Maris Stella High Prelim 2006 Paper 1

NEITHER ELECTRONIC CALCULATORS NOR MATHEMATICAL TABLES MAY BE USED IN THIS PAPER.

1. Evaluate

(a) $60 \div 4 \times 2 - 15$,

(b) $1\frac{2}{3} \div 0.5$.

Answer (a)........................... [1]

(b)........................... [1]

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2. Evaluate the following, expressing each answer in standard form.

(a) $3 \times 7.2 \times 10^7$

(b) $\dfrac{2.1}{8.4 \times 10^6}$

Answer (a)........................... [1]

(b)........................... [1]

---

3. The temperature recorded on a thermometer at time 21 00 was 4°C. At 01 00, the temperature had fallen 9°C.

(i) Write down the temperature at 01 00.

(ii) Calculate the average fall in temperature per hour.

Answer (a) (i)....................°C [1]

(ii)..............°C/hour [1]

---

4. (a) The gradient of the line joining the point $(h,-3)$ and $(7,h)$ is $-\frac{1}{3}$. Calculate the value of h.

(b) Find the coordinates of M, the midpoint of the line segment joining $A(-6,0)$ and $B(12,-4)$.

Answer (a).......................... [2]

(b).......................... [1]

---

5. x and y are integers such that $-9 < x \leq 1$ and $-9 \leq y < -1$. Calculate

(a) the minimum value of xy,

(b) the greatest value of $\frac{x}{y}$.

Answer (a)............................ [1]

(b)............................[1]

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6. Given that $C = \frac{d}{d+r}$,

(a) find the value of C when $d = \frac{1}{3}$ and $r = \frac{7}{9}$,

(b) express r in terms of C and d.

Answer (a) $C =$........................ [1]

(b)............................ [1]

7. A map is drawn to scale of 1:20 000.

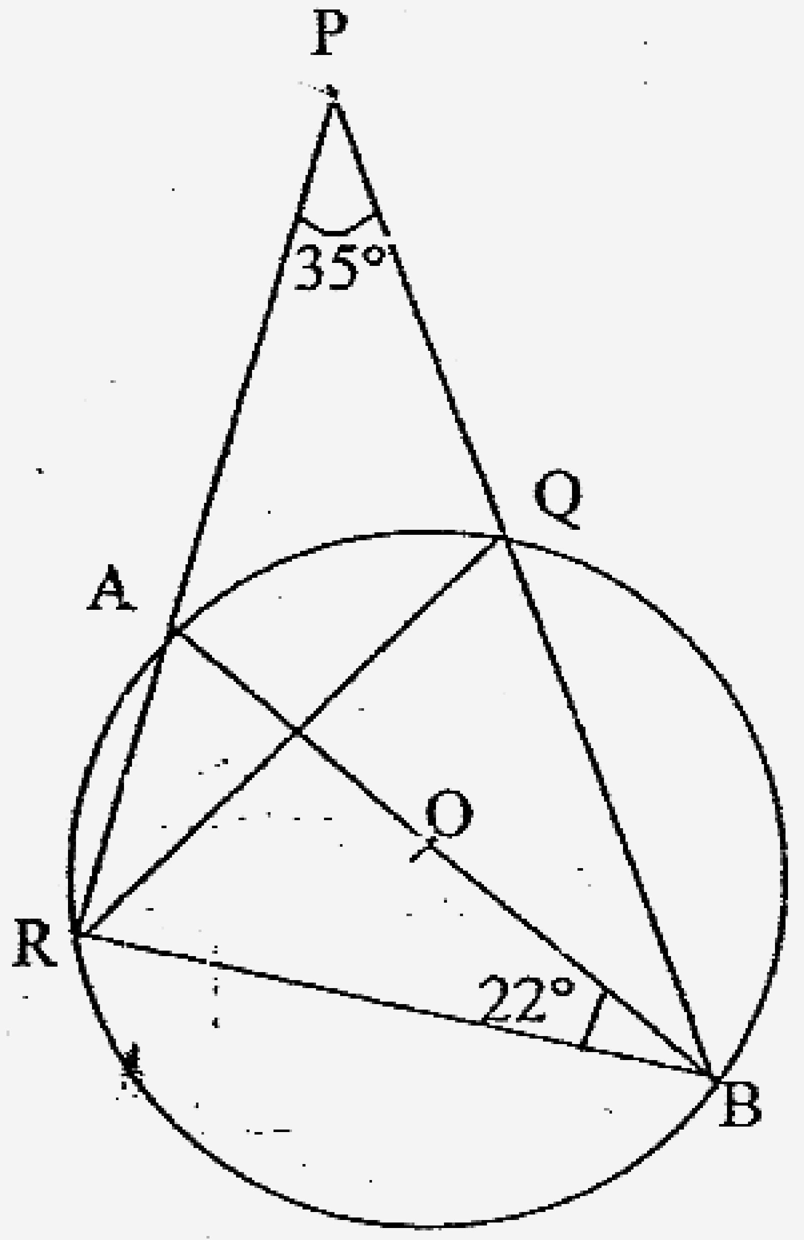
(a) Two towns are 25cm apart on the map. Calculate the actual distance of the two towns in km.

(b) A golf course has an area of $4km^2$. Calculate, in cm^2, the area of the golf course on the map.

Answer (a)...................km [1]

(b)...................cm^2 [2]

8.



In the diagram, AB is a diameter of a circle of centre O. $\angle APQ = 35°$ and $\angle ABR = 22°$.

Find (a) $\angle AOR$,
(b) $\angle ABQ$,
(c) $\angle QRA$.

Answer (a)......................° [1]

(b)......................° [2]

(c)......................° [1]

9. (a) Calculate the number of sides of a regular polygon if the interior angle is 144°.

(b) An n-sided polygon has two of its exterior angles at 45° and 75°. If the remaining exterior angles are each 20°, calculate the value of n.

Answer (a)..................... sides [1]

(b)............................ [2]

---

10. Solve the simultaneous equations

$3x - 4y = 25$

$4x - 5y = 32$

Answer x =.........y=............[2]

11. Given that P varies inversely as $(Q^2 + 1)$ and that $P = 13$ when $Q = 1$.

(a) Express P in terms of Q.

(b) Find the values of Q when $P = 1$.

Answer (a)......................... [2]

(b)......................... [2]

---

12. (a) Solve the equation $(t-6)^2 = 25$.

(b) Given that $25 - 3x \le 14$, find the smallest prime number x.

Answer (a) t =...................... [2]

(b) x =...................... [2]

13. (a) A home maker has enough money to run a household for 3 weeks if she spends \$24 a day. How many more days can the same amount of money last if she spends \$3 less a day?

(b) Given that $(2x+3y):(3x-4y)=7:2$, find the value of $\frac{3x}{y}$.

Answer (a)......................days [1]

(b)............................ [2]

---

14. Factorise completely

(a) $6x^2+7x-5$

(b) $4c^2-4bc+bd-cd$

Answer (a)............................ [1]

(b)............................ [2]

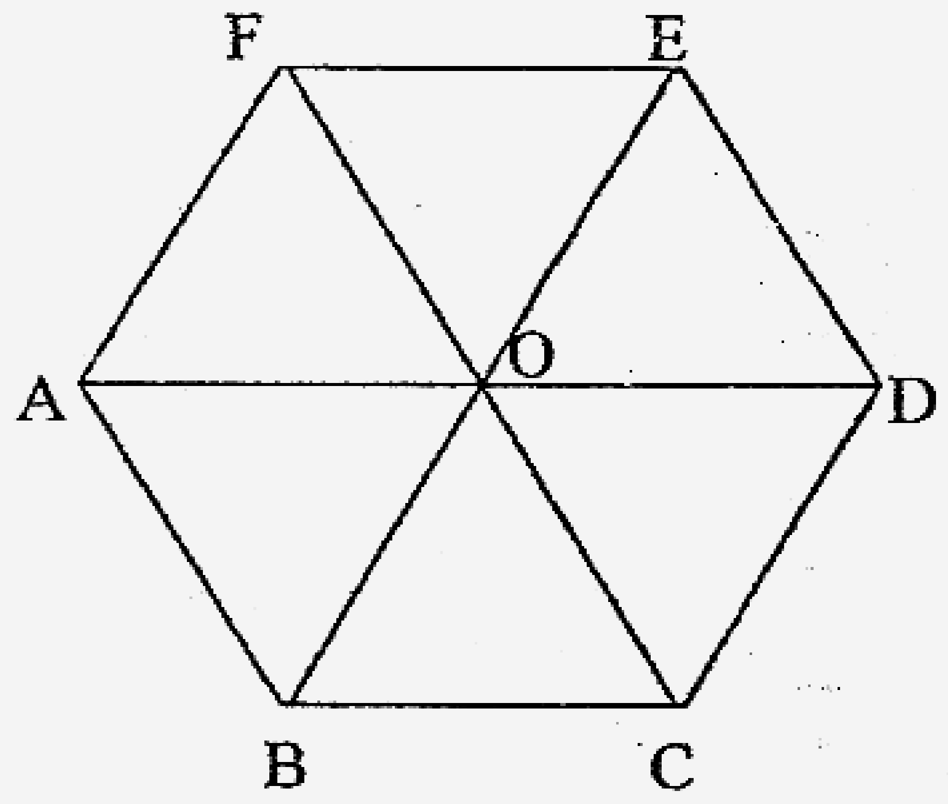
15. ABCDEF is a regular hexagon. R is an anticlockwise rotation of 60° about O. L is the reflection in the line BE, and M is a reflection in the line AD.

(a) Using letters given in the diagram, name the image of $ABOF$ under the transformation

(i) R^{-1}.

(ii) R^2.

(b) By identifying the image of ΔAOB under LM, or otherwise, describe a single transformation which is equivalent to LM.



