AQA

Level 3 Technical Level
IT: PROGRAMMING

Unit 5 Mathematics for programmers

Tuesday 12 June 2018   Afternoon   Time allowed: 2 hours

Materials
For this paper you must have:
- a ruler
- scientific calculator (non-programmable)
- stencils or other drawing equipment (eg flowchart stencils)
- formula sheet.

Instructions
- Use black ink or black ball-point pen.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.
- If you need more space use the additional pages at the back of this booklet.
- Include units in all answers, where required, as marks are given for units in some questions.

Information
- The marks for questions are shown in brackets.
- There are 50 marks in **Section A** and 30 marks in **Section B**. Both sections should be attempted.

Advice
- In all calculations, show clearly how you work out your answer.
- Use diagrams, where appropriate, to clarify your answers.
- You are expected to use a calculator where appropriate.
- You are reminded of the need for good English and clear presentation in your answers.

For Examiner's Use	
Question	Mark
1–5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
TOTAL	

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Section A

Answer **all** questions in this section.

In multiple choice questions only **one** answer per question is allowed.

For each answer completely fill in the circle alongside the appropriate answer.

CORRECT METHOD [●]   WRONG METHODS [⊗] [◉] [⊜] [⊘]

If you want to change your answer you must cross out your original answer as shown. [⊠]

If you wish to return to an answer previously crossed out, ring the answer you now wish to select as shown. [⊗]

You may do your working in the blank space around each question but this will not be marked. Do **not** use additional sheets for this working.

| 0 | 1 | What is the minimum number of binary bits required to represent the decimal number 35?

[1 mark]

A 4 ○

B 5 ○

C 6 ○

D 7 ○

| 0 | 2 | Which **one** of the following hexadecimal numbers is equivalent to the binary number 010111000111?

[1 mark]

A 5C7 ○

B 2707 ○

C A3E ○

D 7C5 ○

0 3 If the probability of an event happening is 0.2, what is the probability of it **not** happening?

[1 mark]

A   99.8   ⬭

B   0.8   ⬭

C   9.8   ⬭

D   0   ⬭

0 4 What two-input (A, B) and one-output (P) logic operator does the truth table in **Table 1** represent?

Table 1

A	B	P
0	0	0
0	1	0
1	0	0
1	1	1

[1 mark]

A   OR   ⬭

B   AND   ⬭

C   XOR   ⬭

D   NOT   ⬭

Turn over ▶

0 5   Which **one** of the infix expressions below does the postfix expression 8 2 * 5 + represent?

[1 mark]

A   8 * 2 + 5   ⬭

B   8 + 2 * 5   ⬭

C   5 * 8 + 2   ⬭

D   5 + 2 * 8   ⬭

5

0 6 . 1 Perform the following subtraction operation on two signed 8-bit binary numbers:

00000111 – 00001010

[1 mark]

_____

_____

0 6 . 2 Convert the following signed binary number into two's complement form:

10001000

[1 mark]

_____

_____

0 6 . 3 Explain the effect of the conversion in Question **06.2** on the signed number.

[1 mark]

_____

_____

3

Turn over for the next question

Turn over ▶

0 7 There are three coloured balls in a bag: one blue, one red and one green.

0 7 . 1 What is the probability of taking out a red ball?

[1 mark]

_____

_____

0 7 . 2 What is the probability of taking out a green ball followed by a red ball without replacing the green ball?

[2 marks]

_____

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0 8 **Table 2** represents a sorted list of numbers in an array.

Table 2

0	4	7	10	14	23	45	47	53

A binary search is used to search for number 47.

Describe the first three steps of this search.

[3 marks]

_____

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3

Turn over for the next question

Turn over ▶

0 9   Let set N = {1, 2, 3, 4, 5, 6}, set A = {2, 3} and set B = {3, 4, 5}

Using Venn diagrams, demonstrate that (A U B)' = A' ∩ B' where:

- U is the set union operator and
- ∩ is the set intersection operator.

Clearly identify the sets in your diagrams.

[4 marks]

4

1 0 In the Linux operating system, file access permissions are represented in three groups of 3-bit values as shown in **Table 3**. The groups are identified as **A**, **B** and **C** from left to right.

Table 3

A	B	C
`rwx – rwx – rwx`		

- if r=1 the file can be read
- if w=1 the file can be written to
- if x=1 the file can be executed,

otherwise they are set to 0 (zero).

The Linux command 'chmod' uses this information as three octal digits to modify file access rights. A programmer wishes to set:

- group **A** as read, write and execute
- group **B** as read and execute
- group **C** as read only.

1 0 . 1 Work out the bit values for **A**, **B** and **C**.

[1 mark]

1 0 . 2 What are the three octal digits that make up the chmod value?

[1 mark]

2

1 1 . 1 Give the unit (or identity) matrix for a '2 by 2' matrix.

[1 mark]

1 1 . 2 The square matrix $\mathbf{A} = \begin{pmatrix} 8 & 3 \\ 5 & 2 \end{pmatrix}$. If the inverse matrix, $\mathbf{A}^{-1}$,

is expressed as $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then calculate the values a, b, c and d and give the inverse matrix.

