Level 3 Technical Level

IT: PROGRAMMING

Unit 5 Mathematics for programmers

| Wednesday 17 January 2018 | Morning | Time allowed: 2 hours |

Materials
For this paper you must have:
- simple drawing instruments
- scientific calculator (non-programmable).

Instructions
- Use black ink or black ball-point pen.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.
- If you need more space use the additional pages at the back of this booklet.
- Include units in all answers, where required, as marks are given for units in some questions.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80. There are 50 marks for **Section A** and 30 marks for **Section B**.
- Both sections should be attempted.
- In all calculations, show clearly how you work out your answer.
- Use diagrams, where appropriate, to clarify your answers.
- You are expected to use a calculator where appropriate.
- You are reminded of the need for good English and clear presentation in your answers.

Advice
Do not spend too long on one question. Read all questions thoroughly before starting your answer. Show all working in the spaces provided.

Section A

Answer **all** questions in this section.

In the multiple choice questions, only **one** answer per question is allowed.

For each question completely fill in the circle alongside the appropriate answer.

CORRECT METHOD ⬛ WRONG METHODS ⊗ ⊙ ⬭ ⊘

If you want to change your answer you must cross out your original answer as shown. ⊠

If you wish to return to an answer previously crossed out, ring the answer you now wish to select as shown. ⊗

| 0 | 1 | Which **one** of the following is a hexadecimal number?

A 37002_8 ⬭

B $75a9f0_{16}$ ⬭

C 101110_2 ⬭

D 4203_5 ⬭

[1 mark]

1

| 0 | 2 | What is the maximum number that can be represented by eight binary bits?

A 255 ⬭

B 256 ⬭

C 127 ⬭

D 128 ⬭

[1 mark]

1

Do not write outside the box

0 3 Which is the correct description for the following arithmetic expression?

+ * 3 2 7

A Infix expression. ⬭

B Postfix expression. ⬭

C Prefix expression. ⬭

D Leftfix expression. ⬭

[1 mark]

1

0 4 Which **one** below accurately describes the operation of a 2-input and 1-output AND logic gate?

A The output is a 1 only if both inputs are 1. ⬭

B The output is a 1 as long as one of the inputs is a 1. ⬭

C The output is a 1 as long as one of the inputs is a 0. ⬭

D The output is a 1 only if both inputs are 0. ⬭

[1 mark]

1

0 5 The correct answer to the expression $2E_{16} + 12_{16}$ is

A 39_{16} ⬭

B 40_{16} ⬭

C 41_{16} ⬭

D $3E_{16}$ ⬭

[1 mark]

1

Turn over ▶

| 0 | 6 | . | 1 | Explain why digital computers prefer to work in binary numbers.

[2 marks]

_____

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_____

_____

| 0 | 6 | . | 2 | Convert the following binary number into two's complement form.

11010010

[1 mark]

_____

_____

<div align="right">

| 3 |

</div>

| 0 | 7 | . | 1 | Convert the following **signed** 8-bit binary number to its decimal equivalent.

10000111

[1 mark]

_____

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| 0 | 7 | . | 2 | Convert the following **signed** 8-bit binary number to its decimal equivalent.

00000111

[1 mark]

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| 2 |

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Do not write outside the box

| 0 | 8 |

A 16-bit diagnostic data area in a printer's memory contains the following value

15   0 ⟵ bit number

```
            0xF2E5
```

Values of bits 4 and 5 of this data indicate one of the fault conditions as listed in the table below.

bit 4	bit 5	Printer fault
0	0	No error
0	1	CPU error
1	0	Memory error
1	1	Unknown error

Work out what fault condition is indicated in the above value.

[3 marks]

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| |
| 3 |

Turn over for the next question

Turn over ▶

| 0 | 9 | Even parity checking logic with 2 inputs and 1 output has the following characteristics.

If both inputs **P** and **Q** are 1 then the output **R** is a 0.
If both inputs **P** and **Q** are 0 then the output **R** is a 0.
Otherwise the output is a 1.

| 0 | 9 | . | 1 | Fill in the truth table below for the even parity logic.

[1 mark]

P	Q	R

| 0 | 9 | . | 2 | Write down the corresponding logic equation.

[2 marks]

_____

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_____

3

| 1 | 0 |

In an electronic display, eight LEDs (Light Emitting Diodes) are controlled by an 8-bit register **R**.

Setting bits 0 to 7 of **R** to 1 will **turn on** LED0 to LED7.
Setting bits 0 to 7 of **R** to 0 will **turn off** LED0 to LED7.

For example, if **R** = 10011000 then LEDs 3, 4 and 7 are **on** and LEDs 0, 1, 2, 5 and 6 are **off**.

Assume that initially all LEDs are **off**.

Write down the logical operations needed on register **R** in order to

| 1 | 0 | . | 1 |

turn on **only** LED0 and LED5.

[2 marks]

_____

_____

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_____

| 1 | 0 | . | 2 |

turn off **only** LED5.

