Level 3 Technical Level
DESIGN ENGINEERING
MECHATRONIC ENGINEERING
J/506/5953

Unit 3: Mathematics for engineers

Mark scheme
January 2018

Version: 1.0 Final

181AJ/506/5953/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Level of response marking instructions

Level of response mark schemes are broken down into levels, each of which has a descriptor. The descriptor for the level shows the average performance for the level. There are marks in each level.

Before you apply the mark scheme to a student's answer read through the answer and annotate it (as instructed) to show the qualities that are being looked for. You can then apply the mark scheme.

Step 1 Determine a level

Start at the lowest level of the mark scheme and use it as a ladder to see whether the answer meets the descriptor for that level. The descriptor for the level indicates the different qualities that might be seen in the student's answer for that level. If it meets the lowest level then go to the next one and decide if it meets this level, and so on, until you have a match between the level descriptor and the answer. With practice and familiarity you will find that for better answers you will be able to quickly skip through the lower levels of the mark scheme.

When assigning a level you should look at the overall quality of the answer and not look to pick holes in small and specific parts of the answer where the student has not performed quite as well as the rest. If the answer covers different aspects of different levels of the mark scheme you should use a best fit approach for defining the level and then use the variability of the response to help decide the mark within the level, ie if the response is predominantly level 3 with a small amount of level 4 material it would be placed in level 3 but be awarded a mark near the top of the level because of the level 4 content.

Step 2 Determine a mark

Once you have assigned a level you need to decide on the mark. The descriptors on how to allocate marks can help with this. The exemplar materials used during standardisation will help. There will be an answer in the standardising materials which will correspond with each level of the mark scheme. This answer will have been awarded a mark by the Lead Examiner. You can compare the student's answer with the example to determine if it is the same standard, better or worse than the example. You can then use this to allocate a mark for the answer based on the Lead Examiner's mark on the example.

You may well need to read back through the answer as you apply the mark scheme to clarify points and assure yourself that the level and the mark are appropriate.

Indicative content in the mark scheme is provided as a guide for examiners. It is not intended to be exhaustive and you must credit other valid points. Students do not have to cover all of the points mentioned in the Indicative content to reach the highest level of the mark scheme.

An answer which contains nothing of relevance to the question must be awarded no marks.

Question	Marking Guidance	Marks
01.1	**Determine the volume of one bucket in m^3. Show your answer in engineering notation.** $Volume = \frac{1}{3}\pi h(R^2 + Rr + r^2)$ $Volume = \frac{1}{3} \times \pi \times 0.275(0.14^2 + (0.14 \times 0.1) + 0.1^2)$ Volume $= 12.6 \times 10^{-3}\ m^3$. **1 mark** for the correct values. **1 mark** for the correct calculation. **1 mark** for the correct answer in m^3 **1 mark** for the correct answer in engineering notation.	**4**
01.2	**Determine the surface area of the steel necessary to manufacture the 100 buckets. Your answer must be in standard SI units and in engineering notation.** Surface area of 1 bucket $= \pi s(R + r) + \pi r^2$ $s = \sqrt{(R - r)^2 + h^2}$ inserting the values, we have: $s = \sqrt{(0.14 - 0.1)^2 + 0.275^2} = 0.277893 \ldots . m$ Now, using the surface area formula, we have: $Surface\ area = \pi s(R + r) + \pi r^2$ inserting the values, we have: $Surface\ area = \pi s(R + r) + \pi r^2 = \ \pi \times 0.277893 \ldots (0.14 + 0.1) + \pi \times 0.1^2$ Surface area $= 0.241\ m^2 or\ 241 \times 10^{-3}\ m^2$. Surface area for 100 buckets $= 24.1\ m^2$. **1 mark** for the correct formula for *s* (correct use of Pythagoras). **1 mark** for correct use of $(R - r)^2$ **1 mark** for the correct answer. **1 mark** for the correct values in surface area formula. **1 mark** for the correct answer in engineering notation.	**7**
	Total marks for Question 1	**11**

