin{pmatrix}8a+5b & 3a+2b\\8c+5d & 3c+2d\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ **A.** Answer which only shows the left side of the equation. From above 8a + 5b = 1, 3a + 2b = 0, 8c + 5d = 0 and 3c + 2d = 1 Solving the above equations yields a = 2, b = –3, c = –5 and d = 8	**4**
11.3	**1 mark** for $\begin{pmatrix}2 & -3\\-5 & 8\end{pmatrix} \text{x} \begin{pmatrix}8 & 3\\5 & 2\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ **A.** Answer which only shows the two matrices being multiplied (ie which does not show unit matrix).	**1**

Question	Guidance	Mark
12.1	**1 mark** for each side of the expression $i_s(n) = \frac{1}{0.8+\frac{0.8}{n}}$	**2**
12.2	**3 marks** for correct answer 1.2 // 0.2 in 1 // 0.2 // 20% // 1 in 5 **A.** Any equivalent answer in words or numbers. **A.** Maximum of 2 marks if candidate has used the wrong expression from Question 12.1 or Question 12, but then used it correctly to work out Question 12.2. If answer given is not correct then award working marks as follows: Maximum of 2 from: showing the correct substitution for either equation; showing the correct result for either equation; answer of 1.2 : 1 or equivalent; $i_s(2) = \frac{1}{0.8+\frac{0.8}{2}} = \frac{1}{1.2}$ $i_s(4) = \frac{1}{0.8+\frac{0.8}{4}} = 1$	**3**

<table>
<tr><th>Question</th><th>Guidance</th><th>Mark</th></tr>
<tr><td>13.1</td><td>
1 mark for S column correct
1 mark for C column correct
<table>
<tr><td>A</td><td>B</td><td>S</td><td>C</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>1</td><td>0</td><td>1</td></tr>
</table>
</td><td>2</td></tr>
<tr><td>13.2</td><td>
1 mark for S logic circuit correct
1 mark for C logic circuit correct


</td><td>2</td></tr>
<tr><td>13.3</td><td>
1 mark for distributive law of logic

P = A(B + C)

A. Without P =.

1 mark for De Morgan’s law

$\overline{P} = \overline{A(B + C)}$
$\overline{P} = \overline{A} + \overline{(B + C)}$

A. Either of the above steps or equivalent.

1 mark for conclusion

Since $\overline{(B + C)}$ is $\overline{B}.\overline{C}$ by De Morgan's law $\overline{P} = \overline{A} + \overline{B}.\overline{C}$

A. Without $\overline{P}$ = part of above.
</td><td>3</td></tr>
</table>

Question	Guidance	Mark
14.1	**1 mark** for result or series 1 + 2 + 4 + 8 + 16 + 32 + 64 = 127 **A.** Appropriate explanation of the above.	**1**
14.2	**1 mark** for $n^2 - 1$	**1**
14.3	**1 mark** for $9^2 - 1$ / 80	**1**
14.4	Maximum of 2 from: The "base case" is the terminal condition of the recursive function // prevents infinite recursive calls; it is required to terminate/end/stop the recursion/function; when a (pre-defined) condition is met.	**2**
14.5	**1 mark** for The base case S(0) = 0 is present **R.** The base case is present. **1 mark** for The function is in terms of itself/self-calling // recursive with regards to terms S(N) and S(N – 1).	**2**

Question	Guidance	Mark
15.1	**1 mark** for T is directly proportional to CPI // the larger CPI is, the larger the T will be/more time taken.	**1**
15.2	**1 mark** for T is inversely proportional to S // the larger S is/higher the speed, the smaller the T will be/less amount of time taken.	**1**
15.3	**1 mark** for substituting values $T_A = \frac{I \times 4}{3}$ $T_B = \frac{I \times 2}{2}$ **A.** where value for I has been assumed, including 1 **1 mark** for CPU B is faster + with qualifying statement, eg: CPU B is faster as one way of deciding which CPU is faster is to look at the ratio of the two times; CPU B is faster with a calculation/ratio of T_A and T_B shown; **A.** Equation which clearly shows ratio, eg: $\frac{T_A}{T_B} = \frac{I \times 4}{3} \times \frac{2}{I \times 2} = \frac{4}{3}$ **A.** CPU B is faster + partly accurate qualifying statement. **R.** CPU B is faster. **R.** Inaccurate qualifying statement.	**2**
15.4	**1 mark** for 25% **A.** A is 33% slower than B. **R.** 33%.	**1**