[4 marks]

1 1 . 3 Prove that, in Question **11.2**, the matrix $\mathbf{A}^{-1}$ is the inverse of matrix $\mathbf{A}$.

[1 mark]

6

| 1 | 2 |

The equation below gives the relationship between the improvement, i_s, in the speed of a computer system and the number of CPUs, n, when running a program:

$$0.8i_s\left(1 + \frac{1}{n}\right) = 1$$

| 1 | 2 |.| 1 |

Express i_s as a function of n using the function notation.

[2 marks]

| 1 | 2 |.| 2 |

Calculate the ratio of improvements in the execution speeds of the program when the number of CPUs is increased from 2 to 4.

[3 marks]

5

Turn over for the next question

| 1 | 3 |

A half-adder is the basic logic block of a CPU's binary numbers adder hardware.

The half-adder:

- adds two binary bits, A and B
- produces a single bit sum S as output
- produces a single bit carry C as a second output.

The two logic equations are shown below:

$S = \bar{A} . B + A . \bar{B}$
$C = A . B$

Here, $\bar{A} =$ NOT A and $\bar{B} =$ NOT B

| 1 | 3 | . | 1 |

Using the above equations, complete the truth table, **Table 4**.

[2 marks]

Table 4

A	B	S	C
0	0		
0	1		
1	0		
1	1		

1 3 . **2**   Using the truth table in **Table 4**, draw the corresponding logic circuit.

[2 marks]

1 3 . **3**   Using De Morgan's law, prove that the complement of the logic equation

$$P = A . B + A . C \text{ is } \overline{P} = \overline{A} + \overline{B} . \overline{C}$$

[3 marks]

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7

Turn over ▶

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1 4 . 1 What is the result of the following series?

$$R = \sum_{i=0}^{i=6} 2^i$$

Show how you calculated your answer.

[1 mark]

_____

_____

1 4 . 2 Consider the following first six numbers of a sequence:

−1, 0, 3, 8, 15, 24

Deduce the nth term where n = 0 refers to the first term.

[1 mark]

_____

_____

1 4 . 3 Calculate the value of the 10th number in the sequence from Question **14.2**.

[1 mark]

_____

_____

1 4 . 4 Explain the need for a 'base case' in a recursive function.

[2 marks]

_____

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_____

| 1 | 4 | . | 5 | A recursive function S(N), representing the sum of every number from 1 to N, is defined below:

S(0) = 0
For all values of N > 0, S(N) = S(N – 1) + N

Explain what makes this a recursive function.

[2 marks]

_____

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7

Turn over for the next question

1 5 The performance of a CPU can be measured by the following formula:

$$T = \frac{I * CPI}{S}$$

Where:

T is the time it takes to run
I is the number of instructions executed
CPI is the average number of clock cycles per instruction
S is the speed of the CPU.

1 5 . 1 Looking at the above formula, state the relationship between T and CPI.

[1 mark]

_____

_____

1 5 . 2 Looking at the above formula, state the relationship between T and S.

[1 mark]

_____

_____

In an experiment, the performance of two CPUs, **A** and **B**, is compared.
Table 5 gives the relevant values used. Both CPUs run the same number of instructions, ie the same software program.

Table 5

CPU	*S*	CPI
A	3	4
B	2	2

| 1 | 5 |.| 3 |

Determine which CPU, **A** or **B**, runs the instructions faster.
Use the information in **Table 5** and the performance formula on page 16.

[2 marks]

_____

_____

_____

_____

| 1 | 5 |.| 4 |

Calculate how much more quickly the faster CPU runs the instructions than the slower CPU. Give your answer as a percentage.

[1 mark]

_____

_____

5

Turn over for the next question

Turn over ▶

Section B

Answer **all** questions in this section.

1 6 The designer of a CPU decides that each instruction word has the following parts:

x bits	y bits	z bits
Operation Code	**Operand 1**	**Operand 2**

x, **y** and **z** are the lengths of each individual part in number of binary bits.

Operation Code defines the instruction, eg add numbers.

Operand 1 and **Operand 2** contain data on which the **Operation Code** will work, eg the two numbers to add up.

This CPU has 28 unique operation codes, numbered 0–27. The largest number **Operand 1** can contain is 255 and the largest number **Operand 2** can contain is 7.

1 6 . 1 Complete **Table 6** giving the number of bits required to store:

Table 6

	Number of bits
an Operation Code up to a decimal value of 28 (ie the value of x)	
the largest number in Operand 1 (ie the value of y)	
the largest number in Operand 2 (ie the value of z)	

[3 marks]

1 6 . 2 What is the length of each instruction word:

[2 marks]

in bits? _____

in bytes? _____

1 6 . 3 In one of the instructions:

- The Operation Code is 0x1C
- Operand 1 is 160
- Operand 2 is 7.

Calculate the contents of this instruction in hexadecimal form.

[3 marks]

_____

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1 6 . 4 The contents of another instruction are 0x50F3

Determine the values of the Operation Code, Operand 1 and Operand 2.