Question	Marking Guidance	Marks
02.1	**Using the logarithm laws, or otherwise, find the value of the following:** $log_3 9$ $log_3\ 9$ therefore, $3^x = 9$ and $3^2 = 3^x = 9$ Therefore, $x = 2$. **1 mark** for $3^x = 9$. **1 mark** for correct answer. **Or** other suitable response. Must show some working – don't accept the answer only.	**2**
02.2	**Using the logarithm laws, or otherwise, find the value of the following:** $log_{16} 8$ $log_{16}\ 8$ therefore, $16^x = 8$ and $(2^4)^x = 8$ then $(2^4)^x = 2^3$ From the laws of indices, we have: $4x = 3$ therefore $= \frac{3}{4}$. **1 mark** for $(2^4)^x = 8$. **1 mark** for $4x = 3$. **1 mark** for final answer. Allow credit for using the laws of logs. **Or** other suitable response Must show some working – don't accept the answer only.	**3**
02.3	**Using the logarithm laws, or otherwise, find the value of the following:** $log_3 \frac{1}{81}$ $log_3\ \frac{1}{81}$ therefore, $3^x = \frac{1}{81}$ or $x\ log\ 3 = log\left(\frac{1}{81}\right)$ then we have: $x = \frac{log\left(\frac{1}{81}\right)}{log\ 3} = -4.$ **1 mark** for $x\ log\ 3 = log(1/81)$ **1 mark** for $x = log(1/81)/log 3$ **1 mark** for final answer. **Or** other suitable response Must show some working – don't accept the answer only.	**3**

02.4	**Determine the value of v.** $log(v-1)+log(v+1)=log(v-1)(v+1)=log(v^2-1)$ and $2log(v+2)=log(v+2)^2=log(v^2+4v+4)$ $log(v^2-1)=\ log(v^2+4v+4)$ or $v^2-1=v^2+4v+4$ then: $-1=4v+4$ thus: $v=-\frac{5}{4}$.or -1.25 **1 mark** for each simplification **2 marks in total**. **1 mark** for correctly removing brackets. **1 mark** for correct transposition. **1 mark** for correct value of v. Must show some working – don't accept the answer only.	**5**

Total marks for Question 2	**13**

Question	Marking Guidance	Marks
03.1	Express 150^0 in radians, as a fraction multiple of π: $150^0 \times \left(\frac{\pi}{180^0}\right) = \frac{150\pi}{180} = \frac{5\pi}{6}$ Radians. **1 mark** for method. **1 mark** for correct answer.	**2**
03.2	Express 37.5^0 in radians, as a fraction multiple of π: $2 \times 37.5^0 \times \left(\frac{\pi}{360^0}\right) = \frac{75\pi}{360} = \frac{5\pi}{24}$ Radians. **1 mark** for method. **1 mark** for correct answer.	**2**
03.3	Express $\frac{3\pi}{4}$ radians in degrees: $\left(\frac{3\pi}{4}\right) \times \left(\frac{180^0}{\pi}\right) = \frac{540^\circ}{4} = 135^\circ$ **1 mark** for method. **1 mark** for correct answer.	**2**
03.4	Express $\frac{5\pi}{2}$ radians in degrees: $\left(\frac{5\pi}{2}\right) \times \left(\frac{180^0}{\pi}\right) = \frac{900^\circ}{2} = 450^\circ$ **1 mark** for method. **1 mark** for correct answer.	**2**
	Total marks for Question 3	**8**

Question	Marking Guidance	Marks
4	**Determine the resultant vector of the two acceleration vectors in Figure 2.** Using vector addition: $25\ ms^{-2} - 16\ ms^{-2} = 9ms^{-2}$. Therefore, we have: $\mathbf{9\ ms^{-2}}$ →	**2**
	Total marks for Question 4	**2**