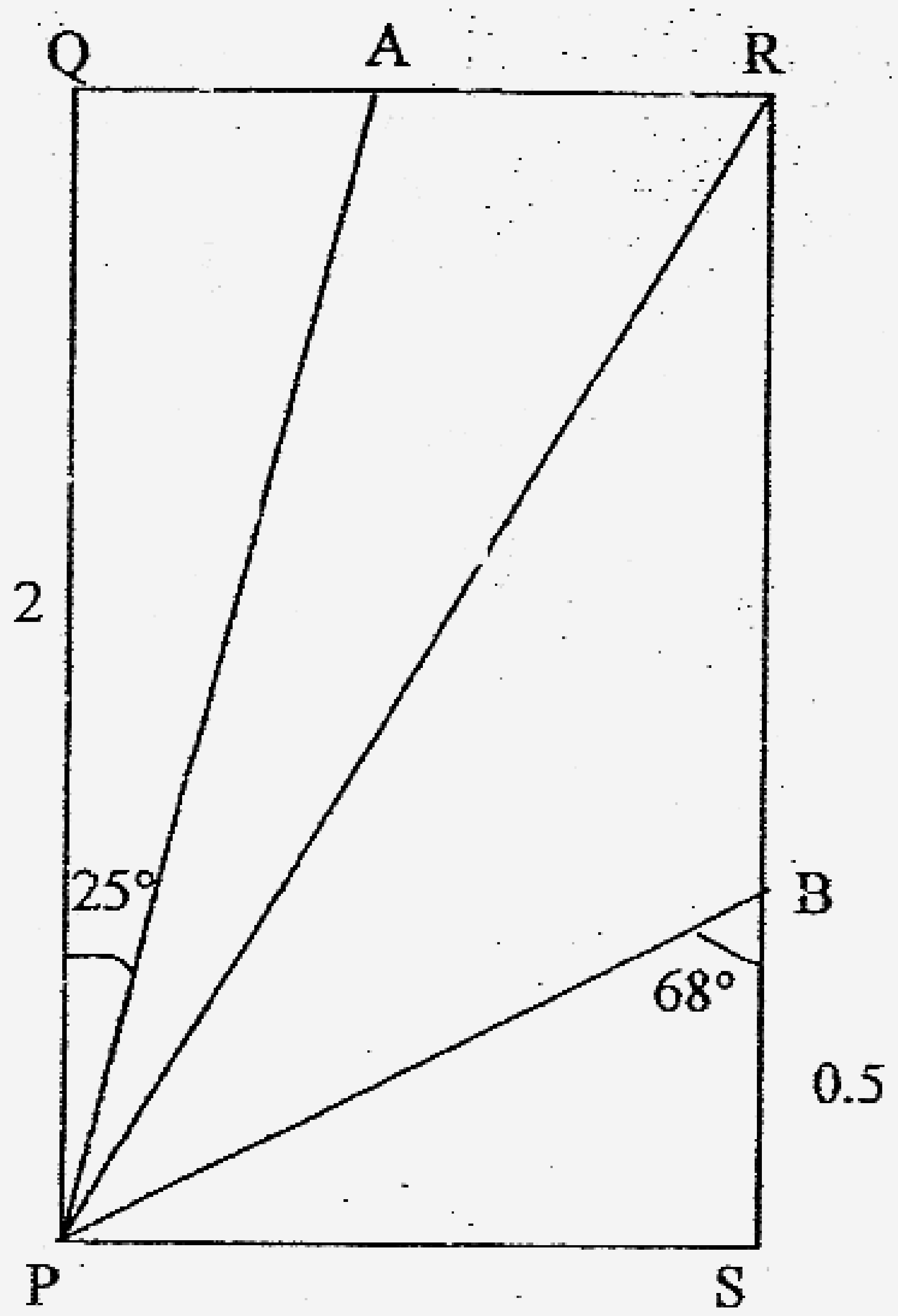
Answer (ai)........................... [1]

(aii)........................... [1]

(b)..

.... [1]

16.



PQRS represents a rectangular gate. *A* lies on *QR* and *B* lies on *RS*, The lines *PA*, *PR* and *PB* represent metal struts. $PQ = 2$ m, $BS = 0.5$ m, $\angle QPA = 25°$ and $\angle PBS = 68°$ using as much information given below as necessary, find

(a) $\sin 155° + \cos 112°$,
(b) the shortest distance from Q to AP,
(c) the length PS.

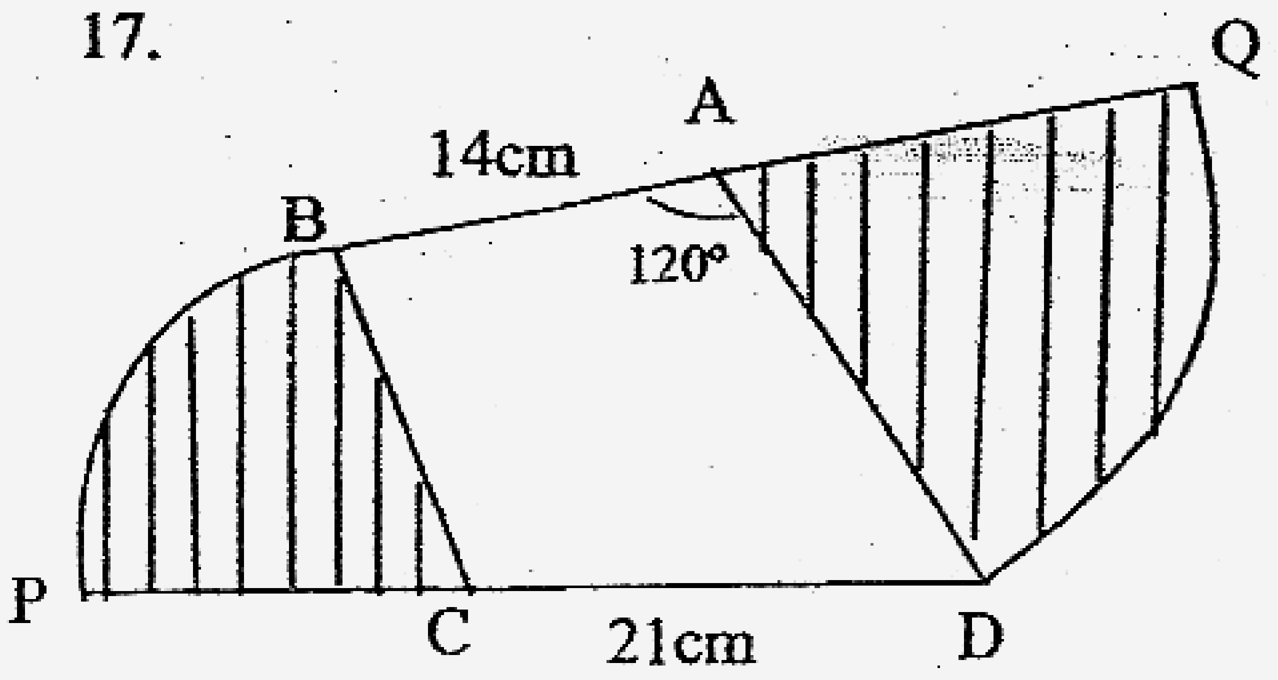
	sine	cosine	Tangent
25°	0.42	0.91	0.47
68°	0.93	0.38	2.48

Answer (a)............................ [1]

(b)..........................m [1]

(c)..........................m [1]

17.



In the diagram ABCD is a kite. AQD and CBP are sectors with centre at A and C respectively. Given that BAQ and PCD are straight lines, AB = BC = 14cm, AD = DC = 21cm and $\angle BAD = 120°$.

(a) Taking $\pi = \dfrac{22}{7}$, calculate

(i) the perimeter of the figure QABPCD,
(ii) the total area of the shaded regions.

(b) Describe fully the symmetry of the kite ABCD.

Answer (a) (i)........................cm [2]

(ii)........................cm^2 [2]

(b)..

....

.... [2]

18. Given that O is the origin. The position vectors of A and B are $\begin{pmatrix}-3\\4\end{pmatrix}$ and $\begin{pmatrix}-15\\2p\end{pmatrix}$ respectively, and M is the mid-point of AB.

(a) Find $|\overline{OA}|$.

(b) Express $\overline{BM}$ as a column vector in terms of p.

(c) Given further that O, C and B lie on a straight line, where C is the point $(-5,2)$, find the value of p.

(d) Hence, state the numerical value of $\dfrac{\text{area of } \Delta OMB}{\text{area of } \Delta OMC}$.

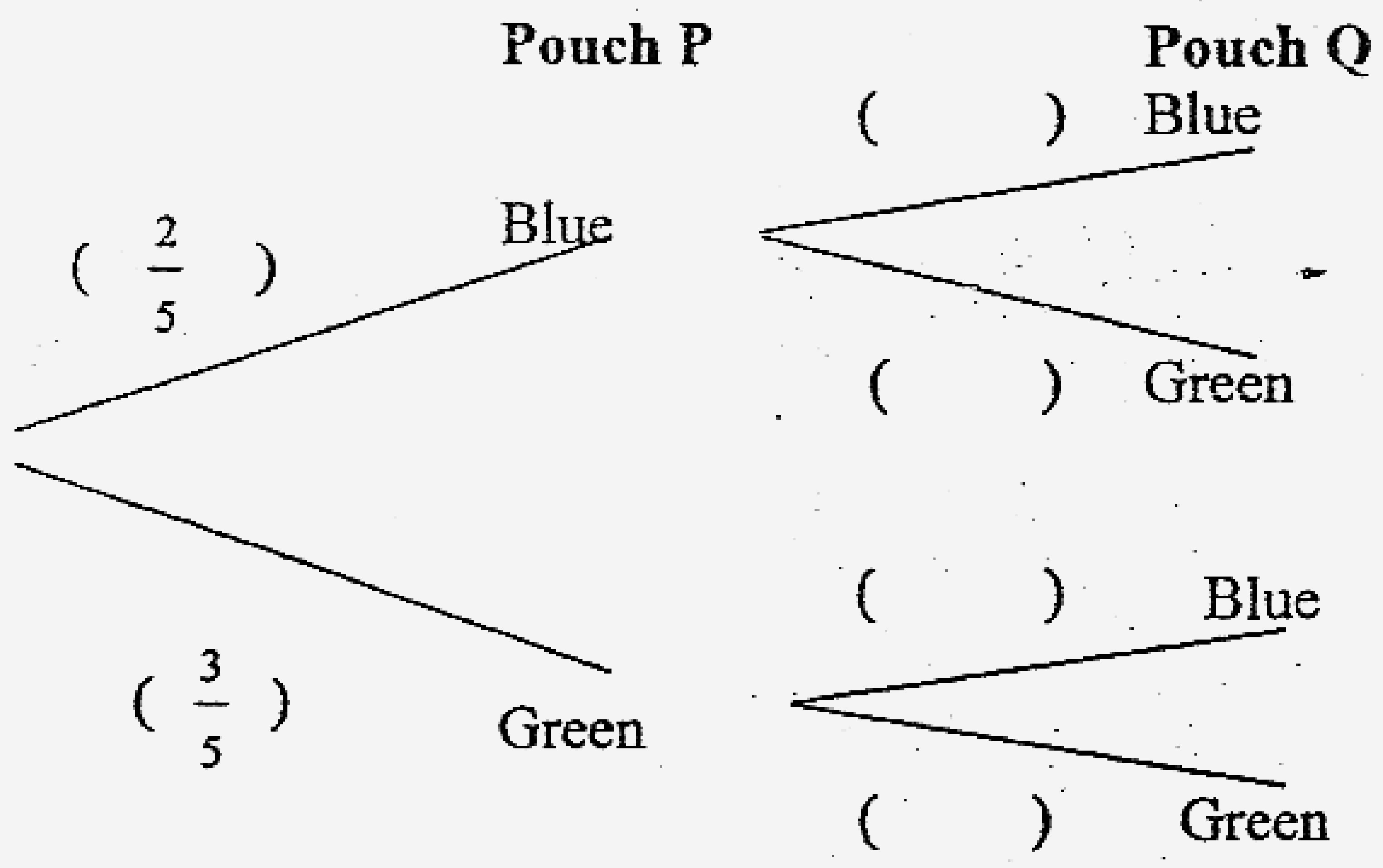
Answer (a)......................units [1]

(b)............................ [1]

(c)............................ [1]

(d)............................ [1]

19. Pouch P contains 20 marbles of which 8 are blue and the rest are green. Pouch Q contains 9 marbles of which x are blue and the rest are green. One marble is first taken from pouch P and put into pouch Q. One marble is then taken from pouch Q.
Complete the tree diagram below in terms of x.

