AQA

Level 3 Technical Level

IT: PROGRAMMING

Y/507/6469

Unit 5: Mathematics for programmers

Mark scheme
January 2018

Version: 1.0 Final

181AY/507/6469/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Level of response marking instructions

Level of response mark schemes are broken down into levels, each of which has a descriptor. The descriptor for the level shows the average performance for the level. There are marks in each level.

Before you apply the mark scheme to a student's answer read through the answer and annotate it (as instructed) to show the qualities that are being looked for. You can then apply the mark scheme.

Step 1 Determine a level

Start at the lowest level of the mark scheme and use it as a ladder to see whether the answer meets the descriptor for that level. The descriptor for the level indicates the different qualities that might be seen in the student's answer for that level. If it meets the lowest level then go to the next one and decide if it meets this level, and so on, until you have a match between the level descriptor and the answer. With practice and familiarity you will find that for better answers you will be able to quickly skip through the lower levels of the mark scheme.

When assigning a level you should look at the overall quality of the answer and not look to pick holes in small and specific parts of the answer where the student has not performed quite as well as the rest. If the answer covers different aspects of different levels of the mark scheme you should use a best fit approach for defining the level and then use the variability of the response to help decide the mark within the level, ie if the response is predominantly level 3 with a small amount of level 4 material it would be placed in level 3 but be awarded a mark near the top of the level because of the level 4 content.

Step 2 Determine a mark

Once you have assigned a level you need to decide on the mark. The descriptors on how to allocate marks can help with this. The exemplar materials used during standardisation will help. There will be an answer in the standardising materials which will correspond with each level of the mark scheme. This answer will have been awarded a mark by the Lead Examiner. You can compare the student's answer with the example to determine if it is the same standard, better or worse than the example. You can then use this to allocate a mark for the answer based on the Lead Examiner's mark on the example.

You may well need to read back through the answer as you apply the mark scheme to clarify points and assure yourself that the level and the mark are appropriate.

Indicative content in the mark scheme is provided as a guide for examiners. It is not intended to be exhaustive and you must credit other valid points. Students do not have to cover all of the points mentioned in the Indicative content to reach the highest level of the mark scheme.

An answer which contains nothing of relevance to the question must be awarded no marks.

Question	Guidance	Mark
01	B	**1**
02	A	**1**
03	C	**1**
04	A	**1**
05	B	**1**

06.1	**Explain why digital computers prefer to work in binary numbers.** **2 marks** for clear explanation, **1 mark** for partial explanation, eg • Binary numbers have only two values, zero or one (1 mark) • These are the same as on or off **switches** (1 mark) • Binary numbers represent true/false or on/off (1 mark) • The logic gates are like switches making binary numbers suitable for computer logic to use easily (2 marks) • Cost/efficiency of implementation (eg much cheaper circuitry) • Technical explanations (eg in terms of electronics/voltage) **Do not allow:** easier to read, more efficient, faster etc, without explanation why.	**2**

06.2	**Convert the following binary number into two's complement form.** **11010010** **1 mark** for two's complement: Inverting all bits and adding a 1 yields **00101110**	**1**

07.1	**Convert the following signed 8-bit binary number to its decimal equivalent.** **10000111** **1 mark** for converting: Most significant bit 1 indicates a negative number: **-128 + 7 = -121**	**1**

07.2	**Convert the following signed 8-bit binary number to its decimal equivalent.** **00000111** **1 mark** for converting: Most significant bit is 0 so a positive number: **+7**	**1**

08

Work out what fault condition is indicated in the above value.

1 mark for recognising the process of conversion.

1 mark for converting from hexadecimal number to binary:

0xF2E5 = $1111\ 0010\ 1110\ 0101_2$

1 mark for interpreting the error:

Bit 4 = 0, bit 5 = 1 therefore 01 is CPU error

3

09.1

Fill in the truth table below for the even parity logic.

1 mark for completing the truth table:

P	Q		R
0	0		0
0	1		1
1	0		1
1	1		0

1

09.2

Write down the corresponding logic equation.

1 mark for identifying one of the cases for R = 1
1 mark for identifying the other case and the relationship between them.