[3 marks]

_____

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_____

Question 16 continues on the next page

Turn over ▶

| 1 | 6 | . | 5 |

The designer decides that one of the instructions will combine Operand 1 and Operand 2 to allow for a bigger value.

How many values can be accommodated by this combination:

[2 marks]

in bytes? _____

in kilobytes (KB)? _____

| 1 | 6 | . | 6 |

The designer wants to increase the number of operation codes.

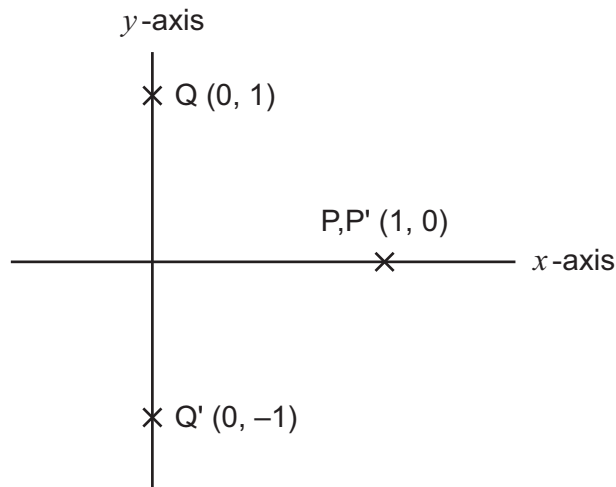How many more operation codes can he have without having to change the instruction structure?

[1 mark]

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14

1 7 In computer graphics, operations on images such as reflection and rotation are frequently used to add realistic shapes and movements.

Consider **Figure 1**.

For reflection in the x-axis, we need to map:

point P $(1, 0)$ into point P' $(1, 0)$ and
point Q $(0, 1)$ into point Q' $(0, -1)$.

Figure 1

1 7 . 1 State the operator matrix required for reflecting any point in the x-axis.

[1 mark]

_____

_____

1 7 . 2 Show the use of the operator in Question **17.1** in reflecting R in the x-axis.

Assume a point R(x, y) is expressed as the matrix R $= \begin{pmatrix} x \\ y \end{pmatrix}$.

[2 marks]

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Question 17 continues on the next page

Turn over ▶

Consider **Figure 2**.

For rotation about the origin O through angle α, we need to map:
- point A $(1, 0)$ into point A' $(\cos \alpha, \sin \alpha)$ and
- point B $(0, 1)$ into point B' $(-\sin \alpha, \cos \alpha)$.

Figure 2
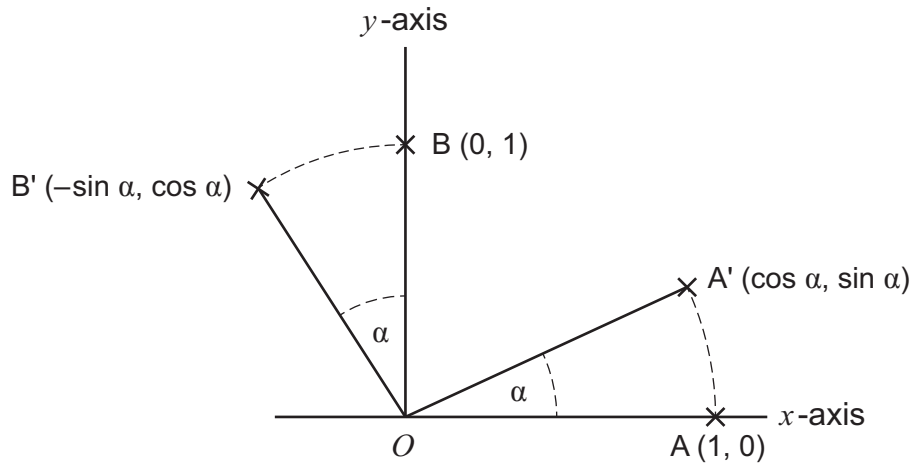
| 1 | 7 | . | 3 | State the operator matrix required for rotating any point about the origin O.

[1 mark]

| 1 | 7 | . | 4 | Show the use of the operator in Question **17.3** in rotating R about the origin O.

Assume a point R(x, y) is expressed as the matrix R = $\begin{pmatrix} x \\ y \end{pmatrix}$.

[2 marks]

1 7 . 5   A point P is mapped into point P' by rotation through an angle α about the origin. The point P' is then mapped into point P" by reflection in the x-axis.

Determine the matrix which maps P into P".

[4 marks]

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1 7 . 6   Show that P will not map into same P" if the operation is reversed, ie reflection followed by rotation.

[3 marks]

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Question 17 continues on the next page

Turn over ▶

1 7 . 7 A point P is mapped into the point P' by the shear $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$. The P' is then mapped into P" by reflection in the line $x = y$.

Determine the matrix equivalent to the double operation, ie shear followed by reflection.

[2 marks]

1 7 . 8 The coordinates of point P are $(1, 1)$. Using your answer from Question **17.7**, calculate the coordinates of point P".

[1 mark]

16

END OF QUESTIONS

If needed, use the following pages to continue your answers. Write the question number beside your answer.

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