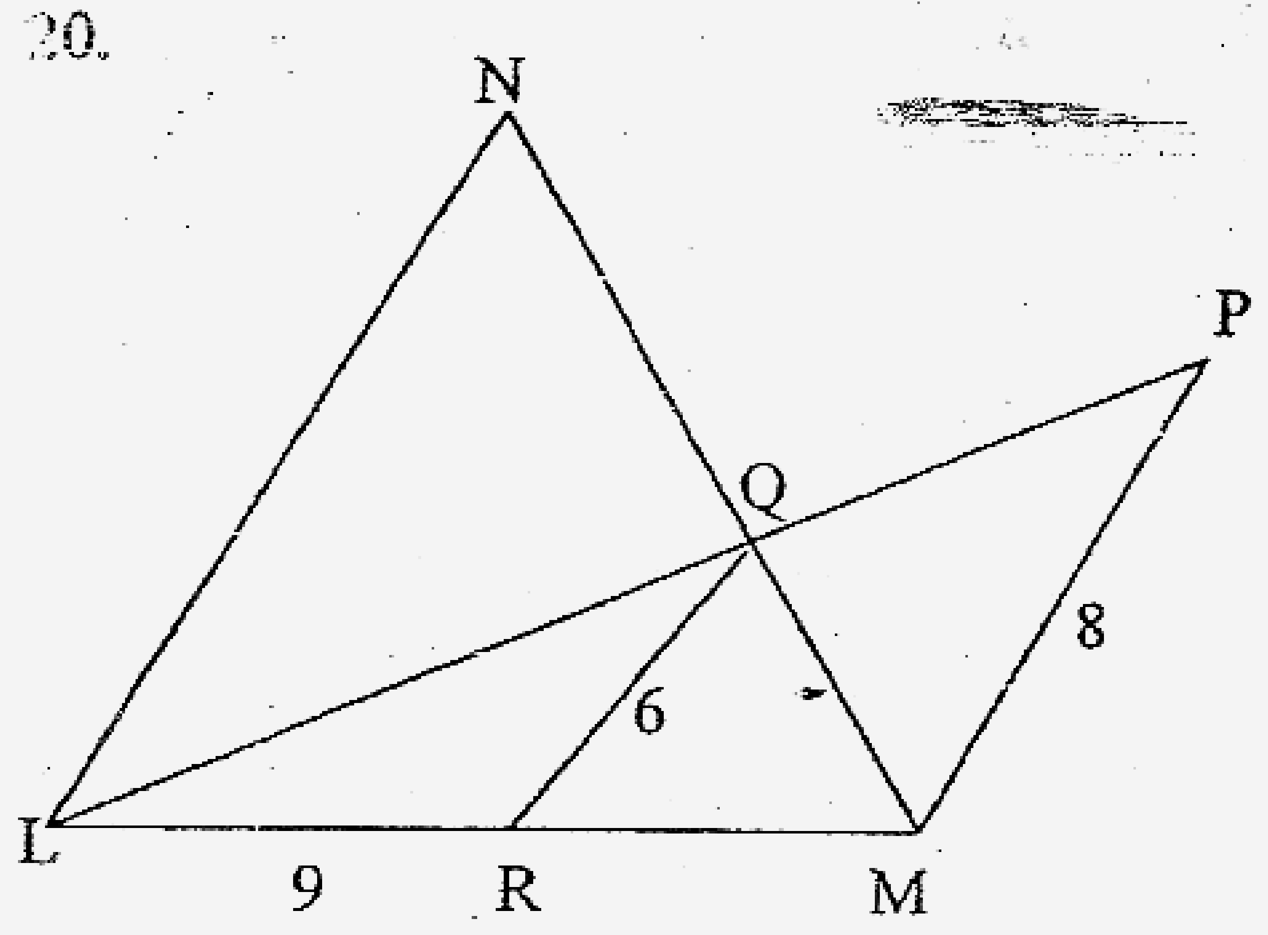


[2]

If the probability that a blue marble taken from pouch Q is $\frac{27}{50}$, find the value of x.

Answer x = ... [2]

20.



In the diagram, LN, RQ and MP are parallel lines, LR = 9cm, RQ = 6cm and MP = 8cm.

(a) Find the length of RM.

(b) Find the value of $\frac{\text{area of } \Delta LMN}{\text{area of } \Delta RLQ}$.

(c) It is further given that the area of the trapezium PQRM is 21cm^2, find the area of ΔPLM.

Answer (a)……………………cm [2]

(b)……………………… [2]

(c)……………………cm^2 [2]

21. The following data shows the marks obtained by 20 students who sat for a Mathematics test.

18	48	25	12	24
35	43	37	31	38
48	37	28	34	41
43	34	40	50	39

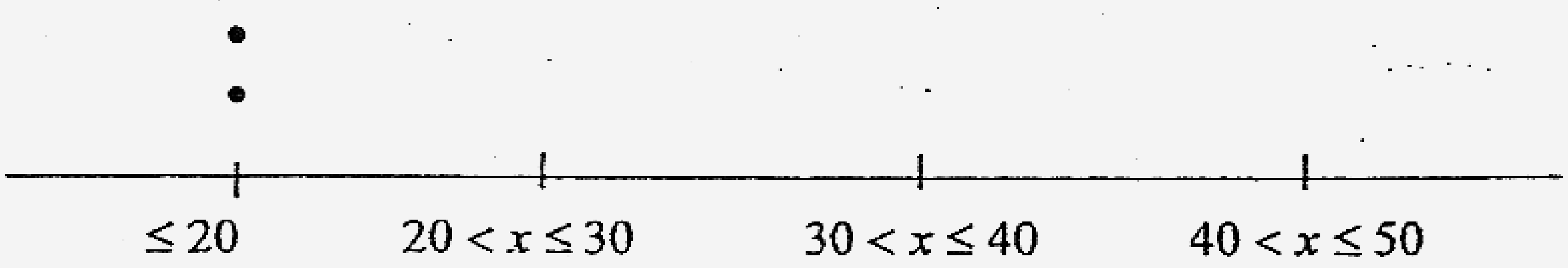
(a) Represent the data set in a single ordered stem and leaf diagram on the given axes below.

[2]

(b) State the median on the answer space (b) below.

(c) The data is subsequently represented on a dot diagram partially drawn below. Complete the dot diagram. Hence, state the modal range in the answer space (c) below.

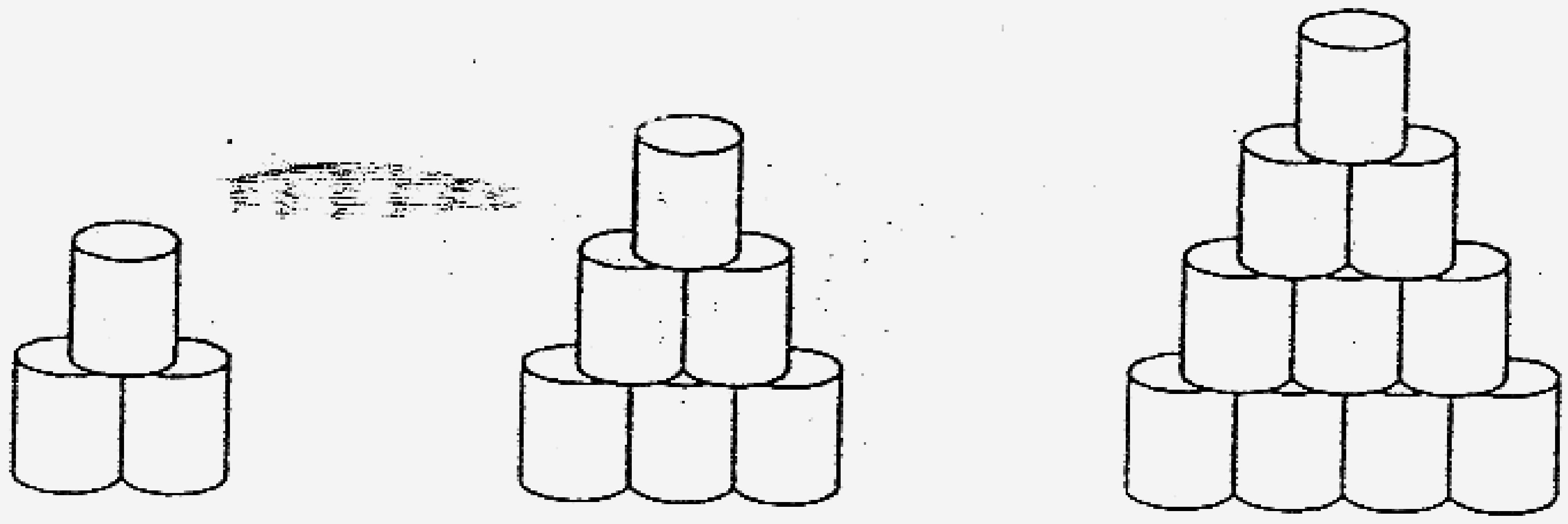
[1]



Answer (b)........................... [1]

(c)................................[1]

22.



The figures above show a display of tins. Let n be the number of tins at the bottom of a particular display, and N be the total number of tins needed to make that display.

n	2	3	4	5	6	...	q
n^2	4	9	16	25	36		q^2
N	3	6	10	15	p		Q

(a) Write down the value of p.

(b) Express Q in terms of q.

(c) Find the total number of tins needed for the display to be 11 tins high.

(d) Explain why it would not be possible for a shop assistant to build a display of this pattern using a total of 25 tins.