Taking into account only the cases where R = 1 the equation is:

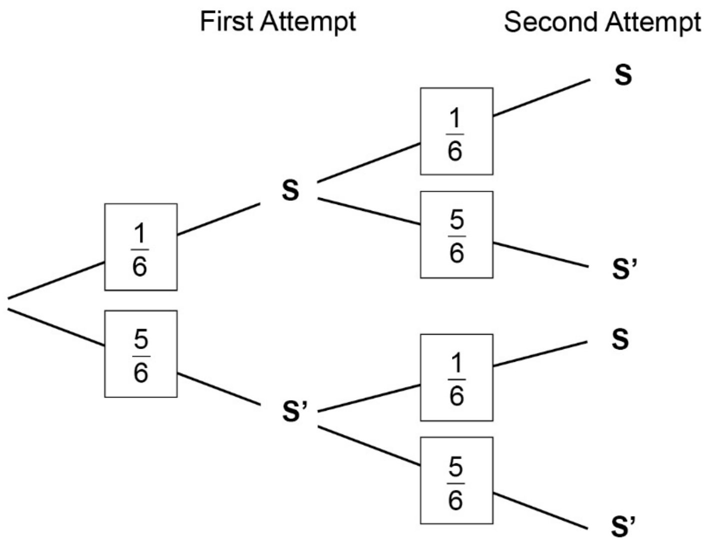
R = P’. Q + P. Q’ (Note: here P’ = NOT P)

$$\mathbf{R = (\overline{P . Q}) . (P + Q)}$$

Allow **1 mark** only for:
R = P XOR Q

10.1	**Write down the logical operations needed on register R in order to turn on only LED0 and LED5.** **1 mark** for the right logical operator **OR**. **1 mark** for the correct bit pattern (accept: without leading zeros; a description). To turn only LED0 and LED5 without changing any of the other bits in R the logical operator to use is the OR operator. The bit pattern requires bits 0 and 5 to be 1 and all other bits 0: **R = R OR** $\mathbf{00100001_2}$	**2**

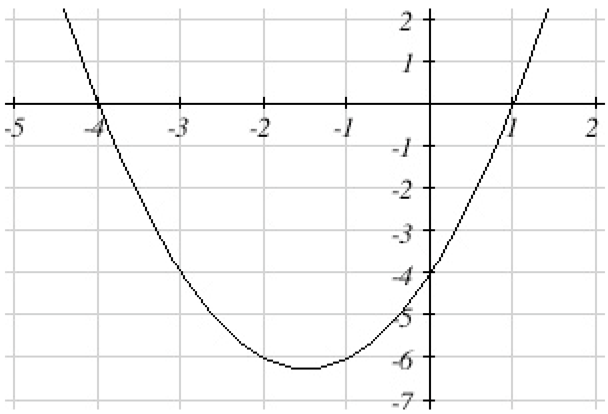
10.2	**Write down the logical operations needed on register R in order to turn off only LED5.** **1 mark** for the right logical operator **AND**. **1 mark** for the correct bit pattern (accept: description or binary pattern, as below). To turn only bit 5 off without affecting any other LED requires the use of an AND operator. We require to set bit 5 of R to 0 without affecting any other bits of R: **R = R AND** $\mathbf{11011111_2}$	**2**

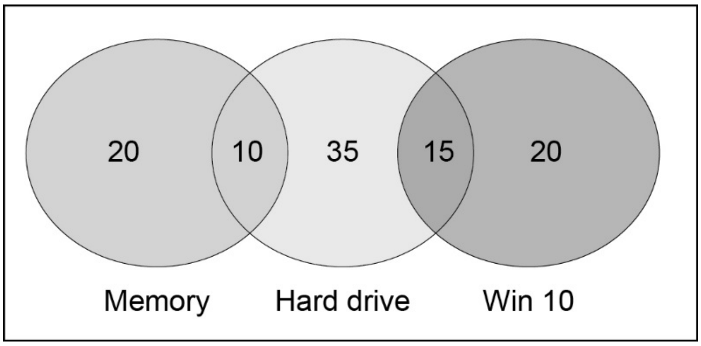
11.1	**Write the probabilities in each of the boxes in the probability tree diagram below.** **1 mark** for identifying the probabilities for each attempt **(max 2 marks)**.  The probability of printing a six is $\frac{1}{6}$ and the probability of printing a number other than a six is $\frac{5}{6}$. Allow: cumulative second attempt, ie 1/36, 5/36, 5/36, 25/36.	**2**

11.2	**The software routine is run twice. What is the probability of printing at least one 6?** **3 marks** for calculating the probability **OR** **1 mark** for correct outcomes (including first three 'cumulative' examples in 11.1) **1 mark** for showing process of adding together three outcomes. The outcomes where one or more sixes are printed in two attempts are **(S,S)** $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$, **(S,S')** $= \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$ and **(S',S)** $= \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$. The resulting probability is therefore the sum of all three $= \frac{1}{36} + \frac{5}{36} + \frac{5}{36} = \frac{11}{36}$. **Allow:** other methods.	**3**