Question	Marking Guidance	Marks
05.1	**Determine the mean resistance of the sample.** $\bar{\Omega} = \frac{\sum \Omega}{n} = \frac{99+100+100+102+97+98+99+101+97+99+102+101+98+99+100+101+99+97+99+100}{20} = 99.4\,\Omega$ **1 mark** for the correct values. **1 mark** for the answer. Must show some working – don't accept the answer only.	**2**
05.2	**Determine the standard deviation of the sample.** The standard deviation can be determined by: $\sigma = \sqrt{\left\{\frac{\sum(x-\bar{x})^2}{n}\right\}}$ $\sigma = \left\{\frac{(-0.4)^2+(0.6)^2+(0.6)^2+(2.6)^2(-2.4)^2+(-1.4)^2+(-0.4)^2+(1.6)^2+(-2.4)^2+(-0.4)^2+(2.6)^2+(1.6)^2+(-1.4)^2+(-0.4)^2+(0.6)^2+(1.6)^2+(-0.4)^2+(-2.4)^2+(-0.4)^2+(0.6)^2}{20}\right\}^{1/2}$ Therefore, we have: $\sigma = 1.50\,\Omega$. **1 mark** for using the correct formula. **1 mark** for using the correct values. **Allow follow-through from part 5.1 if mean is wrong**. **1 mark** for correct use of negative numbers. **1 mark** for the correct answer.	**4**

05.3	**Determine the modal value of the sample.** 6 x 99.0Ω = modal value. Therefore, modal value = 99.0Ω. Must show some working – don't accept the answer only.	**2**

Total marks for Question 5	**8**

Question	Marking Guidance	Marks
06	**Using the process of integration, determine the component's area between the limits of 0 to 4.712 for *L*.** **Your answer must include the correct units and be rounded to the nearest integer.** $H = 5\cos\left(\frac{L}{3}\right)$ $H = 5\cos\left(\frac{L}{3}\right)$ using the correct notation, we have: $\int_0^{4.712} 5\cos\left(\frac{L}{3}\right).dL = \left[\frac{5}{1/3}\sin\left(\frac{L}{3}\right)\right]_0^{4.712}$ and $\left[15\sin\left(\frac{L}{3}\right)\right]_0^{4.712}$ Therefore, this gives us: $\left[15\sin\left(\frac{4.712}{3}\right)\right] - \left[15\sin\left(\frac{0}{3}\right)\right] = 15\ m^2$. **2 mark** for the correct standard integration. **1 mark** for the correct notion using the limits. **2 marks** for the correct use of the limits. **1 mark** for the subtraction. **1 mark** for the correct units. **1 mark** for the correct answer to the nearest integer.	**8**
	Total marks for Question 6	**8**

Question	Marking Guidance	Marks
07	**Determine the value of the resultant position vector AD.** Resolving horizontally: $-20 - 8\cos 40^0 + 10\cos 60^0 = -21.12835555\ m$ Resolving vertically: $0 + 8\sin 40^0 + 10\cos 30^0 = 13.80255492\ m$ Magnitude $= \sqrt{(-21.12835555)^2 + (13.80255492)^2} = 25.237 \ldots = 25.2\ m$ $\theta = tan^{-1}\left(\frac{13.80255492}{-21.12835555}\right) = -33.155 \ldots = -33.1^0$ **NB allow all trigonometric variants for resolving horizontally and vertically. Not just those above.** **3 marks** for resolving each component and their answers **– total 6 marks.** **2 marks** for correctly determining the magnitude. **2 marks** for correctly determining the angle. Also accept $146.9°$	**10**
	Total marks for Question 7	**10**