Answer (a) $p =$ .... [1]

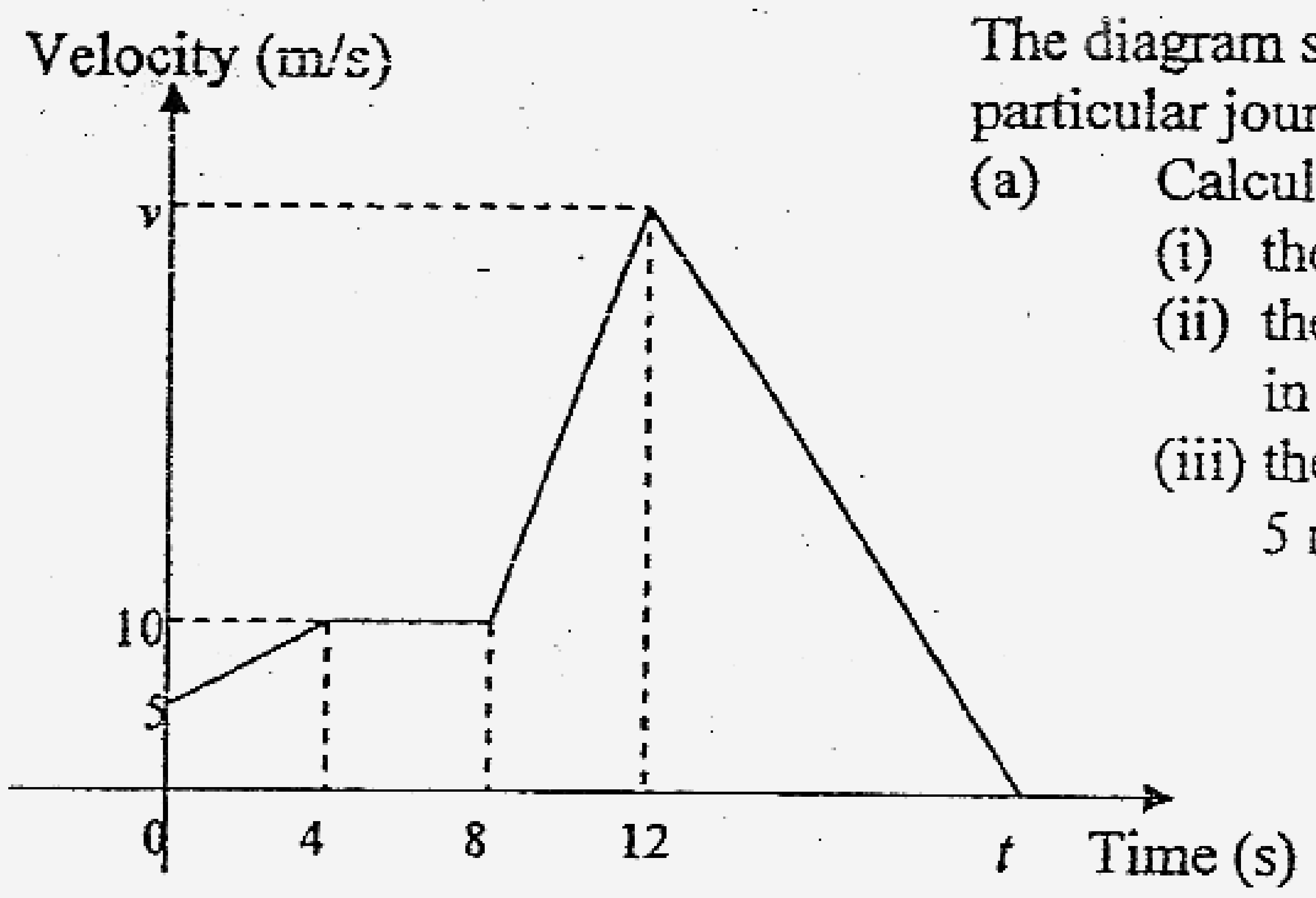
(b) $Q =$.................................. [1]

(c) .... [1]

(d)..

.... [2]

23.



The diagram shows the velocity-time graph for a particular journey over a period of t seconds.

(a) Calculate

(i) the acceleration during the first 4 seconds,

(ii) the value of v if the total distance travelled in the first 12 seconds is 150m,

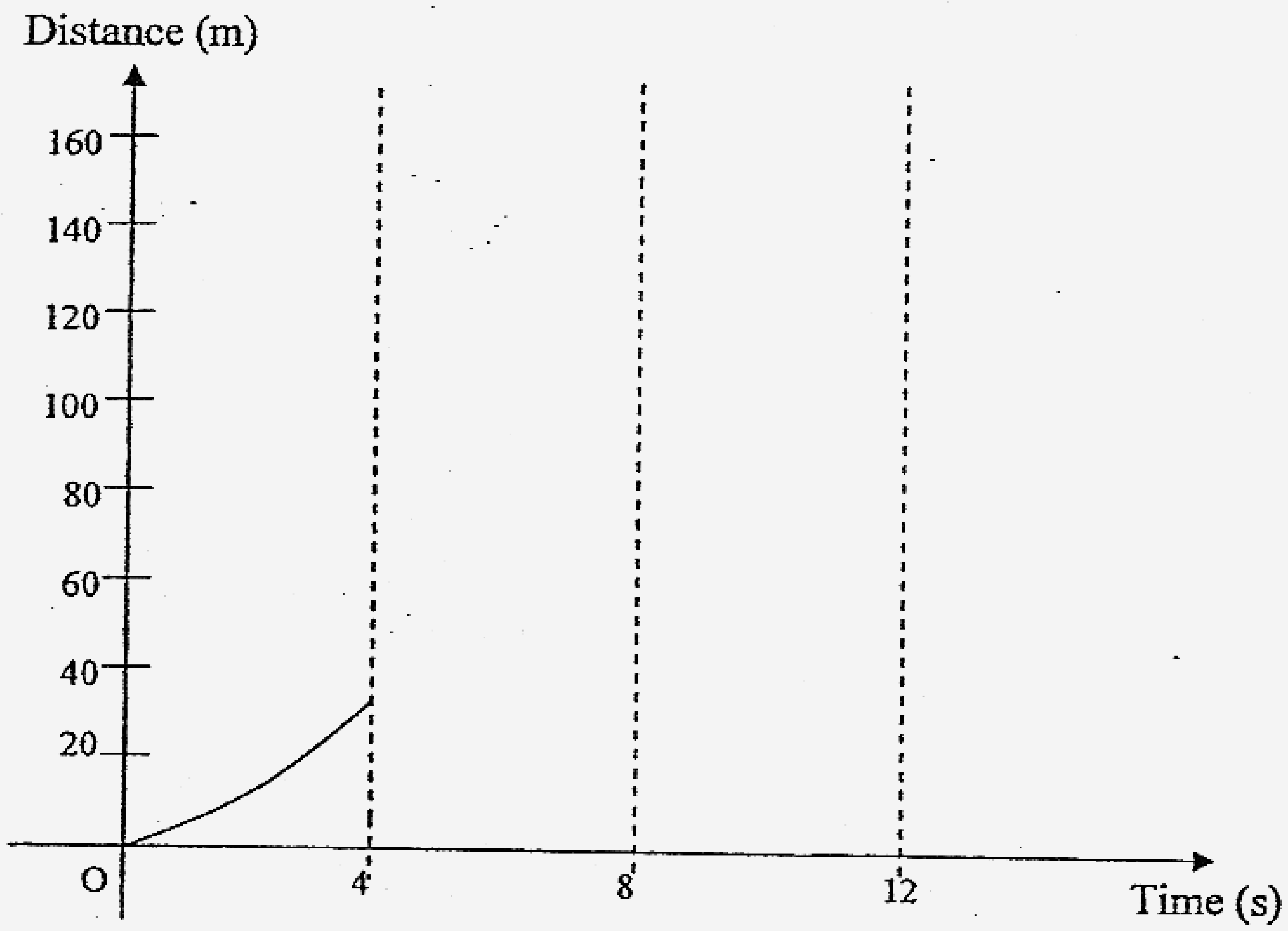
(iii) the value of t, given that the retardation is 5 m/s^2.

Answer (a) (i).......................................m/s^2 [1]

(ii)....................................... [1]

(iii) t =.................................. [1]

(b) Complete the sketch of the distance-time graph given below, for the first 12 seconds of the journey.



[2]

--*End of Paper*--

SoLN

Maris Stella High Prelim 2006 Paper 1

NEITHER ELECTRONIC CALCULATORS NOR MATHEMATICAL TABLES MAY BE USED IN THIS PAPER.

1. Evaluate

(a) $60 \div 4 \times 2 - 15$,

(b) $1\frac{2}{3} \div 0.5$.

(a) $60 \div 4 \times 2 - 15$

$= 60 \times \frac{1}{4} \times 2 - 15$

$= 30 - 15$

$= 15$

(b) $1\frac{2}{3} \div 0.5$

$= \frac{5}{3} \times \frac{2}{1}$

$= \frac{10}{3}$

$= 3\frac{1}{3}$

Answer (a) 15 ✓ [1]

(b) $3\frac{1}{3}$ ✓ [1]

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2. Evaluate the following, expressing each answer in standard form.

(a) $3 \times 7.2 \times 10^7$

(b) $\frac{2.1}{8.4 \times 10^6}$

(a) $3 \times 7.2 \times 10^7$

$= 21.6 \times 10^7$

$= 2.16 \times 10^8$

(b) $\frac{2.1}{8.4 \times 10^6}$

$= \frac{21}{84} \times 10^{-6}$

$= \frac{1}{4} \times 10^{-6}$

$= 0.25 \times 10^{-6}$

$= 2.5 \times 10^{-7}$

Answer (a) 2.16×10^8 ✓ [1]

(b) 2.5×10^{-7} ✓ [1]

---

3. The temperature recorded on a thermometer at time 21 00 was 4°C. At 01 00, the temperature had fallen 9°C.

(i) Write down the temperature at 01 00.

(ii) Calculate the average fall in temperature per hour.

(i) Temp at 0100 = 4°c − 9°c

= −5°c

(ii) Avg fall in temp = $\frac{9°c}{4h}$

= 2.25 °c / h

Answer (a) (i)........−5........°C [1]

(ii)........2.25........°C/hour [1]

---

4. (a) The gradient of the line joining the point $(h,-3)$ and $(7,h)$ is $-\frac{1}{3}$. Calculate the value of h.

(b) Find the coordinates of M, the midpoint of the line segment joining $A\ (-6,0)$ and $B(12,-4)$.

(b) Coord M = $\left(\frac{-6+12}{2}, \frac{0+(-4)}{2}\right)$

= $\left(\frac{6}{2}, -\frac{4}{2}\right)$

= (3, −2)

a) Given gradient = $-\frac{1}{3}$

⇒ $\frac{-3-h}{h-7} = -\frac{1}{3}$

⇒ 9 + 3h = h − 7

2h = −16

h = −8

Answer (a)........h = −8........ [2]

(b)........(3, −2)........ [1]

---

5. x and y are integers such that $-9 < x \leq 1$ and $-9 \leq y < -1$. Calculate

(a) the minimum value of xy,

(b) the greatest value of $\frac{x}{y}$.