12.1	**Simplify the following algebraic expression.** $\frac{x^2-4}{3} \times \frac{6x}{2(x-2)}$ **1 mark** for simplification (allow numerator of $x^3 - 4x$): $\frac{x^2-4}{1} \times \frac{x}{(x-2)} = \frac{\boldsymbol{x(x^2-4)}}{\boldsymbol{x-2}}$ **1 mark** for further simplification by factorisation yields: $\frac{x(x^2-4)}{x-2} = \frac{x(x-2)(x+2)}{x-2} = \boldsymbol{x(x+2)}$ Allow: $\mathbf{x^2 + 2x}$ **Allow:** other methods (eg factorisation first).	**2**

12.2	**Solve the following equation by factorisation method.** $x^2 + 3x - 4 = 0$ **1 mark** for factorising correctly. Factorising yields: $(x+4)(x-1) = 0$ **1 mark** for identifying the solution for x. Finding the solution: $x + 4 = 0$ and $x - 1 = 0$ Therefore the two values of x are **-4** and **+1** as a solution.	**2**

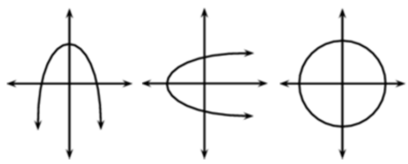
12.3	**Make a rough sketch of the graph for the equation in 12.2.** **1 mark** for sketching the graph. When the graph is drawn on x and y axis then: • the graph will intersect the x axis at -4 and +1 where y is 0 • the graph will also intersect the y axis at -4 where x is 0. 	**1**

13.1	**Draw the Venn diagram that represents the above observation.** **2 marks** for accurate Venn diagram OR **1 mark** for identification of the two intersections. The Venn diagram is shown below. The two intersections include 10 and 15 computers.  	**2**

13.2	**Use the Venn diagram to work out how many computers had 2 TB of hard drive, no 4 GB of memory and no Windows 10 installed.** **1 mark** for the correct calculation. From the Venn diagram it can be seen that the number of computers with 2 TB drives and no 4GB memory and no Windows 10 installed is $60 - 10 - 15 = \mathbf{35}$	**1**

14.1	**Functions can have one-to-one or many-to-one relationships. Explain what is meant by this statement.** **1 mark** for an explanation along the following lines: A single input value to a function can yield a single output value. More than one input value to a function can yield a single (same) output value.	**1**

14.2	**A function cannot have a one-to-many relationship. Explain why this is the case.** **1 mark** for correct explanation along the following lines: • A single input value to a function cannot yield more than one output value. • It can only have one output.	**1**

14.3	**Identify which one of the following is function and explain why.** **A** **B** **C**  **1 mark** for identifying the graph. **1 mark** for an explanation of why it is a function. **Graph A** is the correct answer. If a vertical line parallel to the y axis is drawn it will cut the graph in only one point giving a single solution (1 mark). This is not the case with the other two graphs as there will be two solutions (1 mark) to the same input and this violates the definition of a function.	**2**

14.4	**Given this information expand the following function $\boldsymbol{g(f(x))}$** **1 mark** for correct substitution. **1 mark** for correct expanding. **1 mark** for simplification. Substituting function f in function g we get: $f(g(x)) = 4(x+1)^2 + 8(x+1) - 7$ $= 4(x^2 + 2x + 1) + 8x + 8 - 7$ $= 4x^2 + 8x + 4 + 8x + 1$ $= \mathbf{4x^2 + 16x + 5}$	**3**

15.1	**What is meant by an 'x-by-y' matrix?** **1 mark** for stating: x stands for the number of rows of the matrix. y stands for the number of columns of the matrix. **Allow:** column matrix.	**1**

15.2	**What is a square matrix?** **1 mark** for stating: Square matrix has an equal number of rows and columns. **Allow:** example.	**1**

15.3	**What is the required relationship between x1, y1, x2 and y2 in order to allow addition of the two matrices?** **1 mark** for stating the relationship: To be able to add two matrices we need the two matrices to have the same number of rows and columns, ie **x1 = x2 and y1 = y2**.	**1**

15.4	**What is the required relationship between x1, y1, x2 and y2 in order to allow multiplication of the two matrices?** **1 mark** for stating the relationship: To be able to multiply the two matrices we need the number of columns of the first matrix to be the same as the number of rows of the second matrix, ie **y1 = x2**.	**1**