<table>
<tr><th>Question</th><th>Guidance</th><th>Mark</th></tr>
<tr><td>16.1</td><td>
1 mark for
x = 5
1 mark for
y = 8
1 mark for
z = 3
A. The numbers 5, 8 and 3 in the correct order.
Table 6
<table>
<tr><th></th><th>Number of bits</th></tr>
<tr><td>an Operation Code up to a decimal value of 28
(ie the value of x)</td><td>$2^x - 1 >= 28$ therefore:

x = 5</td></tr>
<tr><td>the largest number in Operand 1 (ie the value of y)</td><td>$2^y - 1 >= 255$ therefore

y = 8</td></tr>
<tr><td>the largest number in Operand 2 (ie the value of z).</td><td>$2^z - 1 >= 7$ therefore

z = 3</td></tr>
</table>
</td><td>3</td></tr>
<tr><td>16.2</td><td>
1 mark for
16 bits / 5 + 8 + 3
A. 16 binary digits.
1 mark for
2 bytes / attempt to divide bits by 8
A. If incorrect number of bits is converted accurately to bytes.
A. Maximum one mark if 16 and 2 are given, but the wrong way around.
</td><td>2</td></tr>
</table>

Question	Guidance	Mark

16.3	**3 marks** for correct answer
	
	`0xE507` // `E507`
	
	If answer given is not correct then award working marks as follows:
	
	1 mark for converting to binary
	
	1 mark for assigning the correct number of bits for two fields of the instruction
	
	`11100  10100000  111`
		3

16.4	Maximum of 3 from: 10 // `0x0A` // `0A`; 30 // `1E`; 3 // `03`; If answer given is not correct then award working marks as follows: **2 marks** for correctly partitioning the bits to each field of the instruction **0101 0**000 1111 0**011** **A. 1 mark** for the above with two fields correct.	**3**
16.5	Operand 1 and Operand 2 => 11 bits or 2^{11} bytes = 2048 bytes or 2 kB **1 mark** for 2048 / 2048 bytes / 2048B **A.** 2047 **1 mark** for 2 / 2kB / 2KB // attempt to divide by 1000 or 1024 **A.** If incorrect number of bytes is converted accurately to kilobytes.	**2**
16.6	Max no of Operation Codes = 2^5 = 32. Therefore, he can have 32 – 28 = 4 more Operation Codes. **1 mark** for 4 **A.** 3	**1**

Question	Guidance	Mark
17.1	**1 mark** for $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	**1**
17.2	Reflection in the x-axis is done by applying the mapping operator on any point $\binom{x}{y}$ $$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$ The point (x, y) is mapped onto $(x, -y)$ by reflection in the x-axis. **1 mark** for identifying the equations **1 mark** for the result	**2**
17.3	**1 mark** for $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$	**1**
17.4	**Show the use of the operator in Question 17.3 in rotating R about the origin *O*. Assume a point R(x, y) is expressed as the matrix R $= \binom{x}{y}$.** Rotation about the origin O is done by applying the mapping operator on any point $\binom{x}{y}$ $$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x\cos\alpha - y\sin\alpha \\ x\sin\alpha + y\cos\alpha \end{pmatrix}$$ The point (x, y) is mapped onto point $(x\cos\alpha - y\sin\alpha,\ x\sin\alpha + y\cos\alpha)$ by rotation about the origin O. **1 mark** for identifying the equations **1 mark** for the result	**2**

Question	Guidance	Mark
17.5	The operation for rotation about the origin is $\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$ The operation for reflection in the x-axis is $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ **1 mark** for identifying the correct operators Therefore, rotation followed by reflection on point P (x, y) is given by $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & -\cos\alpha\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$ **1 mark** for applying the operators in the correct order **1 mark** for the result Therefore, the mapping matrix is $\begin{pmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & -\cos\alpha\end{pmatrix}$ in this case. **1 mark** for identifying the mapping matrix	4
17.6	If the operations are reversed the operator is $\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & -\cos\alpha\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$ **1 mark** for applying the operators in the correct order **1 mark** for the result Therefore, the two operations are not the same. **1 mark** for identifying the difference $\begin{pmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & -\cos\alpha\end{pmatrix}$ is not the same as $\begin{pmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & -\cos\alpha\end{pmatrix}$	3

Question	Guidance	Mark
17.7	The matrix corresponding to reflection in the line $x = y$ is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ **1 mark** for identifying the operator matrix for reflection in the line $x = y$ The double operation is therefore equivalent to $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ **1 mark** for identifying the double operation matrix. **A.** either side of equation. **R.** left side of equation only and the matrices the wrong way round.	**2**
17.8	$\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ **1 mark** for P'' = (3, 1) **A.** (3,1) / 3 and 1 in the right order.	**1**

Assessment Objectives						
Question	**AO1**	**AO2**	**AO3**	**AO4**	**AO5**	**Question Total**
Section A						
1	1c (1)					1
2	1b (1)					1
3			3c (1)			1
4		2a (1)				1
5				4c (1)		1
6	1bc (3)					3
7			3c (3)			3
8			3c (3)			3
9		2bc (2)	3c (2)			4
10	1ab (2)					2
11					5ab (6)	6
12				4de (5)		5
13		2bcd (7)				7
14			3bc (7)			7
15				4d (5)		5
Section B						
16	1bc (14)					14
17					5bc (16)	16
Totals	21	10	16	11	22	80