(a) Min $xy = (1)(-9)$

$= -9$

(b) Greatest $\frac{x}{y} = \frac{-9}{-1}$

$= 9$

Answer (a) ....-9.......... [1]

(b) ....9.......... [1]

---

6. Given that $C = \frac{d}{d+r}$,

(a) find the value of C when $d = \frac{1}{3}$ and $r = \frac{7}{9}$,

(b) express r in terms of C and d.

(a) $C = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{7}{9}}$

$= \frac{3}{3+7}$

$= \frac{3}{10}$ or 0.3

(b) $C = \frac{d}{d+r}$

$d + r = \frac{d}{C}$

$r = \frac{d}{C} - d$

Answer (a) $C =$ ....$\frac{3}{10}$ or 0.3.......... [1]

(b) ....$r = \frac{d}{C} - d$.......... [1]

---

7. A map is drawn to scale of 1:20 000.

(a) Two towns are 25*cm* apart on the map. Calculate the actual distance of the two towns in *km*.

(b) A golf course has an area of $4km^2$. Calculate, in cm^2, the area of the golf course on the map.

Scale 1 : 20 000

⇒ 1cm : 0.2 km

⇒ $1cm^2$: $0.04 km^2$

(a) Actual dist = 25 × 0.2

= 5 km

(b) Area on Map = $\frac{4}{0.04}$

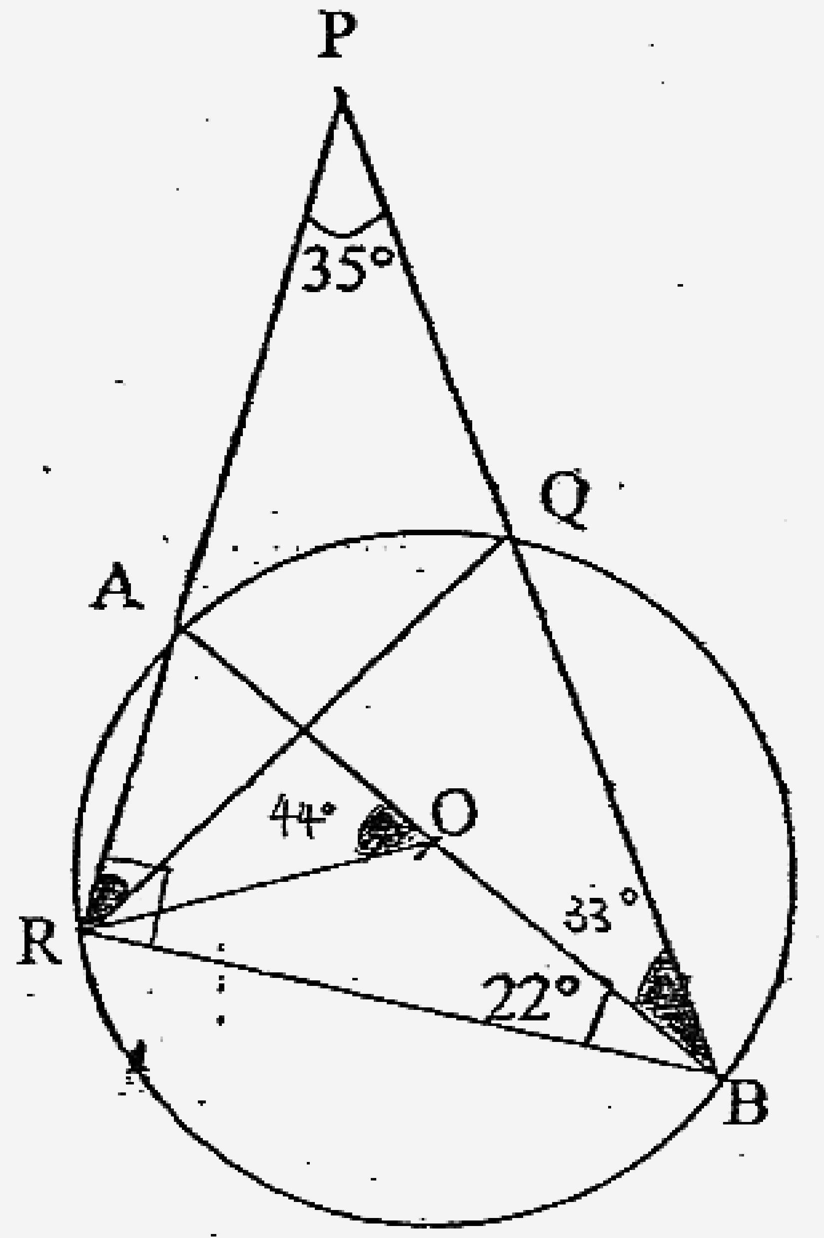
= $100 cm^2$

Answer (a) ......5...... km [1]

(b) ......100...... cm^2 [2]

---

8. In the diagram, AB is a diameter of a circle of centre O. $\angle APQ = 35°$ and $\angle ABR = 22°$.



Find (a) $\angle AOR$,
(b) $\angle ABQ$,
(c) $\angle QRA$.

(b) ∠ABQ = 180° − 35° − 22° − ∠ARB (∠ sum △BPR)

= 180° − 35° − 22° − 90° (∠ in semicircle)

= 33°

(c) ∠QRA = ∠QBA (∠s in same seg)

= 33°

(a) ∠AOR = 2∠ABR (∠ at ctr = 2∠ at O^ce)

= 2(22°)

= 44°

Answer (a) ......44...... ° [1]

(b) ......33...... ° [2]

(c) ......33...... ° [1]

9. (a) Calculate the number of sides of a regular polygon if the interior angle is 144°.

(b) An n-sided polygon has two of its exterior angles at 45° and 75°. If the remaining exterior angles are each 20°, calculate the value of n.

(a) int ∡ + ext ∡ = 180°

⇒ ext ∡ = 180° − 144°

= 36°

no. of sides = $\frac{\text{sum of ext } ∡}{\text{size of each ext } ∡}$

= $\frac{360°}{36°}$

= 10

(b) Sum of ext ∡ = 360°

45° + 75° + (n−2)(20°) = 360°

n − 2 = $\frac{360° − 45° − 75°}{20°}$

n = 12 + 2

= 14

Answer (a)........10........ sides [1]

(b)........14........ [2]

---

10. Solve the simultaneous equations

$3x - 4y = 25$ ——— ①

$4x - 5y = 32$ ——— ②

① × 4 : 12x − 16y = 100 ——— ③

② × 3 : 12x − 15y = 96 ——— ④

④ − ③ : y = −4 ——— ⑤

⑤ sub → ① : 3x − 4(−4) = 25

3x = 9

x = 3

Answer x =....3.... y=....−4.... [2]

---

11. Given that P varies inversely as $(Q^2 + 1)$ and that $P = 13$ when $Q = 1$.

(a) Express P in terms of Q.

(b) Find the values of Q when $P = 1$.

(a) $P \propto \frac{1}{Q^2+1}$

$P = \frac{k}{Q^2+1}$

Given $13 = \frac{k}{(1)^2+1}$

$\Rightarrow k = 26$

$\therefore P = \frac{26}{Q^2+1}$

(b) when $P = 1$,

$1 = \frac{26}{Q^2+1}$

$Q^2+1 = 26$

$Q^2 = 25$

$Q = \pm 5$

Answer (a) $P = \frac{26}{Q^2+1}$ [2]

(b) -5 or 5 [2]

---

12. (a) Solve the equation $(t-6)^2 = 25$.

(b) Given that $25 - 3x \leq 14$, find the smallest prime number x.

(a) $(t-6)^2 = 25$

$t-6 = \pm\sqrt{25}$

$t-6 = -5$ or $t-6 = 5$

$t = 1$ or $t = 11$

(b) $25 - 3x \leq 14$

$11 \leq 3x$

$\frac{11}{3} \leq x$

$x \geq 3\frac{2}{3}$

$\therefore$ smallest prime no. is 5

Answer (a) $t =$ 1 or 11 [2]

(b) $x =$ 5 [2]

---

13. (a) A home maker has enough money to run a household for 3 weeks if she spends \$24 a day. How many more days can the same amount of money last if she spends \$3 less a day?

(b) Given that $(2x+3y):(3x-4y)=7:2$, find the value of $\frac{3x}{y}$.

(a) No. of extra days

$= \frac{3\times7\times\$24}{\$24-\$3} - 3\times7$

$= \frac{3\times7\times\$24}{\$21} - 21$

$= 24-21$

$= 3$

(b) $\frac{2x+3y}{3x-4y} = \frac{7}{2}$

$4x+6y = 21x-28y$

$34y = 17x$

$\frac{34}{17} = \frac{x}{y}$

$\frac{x}{y} = \frac{2}{1}$

$\frac{3x}{y} = 6$

Answer (a) ....3..........days [1]

(b) ....6.......... [2]

---

14. Factorise completely

(a) $6x^2+7x-5$

(b) $4c^2-4bc+bd-cd$

(a) $6x^2+7x-5$

$= (2x-1)(3x+5)$

$2x$	-1	$-3x$
$3x$	$+5$	$10x$
$6x^2$	-5	$7x$

(b) $4c^2-4bc+bd-cd$

$= 4c(c-b)-d(c-b)$

$= (c-b)(4c-d)$

Answer (a) $(2x-1)(3x+5)$ [1]

(b) $(c-b)(4c-d)$ [2]

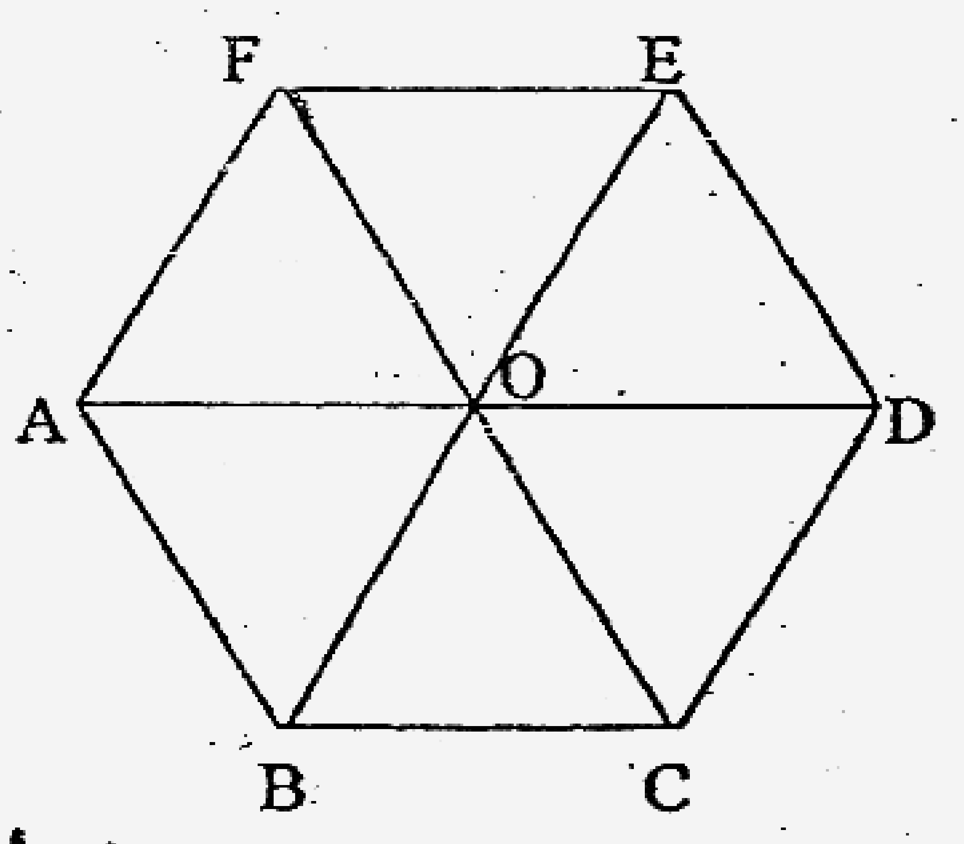
15. ABCDEF is a regular hexagon. R is an anticlockwise rotation of 60° about *O*. *L* is the reflection in the line *BE*, and *M* is a reflection in the line *AD*.