16.1	**Describe the difference between a sequence and a series.** **1 mark** for a description along the lines below for each of: A sequence is an ordered list of numbers that possess a mathematical relationship (eg formula/pattern) with their neighbouring numbers. A series is the sum of the list of numbers in a sequence.	**2**

16.2	**A sequence has the general formula $\boldsymbol{a_n = 2n + 3}$. Write down the first five numbers of this sequence where** $n >= 1$ **1 mark** for writing down the sequence. Substituting 1 to 5 in place of n in the formula we get: 5, 7, 9, 11, 13.	**1**

16.3	**Consider the following first four numbers of a sequence 4, 9, 16, 25. Write down the nth term.** **1 mark** for working out the nth term of the sequence. Observing the sequence and how the numbers are related to each other we deduce the pattern $4 = 2^2, 9 = 3^2, 16 = 4^2$ and $25 = 5^2$. This pattern suggests that the nth term is given by $\boldsymbol{(n + 1)^2}$.	**1**

16.4	**Explain what the following series notation represents.** **1 mark** for summation or sigma and **1 mark** for description or numeric representation of sequence. This notation is the summation notation. In this case the sum of all terms with 2 to the power of i where the value of i changes from 0 to 7 is calculated as follows: **Sum** $\boldsymbol{= 2^0 + 2^1 + 2^2 + 2^3 +}$ **etc**.	**2**

17.1	**The designers need to map out the relationship between the inputs and the output. This requires a truth table to be constructed. Help them out by filling in the following truth table from the above logic.** **1 mark** for deriving the correct outputs through the logic. **1 mark** for correctly filling in all combinations of inputs.	2

A	B	C		P
0	0	0		1
0	0	1		0
0	1	0		0
0	1	1		0
1	0	0		1
1	0	1		1
1	1	0		0
1	1	1		0

17.2	**Now that the truth table is done, write down the logic equation from this truth table.** **1 mark** for writing the equation P= as the sum of three products. **1 mark** (max 3 marks) for each accurate product. The logic equation is therefore: **P = A'B'C' + A B'C' + A B'C** (Note: A' = NOT A)	4

17.3

Next, the designers wish to simplify their initial logic. They do this by using the Karnaugh map method. Fill in the following Karnaugh map (K-map) by referring to the truth table in Question 17.1.

1 mark for filling in the K-map (based on 17.1). Allow without 0s.

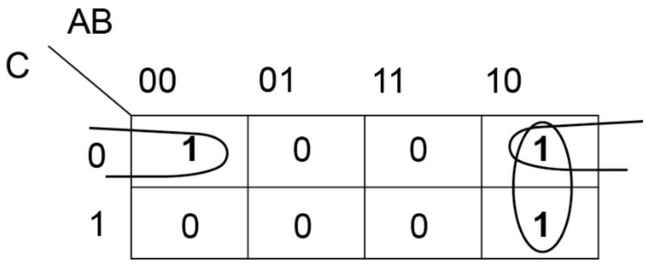
C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	1

Marks: **1**

17.4

Identify the relevant adjacent cells for simplification.

1 mark for identifying the relevant adjacent cells for simplification, ie 100, 101; 100, 000.



Marks: **1**

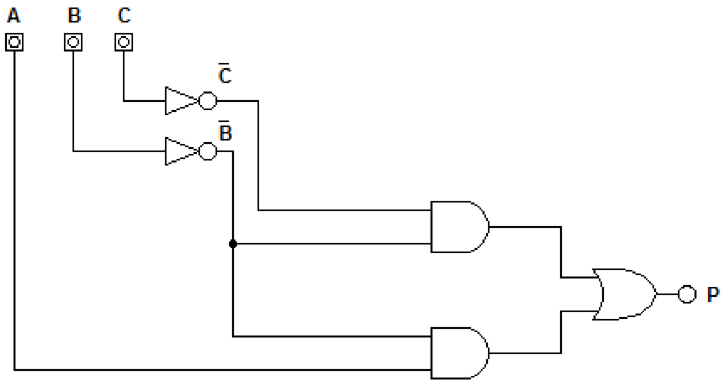
17.5

Write down the simplified logic equation using the information in the K-map in 17.3. You need to explain how you arrived at the simplified equation.

1 mark for working out the two simplifications.
1 mark for writing the equation as the sum of two products.
1 mark for explanation.