[2 marks]

_____

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_____

_____

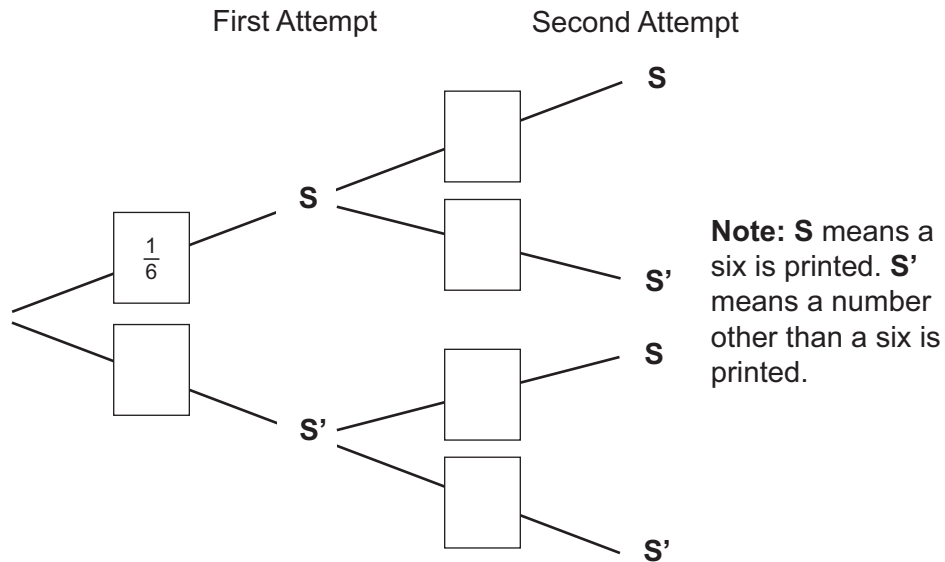| |
| 4 |

Turn over for the next question

Turn over ▶

1 1 A software routine can print random whole numbers in the range 1 to 6. We wish to find the probability of this routine printing one or more sixes in two successive attempts.

1 1 . 1 Write the probabilities in each of the boxes in the probability tree diagram below.

[2 marks]

First Attempt   Second Attempt

☐ — S

S

$\frac{1}{6}$ ☐ — S'

☐   **Note: S** means a six is printed. **S'** means a number other than a six is printed.

☐ — S

S'

☐ — S'

1 1 . 2 The software routine is run twice. What is the probability of printing at least one 6?

[3 marks]

5

1 2 . **1** Simplify the following algebraic expression.

$$\frac{x^2 - 4}{3} \times \frac{6x}{2(x-2)}$$

[2 marks]

1 2 . **2** Solve the following equation by factorisation method.

$$x^2 + 3x - 4 = 0$$

[2 marks]

1 2 . **3** Make a rough sketch of the graph for the equation in **1 2** . **2**

[1 mark]

5

Turn over ▶

| 1 | 3 |

In a random sample of 100 computers at a school the following were observed.

30 had 4 GB of memory.
60 had 2 TB of hard drive.
35 had Windows 10 installed.
10 had 4 GB of memory and 2 TB of hard drive.
15 had 2 TB of hard drive and Windows 10 installed.

| 1 | 3 | . | 1 |

Draw the Venn diagram that represents the above observation.

[2 marks]

| 1 | 3 | . | 2 |

Use the Venn diagram to work out how many computers had 2 TB of hard drive, **no** 4 GB of memory and **no** Windows 10 installed.

[1 mark]

3

1 4 . 1 Functions can have **one-to-one** or **many-to-one** relationships.

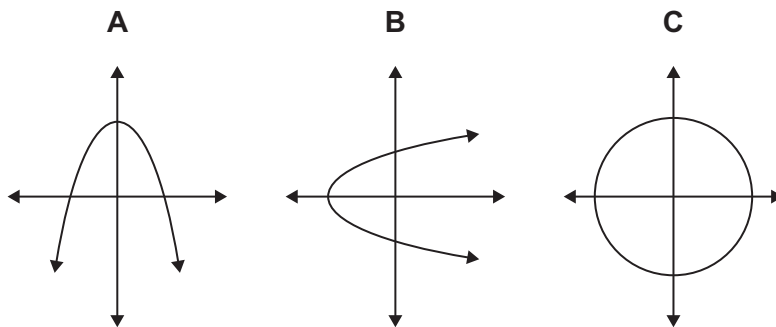Explain what is meant by this statement.

[1 mark]

_____

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1 4 . 2 A function cannot have a **one-to-many** relationship.

Explain why this is the case.

[1 mark]

_____

_____

1 4 . 3 Identify which **one** of the following is a function and explain why.

[2 marks]

A   **B**   **C**

_____

_____

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1 4 . 4 Two functions g and f are defined as below.

$g(x) = 4x^2 + 8x - 7$

$f(x) = x + 1$

Given this information expand the following function:

$g(f(x))$

[3 marks]

7

1 5 . 1 What is meant by an '*x*-by-*y*' matrix?