(a) Using letters given in the diagram, name the image of *ABOF* under the transformation

(i) R^{-1}.

(ii) R^2.

(b) By identifying the image of ΔAOB under *LM*, or otherwise, describe a single transformation which is equivalent to *LM*.



Regular hexagon

$\Rightarrow \angle$ at ctr $= \frac{360°}{6} = 60°$ each.

(a) (i) R^{-1} is a clockwise rotation of 60° about pt. O

$R^{-1}(ABOF) = FAOE$

(ii) R^2 is an anticlockwise rotation of 120° about pt. O

$R^2(ABOF) = CDOB$

(b) $LM(\Delta AOB) = L(\Delta AOF)$

$= \Delta COD$

which is equivalent to an anticlockwise rotation of 120° about pt. O

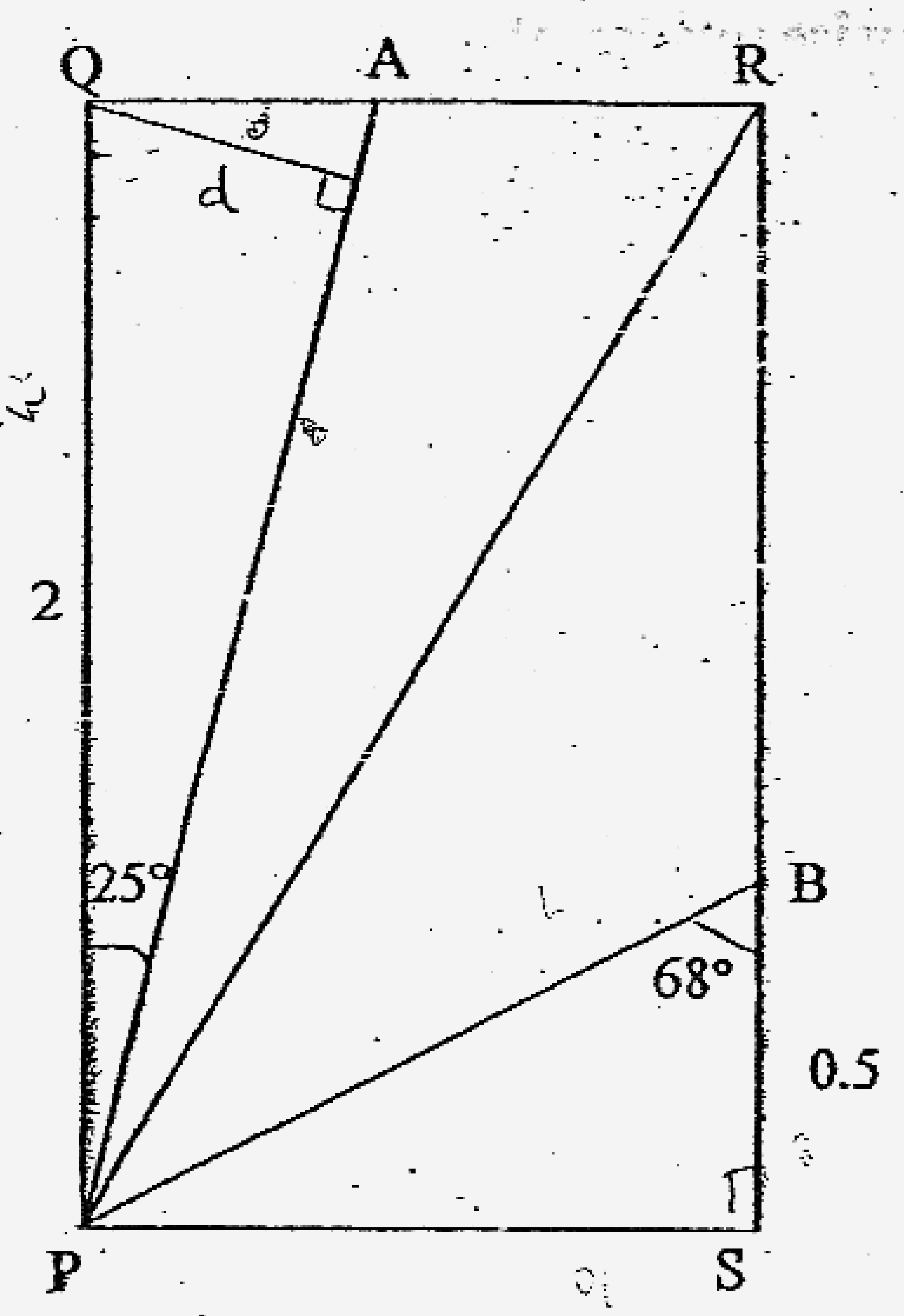
Answer (ai) FAOE [1]

(aii) CDOB [1]

(b) An anticlockwise rotation of 120° about pt. O

[1]

16.



PQRS represents a rectangular gate. *A* lies on *QR* and *B* lies on *RS*, The lines *PA*, *PR* and *PB* represent metal struts. *PQ* = 2 m, *BS* = 0.5 m, ∠QPA = 25° and ∠PBS = 68° using as much information given below as necessary, find

(a) sin 155° + cos 112°,
(b) the shortest distance from Q to AP,
(c) the length PS.

	sine	cosine	Tangent
25°	0.42	0.91	0.47
68°	0.93	0.38	2.48

(a) $\sin 155° + \cos 112°$

$= \sin(180° - 25°) + \cos(180° - 68°)$

$= \sin 25° - \cos 68°$

$= 0.42 - 0.38$

$= 0.04$

(b) Let d be the shortest dist from Q to AP

$\sin 25° = \frac{d}{2}$

$d = 2 \sin 25°$

$= 2(0.42)$

$= 0.84$

(c) $\tan 68° = \frac{PS}{0.5}$

$PS = 0.5 \tan 68°$

$= \frac{1}{2}(2.48)$

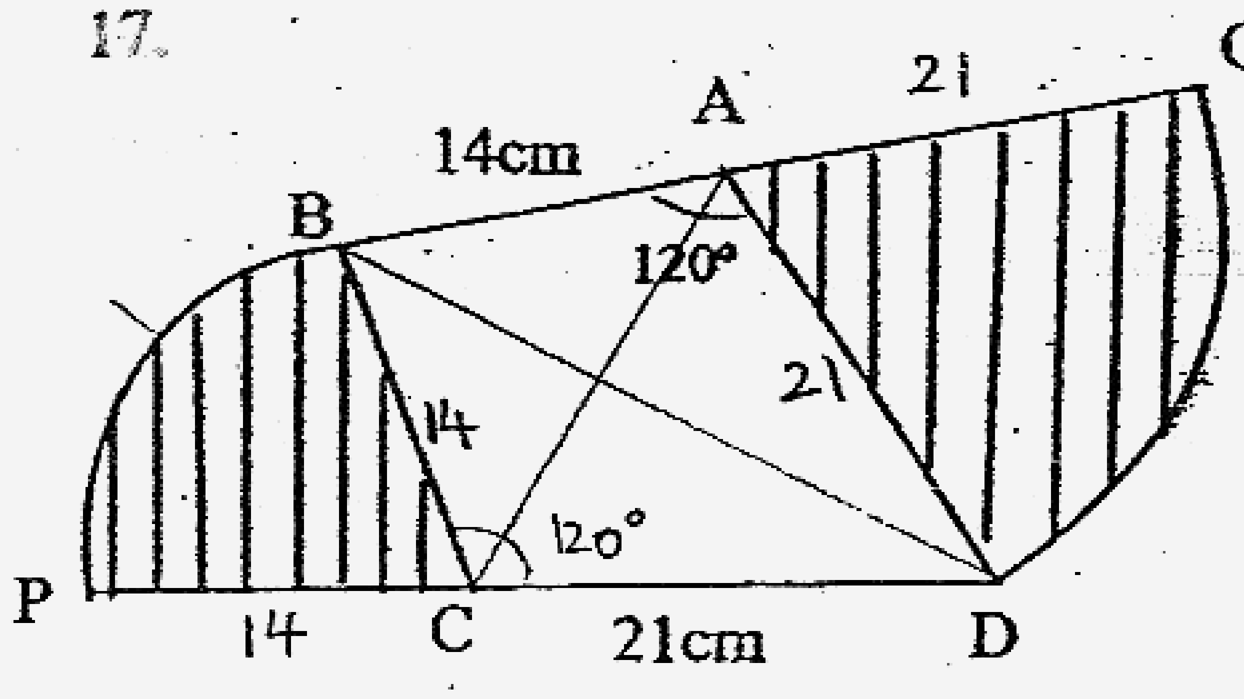
$= 1.24$ m

Answer (a) 0.04 [1]

(b) 0.84 m [1]

(c) 1.24 m [1]

17.



In the diagram ABCD is a kite. AQD and CBP are sectors with centre at A and C respectively. Given that BAQ and PCD are straight lines, AB = BC = 14cm, AD = DC = 21cm and ∠BAD = 120°.

(a) Taking $\pi = \frac{22}{7}$, calculate

(i) the perimeter of the figure QABPCD,

(ii) the total area of the shaded regions.

(b) Describe fully the symmetry of the kite ABCD.