Question	Marking Guidance	Marks

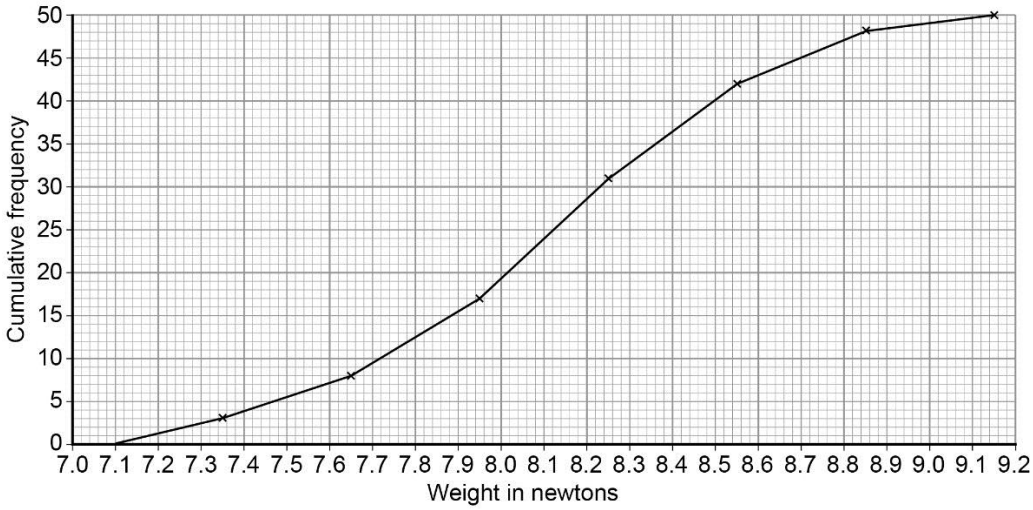
08.1 **Calculate the frequency and cumulative frequencies in the space below.**

For example:

Class	Tally
7.1 to 7.3	111
7.4 to 7.6	~~1111~~
7.7 to 7.9	~~1111~~ 1111
8.0 to 8.2	~~1111~~ ~~1111~~ 111
8.3 to 8.5	~~1111~~ ~~1111~~ 11
8.6 to 8.8	~~1111~~ 1
8.9 to 9.1	11

4

08.2 **Using the upper class boundary, construct the cumulative frequency distribution graph of the sample on Figure 5.**



6 marks for a graph similar to the one above – upper class boundary values should be used. Please note that joining lines must be STRAIGHT – not curved.

6

Total marks for Question 8	**10**

Question	Marking Guidance	Marks
09.1	**Using the process of differentiation, determine the angular velocity of the flywheels when $t = 1\ s$** $\theta = 9t^2 - 2t^3$ and $\frac{d\theta}{dt} = 18t - 6t^2$ is the velocity function. Where t = 1 we have: $\frac{d\theta}{dt} = 18(1) - 6(1^2) = 12\ Rad\ s^{-1}$ **1 mark** for each differential; **2 marks** in total. **1 mark** for the correct answer. **1 mark** for the correct units.	**4**
09.2	**Using the process of differentiation, determine the angular acceleration of the flywheels when $t = 1.2\ s$** $\frac{d\theta}{dt} = 18t - 6t^2$ then we have $\frac{d^2\theta}{dt^2} = 18 - 12t$ is the acceleration function. Where $t = 1.2$ seconds we have: $\frac{d^2\theta}{dt^2} = 18 - 12(1.2) = 3.6\ Rad\ s^{-2}$ **1 mark** for each differential – 2 marks in total. **1 mark** for the correct answer. **1 mark** for the correct units.	**4**
09.3	**Determine the time when the angular acceleration is zero.** $\frac{d^2\theta}{dt^2} = 18 - 12t$ and $0 = 18 - 12t$ therefore, we have: $18 = 12t$ and $t = \frac{18}{12} = 1.5$ seconds. **1 mark** for the correct transposition. **1 mark** for the correct answer with units.	**2**
	Total marks for Question 9	**10**

Assessment outcomes coverage

Assessment Outcomes	Marks and % of marks available in section A	Marks and % of marks available in section B	Total Marks
AO1:	0 marks 0%	30 marks 100%	30 marks
AO2:	11 Marks 22%	0 marks 0%	11 marks
AO3:	13 Marks 26%	0 marks 0%	13 marks
AO4:	10 Marks 20%	0 marks 0%	10 marks
AO5:	8 Marks 16%	0 Marks 0%	8 marks
AO6:	8 Marks 16%	0 Marks 0%	8 marks
Total Marks	**50**	**30**	**80**

Question	AO1	AO2	AO3	AO4	AO5	AO6
1		11				
2			8			
3			5			
4				10		
5					8	
6						8
7	10					
8	10					
9	10					
Totals	**30**	**11**	**13**	**10**	**8**	**8**