[1 mark]

_____

_____

1 5 . 2 What is a square matrix?

[1 mark]

_____

_____

A is an '*x*1-by-*y*1' matrix and **B** is an '*x*2-by-*y*2' matrix.

What is the required relationship between *x*1, *y*1, *x*2 and *y*2 in order to allow:

1 5 . 3 addition of the two matrices?

[1 mark]

_____

_____

1 5 . 4 multiplication of the two matrices?

[1 mark]

_____

_____

4

Turn over for the next question

Turn over ▶

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16 . 1 Describe the difference between a sequence and a series.

[2 marks]

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16 . 2 A sequence has the general formula $a_n = 2n + 3$.

Write down the first **five** numbers of this sequence where $n >= 1$.

[1 mark]

_____

_____

16 . 3 Consider the following first **four** numbers of a sequence.

4, 9, 16, 25

Write down the nth term.

[1 mark]

_____

_____

16 . 4 Explain what the following series notation represents.

$$\sum_{i=0}^{i=7} 2^i$$

[2 marks]

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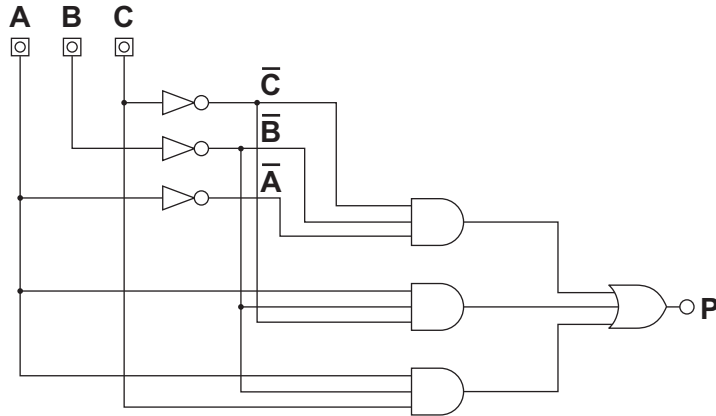_____

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6

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Section B

Answer **both** questions in this section.

| 1 | 7 | Designers of an electronic device initially came up with the following combinational logic. It has three inputs **A**, **B**, **C** and a single output **P**. It includes three inverter gates (ie **NOT** gates) that produce inverted versions of the three inputs, three **AND** gates and a single **OR** gate as shown below.

| 1 | 7 | . | 1 | The designers need to map out the relationship between the inputs and the output. This requires a truth table to be constructed. Help them out by filling in the following truth table from the above logic.

A	B	C	P

[2 marks]

Turn over ▶

17 . 2 Now that the truth table is done, write down the logic equation from this truth table.

[4 marks]

17 . 3 Next, the designers wish to simplify their initial logic. They do this by using the Karnaugh map method.

Fill in the following Karnaugh map (K-map) by referring to the truth table in **17 . 1**

AB

C	00	01	11	10
0				
1				

[1 mark]

17 . 4 Identify the relevant adjacent cells for simplification.

[1 mark]

G/Jan18/Y/507/6469

1 7 . 5 Write down the simplified logic equation using the information in the K-map in **1 7 . 3** . You need to explain how you arrived at the simplified equation.

[3 marks]

1 7 . 6 Draw a rough sketch of the simplified logic diagram in the space below.

[2 marks]

1 7 . 7 State **two** benefits of the simplified logic.

[2 marks]

15

Turn over ▶

| 1 | 8 | | Engineers measured the performance of two different CPUs, **CPU A** and **CPU B**.

They ran two types of instructions, $n1$ and $n2$, on both CPUs.

They recorded the total number of CPU clock cycles it took to complete each run.

Below are their results:

CPU A:
Executed $n1$ million instructions at 3 clock cycles/instruction.
Executed $n2$ million instructions at 4 clock cycles/instruction.
Total CPU clock cycles needed was 10 million.

CPU B:
Executed $n1$ million instructions at 2 clock cycles/instruction.
Executed $n2$ million instructions at 3 clock cycles/instruction.
Total CPU clock cycles needed was 7 million.

| 1 | 8 | . | 1 | Derive the simultaneous equations in terms of $n1$ and $n2$ from the above information.

[2 marks]

| 1 | 8 | . | 2 | Using **matrix** method solve the simultaneous equations to reveal the values of $n1$ and $n2$.

[7 marks]

The time it takes to run instructions on a CPU is given by the formula:

$$T = \frac{C}{S}$$

Here, C is the number of CPU clock cycles and S is the speed of CPU in Hertz (Hz); T is in seconds.

Refer back to the CPU results table at the start of | 1 | 8 |

| 1 | 8 | . | 3 | If **CPU A** takes 80 milliseconds to run then calculate the speed of **CPU A**.

[3 marks]

| 1 | 8 | . | 4 | Calculate the time it takes to run instructions on **CPU B** running at the same speed as **CPU A**.

[3 marks]

15

END OF QUESTIONS

If needed, use the following page to continue your answers. Write the question number beside your answer.

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