The cells under AB = 10 indicate that the logical value of C changes but those of AB remain the same. This simplification yields AB'. The cells C=0 and AB=10 show that B and C logic levels do not change. This therefore yields B'C'. The simplified logic is **P = B'C' + A B'**.

Marks: **3**

17.6	**Draw a rough sketch of the simplified logic diagram in the space below.** **1 mark** for producing inverted inputs. **1 mark** for connecting the inputs and the outputs to the appropriate logic gates. The simplified logic includes two inverters, two AND gates (with only two inputs each) and an OR gate (with two inputs). This is obviously less complex than the original design. 	**2**

17.7	**State two benefits of the simplified logic.** **1 mark** each for the two benefits (**max 2 marks**), eg: • One less AND logic gate • One less inverter (visible in the logic circuit) • Less wiring (visible in the logic circuit) • Lower cost (due to less components) • Less power requirement • Lower temperature (due to lower power) • Clearer to see what inputs will give you outputs • Clearer what is happening at each step.	**2**

18.1	**Derive the simultaneous equations in terms of n1 and n2 from the above information.** **1 mark** for each simultaneous equation (**max 2 marks**) From CPU A and millions both sides of the equations cancel each other **3n1 + 4n2 = 10** From CPU B and similarly millions both sides of the equations cancel each other **2n1 + 3n2 = 7**	**2**

18.2	**Using matrix method solve the simultaneous equations to reveal the values of n1 and n2.** **1 mark** for expressing the simultaneous equations in matrix form. **Do not penalise further** error carried through from 18.1 if answer in 18.1 can be expressed as matrix. $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} n1 \\ n2 \end{pmatrix} = \begin{pmatrix} 10 \\ 7 \end{pmatrix}$ **2 marks** for finding the inverse matrix **or 1 mark** for stating inverse matrix is required. Need to derive the inverse matrix first. To do this we need to find the difference between the product of the elements of the main diagonal and the product of the elements of its subsidiary diagonal of the matrix. Therefore for $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ this is (3 x 3) – (4 x 2) = 1. As a result we can write the inverse matrix by using the rule "interchange the elements of the main diagonal and change the signs of the subsidiary diagonal" ie inverse of $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ is $\begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$ **4 marks** for the rest of the solution (do not penalise further if inverse matrix is incorrect, ignore carry through of error if working correct). If answer is correct: **1 mark** for stating or showing method. **3 marks** for correct answer. If answer is incorrect: **1 mark** for stating or showing method. **1 mark** if working is shown and one error carried through. First pre-multiply each side of the equation by the above matrix (**1 mark** for stating or showing method): $\begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} n1 \\ n2 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 10 \\ 7 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} n1 \\ n2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\begin{pmatrix} n1 \\ n2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ Therefore from the above equation **n1 = 2 million** instructions and **n2 = 1 million** instructions (**3 marks**). Unit not required.	**7**

18.3	**If CPU A takes 80 milliseconds to run then calculate the speed of CPU A.** **1 mark** for re-arranging the formula. Re-arranging the formula: S = $\frac{C}{T}$ Hz **1 mark** for substituting known values and **1 mark** for calculating the speed of CPU A (accept Hz or MHz). Therefore substituting the values: S = $\frac{10 \times 10^6}{80 \times 10^{-3}}$ Hz = $\frac{10^9}{8}$ Hz = 125 MHz	**3**

18.4	**Calculate the time it takes to run instructions on CPU B running at the same speed as CPU A.** **1 mark** for using the formula: T = $\frac{C}{S}$ seconds **1 mark** for substituting relevant values for CPU B into numerator (C) and substituting answer for 18.3 into denominator (S), ie 7/125. **1 mark** for correct answer **based on answer given for 18.3.** T = $\frac{7 \times 10^6}{125 \times 10^6}$ seconds = $\frac{7 \times 10^3}{125}$ milliseconds = 56 milliseconds or 0.056 seconds	**3**

Assessment Outcomes						
Question	**AO1**	**AO2**	**AO3**	**AO4**	**AO5**	**Question Total**
Section A						
1	1a(1)					1
2	1c(1)					1
3		2a(1)				1
4				4d(1)		1
5	1b (1)					1
6	1c(3)					3
7	1b(2)					2
8	1c(3)					3
9		2c(3)				3
10		2b(4)				4
11			3c(5)			5
12				4d(5)		5
13		2b(3)				3
14				4a(7)		7
15					5a(4)	4
16			3b(6)			6
Section B						
17		2acd(15)				15
18				4d(4)	5bc(11)	15
Totals	11	26	12	16	15	80