(a) Perimeter

$= 2(14) + 2(21) + \left[\frac{60°}{360°} \times 2 \times \frac{22}{7} \times 14\right] + \left[\frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21\right]$

$= 70 + \frac{1}{6} \times 2 \times \frac{22}{7} \times 35$

$= 70 + \frac{110}{3}$

$= 70 + 36\frac{2}{3}$

$= 106\frac{2}{3}$ cm

(b) Total area of shaded regions

$= \frac{60°}{360°} \times \frac{22}{7} \times 14 \times 14 + \frac{60°}{360°} \times \frac{22}{7} \times 21 \times 21$

$= \frac{1}{6} \times \frac{22}{7} \times 14 \times 14 + \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$

$= \frac{308}{3} + 231$

$= 102\frac{2}{3} + 231$

$= 333\frac{2}{3}$ cm²

Answer (a) (i) .... $106\frac{2}{3}$ .... cm [2]

(ii) .... 333 .... cm² [2]

(b) Line symmetry of order 1 about the line BD

Rotational symmetry of order 1

.... [2]

18. Given that O is the origin. The position vectors of A and B are $\begin{pmatrix}-3\\4\end{pmatrix}$ and $\begin{pmatrix}-15\\2p\end{pmatrix}$ respectively, and M is the mid-point of AB.

(a) Find $|\overrightarrow{OA}|$.

(b) Express $\overrightarrow{BM}$ as a column vector in terms of p.

(c) Given further that O, C and B lie on a straight line, where C is the point $(-5,2)$, find the value of p.

(d) Hence, state the numerical value of $\dfrac{\text{area of } \Delta OMB}{\text{area of } \Delta OMC}$.

(a) $|\overrightarrow{OA}| = \sqrt{(-3)^2+(4)^2}$

$= \sqrt{25}$

$= 5$ units

(b) $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BA}$

$= \frac{1}{2}[\overrightarrow{OA} - \overrightarrow{OB}]$

$= \frac{1}{2}\left[\begin{pmatrix}-3\\4\end{pmatrix} - \begin{pmatrix}-15\\2p\end{pmatrix}\right]$

$= \frac{1}{2}\begin{pmatrix}12\\4-2p\end{pmatrix}$

$= \begin{pmatrix}6\\2-p\end{pmatrix}$

(c) Given O, C, B are collinear

$\Rightarrow \overrightarrow{OC} \parallel \overrightarrow{OB} \Rightarrow \overrightarrow{OC} = k\overrightarrow{OB}$

$\Rightarrow \begin{pmatrix}-5\\2\end{pmatrix} = k\begin{pmatrix}-15\\2p\end{pmatrix}$

$\Rightarrow \begin{cases}-5 = -15k\\2 = 2kp\end{cases}$

Hence $k = \frac{1}{3}$

$\Rightarrow 2 = 2(\frac{1}{3})p$

$\therefore p = 3$

(d) Since $\overrightarrow{OC} \parallel \overrightarrow{OB}$ and $\overrightarrow{OC} = \frac{1}{3}\overrightarrow{OB}$

$\Rightarrow \overrightarrow{OB} = 3\overrightarrow{OC}$

$\dfrac{\text{Area } \Delta OMB}{\text{Area } \Delta OMC} = \dfrac{\frac{1}{2} \times OB \times h}{\frac{1}{2} \times OC \times h}$ (Both Δs share common height)

$= \dfrac{3}{1}$

Answer (a) 5 units [1]

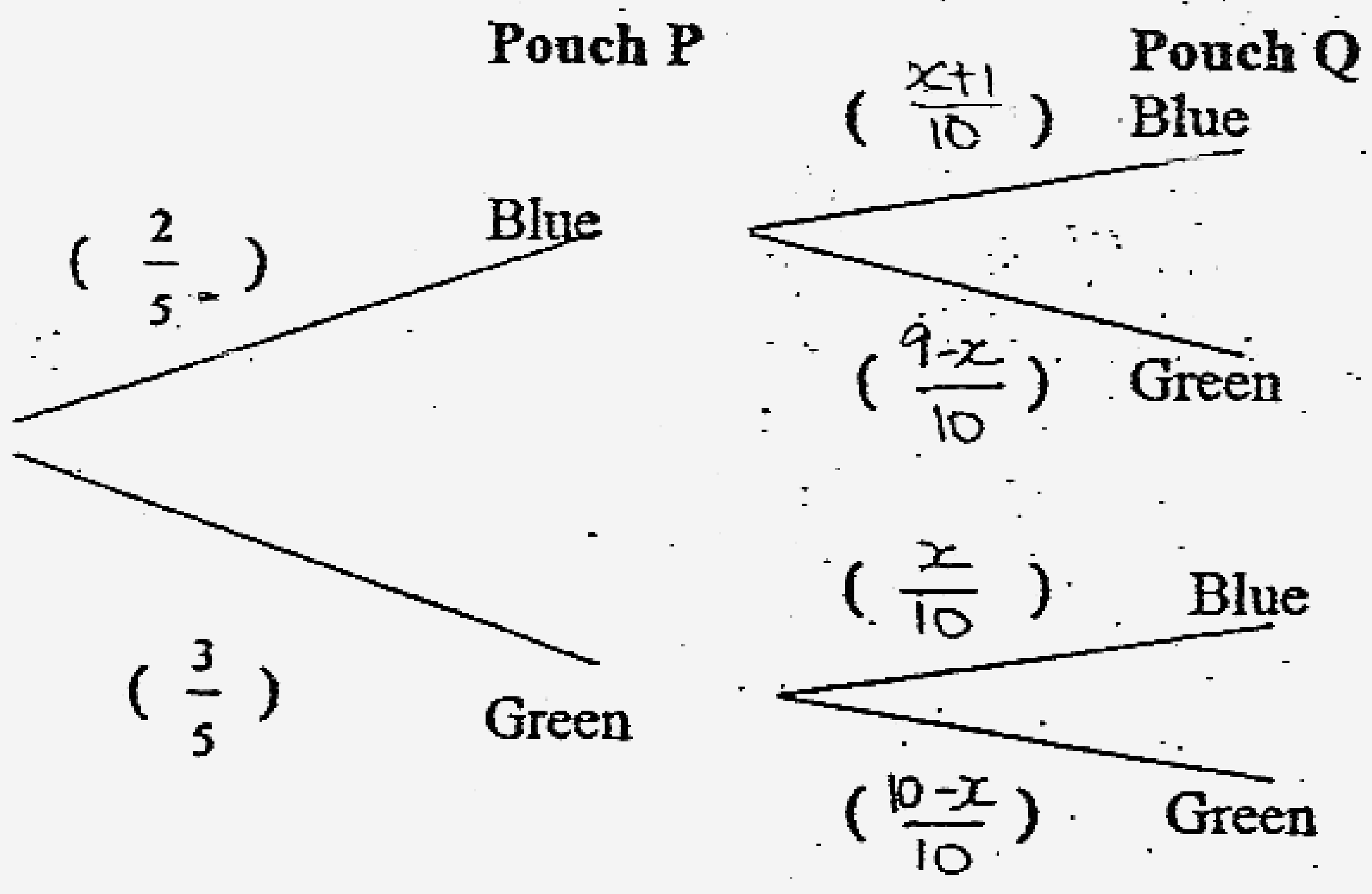
(b) $\begin{pmatrix}6\\2-p\end{pmatrix}$ [1]

(c) $p = 3$ [1]

(d) $\frac{3}{1}$ [1]

$\frac{8}{20} = \frac{2}{5}$ $\frac{12}{20} = \frac{3}{5}$

19. Pouch P contains 20 marbles of which 8 are blue and the rest are green. Pouch Q contains 9 marbles of which x are blue and the rest are green. One marble is first taken from pouch P and put into pouch Q. One marble is then taken from pouch Q.
Complete the tree diagram below in terms of x.



[2]

If the probability that a blue marble taken from pouch Q is $\frac{27}{50}$, find the value of x.

$P(\text{Blue}) = \frac{27}{50}$

$\Rightarrow P(\text{BB or GB}) = \frac{27}{50}$

$\Rightarrow \left(\frac{2}{5} \times \frac{x+1}{10}\right) + \left(\frac{3}{5} \times \frac{x}{10}\right) = \frac{27}{50}$

$2(x+1) + 3x = 27$

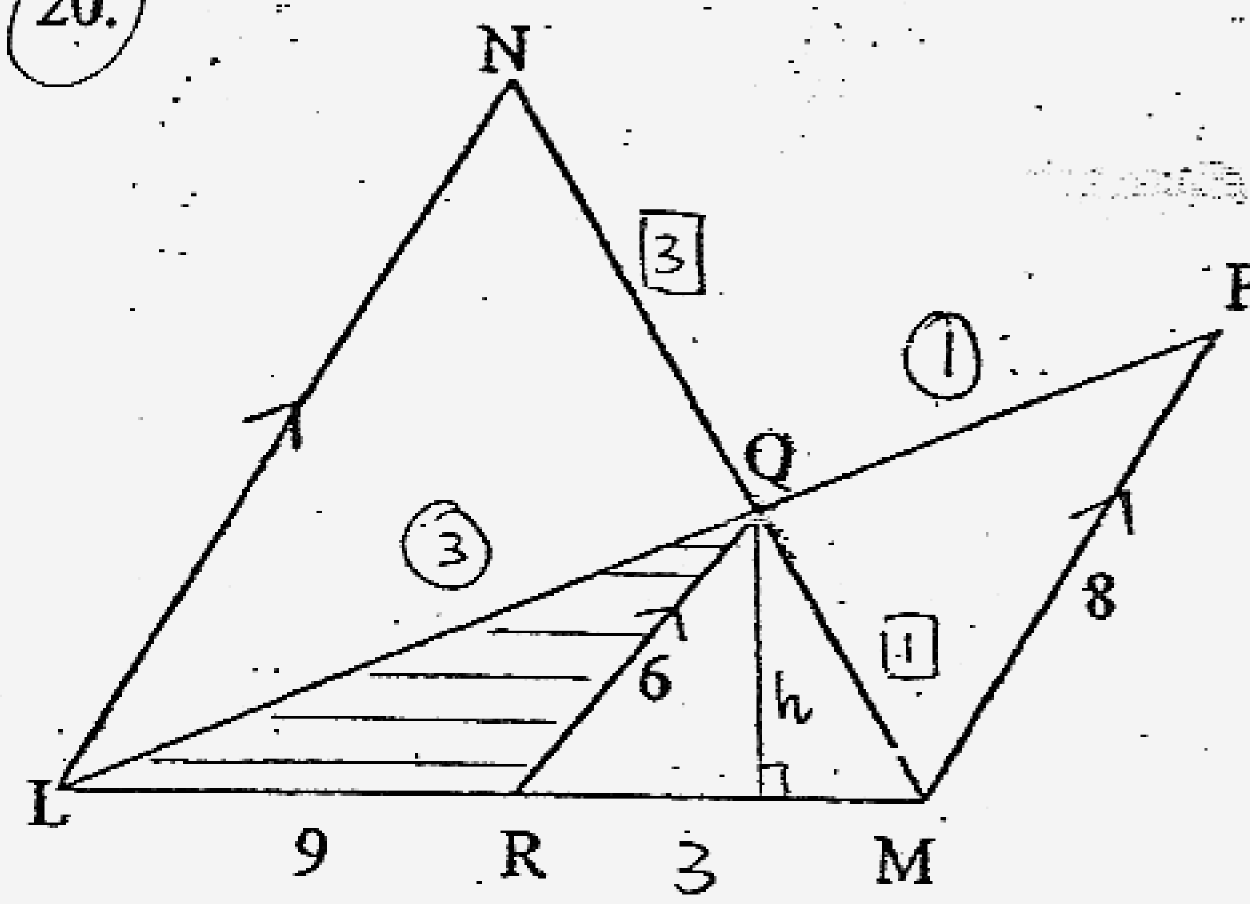
$2x + 2 + 3x = 27$

$5x = 25$

$x = 5$

Answer $x =$ ......5...... [2]

20.



In the diagram, LN, RQ and MP are parallel lines, LR = 9cm, RQ = 6cm and MP = 8cm.

(a) Find the length of RM.

(b) Find the value of $\dfrac{\text{area of } \Delta LMN}{\text{area of } \Delta RLQ}$.

(c) It is further given that the area of the trapezium PQRM is 21cm^2, find the area of ΔPLM.

(a) $\dfrac{LM}{LR} = \dfrac{MP}{RQ}$ ($\Delta LRQ \cong \Delta LMP$)

$\dfrac{RM+9}{9} = \dfrac{8}{6}$

$RM + 9 = \dfrac{4}{3} \times 9$

$RM = 12 - 9$

$RM = 3$

(c) $\dfrac{\text{Area } \Delta PLM}{\text{Area } \Delta QLR} = \left(\dfrac{8}{6}\right)^2 = \dfrac{16}{9}$

⇒ Area trapezium PQRM

= 16 − 9 = 7 units

7 units → 21cm^2

16 units → $\dfrac{21}{7} \times 16 = 48\text{cm}^2$

b) $\dfrac{\text{Area } \Delta LMN}{\text{Area } \Delta RMQ} = \left(\dfrac{12}{3}\right)^2 = \dfrac{16}{1}$

$\dfrac{\text{Area } \Delta RLQ}{\text{Area } \Delta RMQ} = \dfrac{9}{3} = \dfrac{3}{1}$

$\therefore \dfrac{\text{Area } \Delta LMN}{\text{Area } \Delta RLQ} = \dfrac{16}{3}$

Answer (a) ......3...... cm [2]

(b) ......$\frac{16}{3}$...... [2]

(c) ......48...... cm^2 [2]

21. The following data shows the marks obtained by 20 students who sat for a Mathematics test.

18	48	25	12	24
35	43	37	31	38
48	37	28	34	41
43	34	40	50	39

(a) Represent the data set in a single ordered stem and leaf diagram on the given axes below.

1	2 8
2	4 5 8
3	1 4 4 5 7 7 8 9
4	0 1 3 3 8 8
5	0

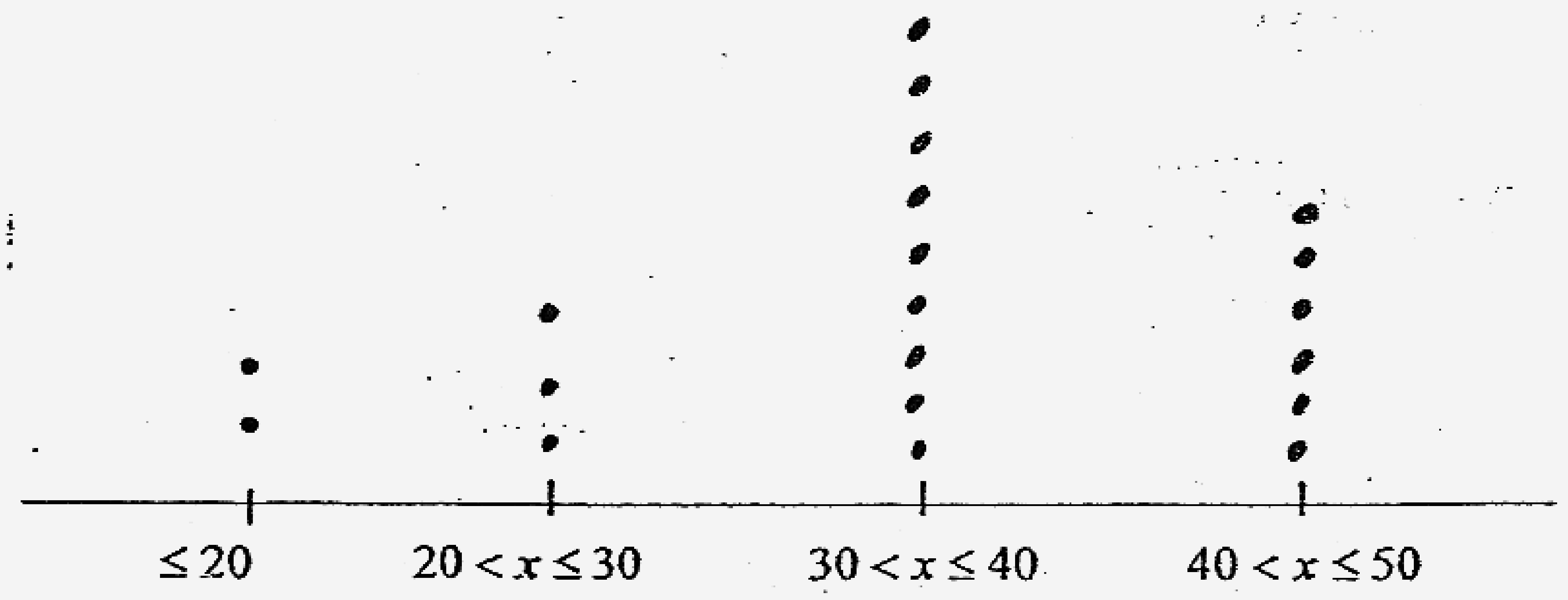
[2]

Median position = $\frac{20+1}{2}$ = 10.5th

(b) State the median on the answer space (b) below. Median = $\frac{7+7}{2}$ = 7 marks

(c) The data is subsequently represented on a dot diagram partially drawn below. Complete the dot diagram. Hence, state the modal range in the answer space (c) below.

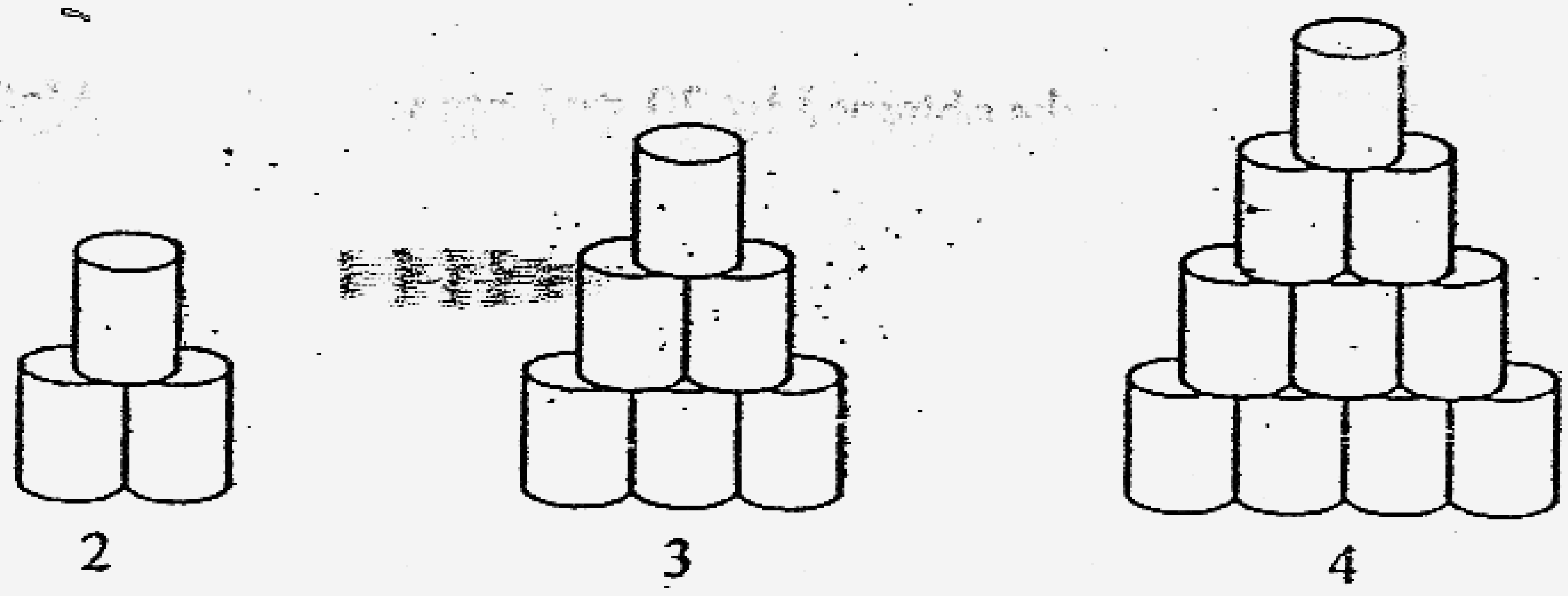
[1]



Answer (b) 7 marks [1]

(c) $30 < x \leq 40$ [1]

22.



The figures above show a display of tins. Let n be the number of tins at the bottom of a particular display, and N be the total number of tins needed to make that display.

n	2	3	4	5	6	...	q
n^2	4	9	16	25	36		q^2
N	3	6	10	15	p		Q

(a) Write down the value of p.

(b) Express Q in terms of q.

(c) Find the total number of tins needed for the display to be 11 tins high.

(d) Explain why it would not be possible for a shop assistant to build a display of this pattern using a total of 25 tins.

(a) $P = 15 + 6 = 21$ OR $P = 36 - 15 = 21$ OR $P = \frac{1}{2}(6+36) = 21$

(b) $Q = \frac{1}{2}(q + q^2)$

(c) No. of tins $= \frac{1}{2}(11 + 11^2)$
$= \frac{1}{2}(11 + 121)$
$= \frac{1}{2}(132)$
$= 66$

(d) when $n = 7$, $N = \frac{1}{2}(7 + 7^2)$
$= \frac{1}{2}(56)$
$= 28$

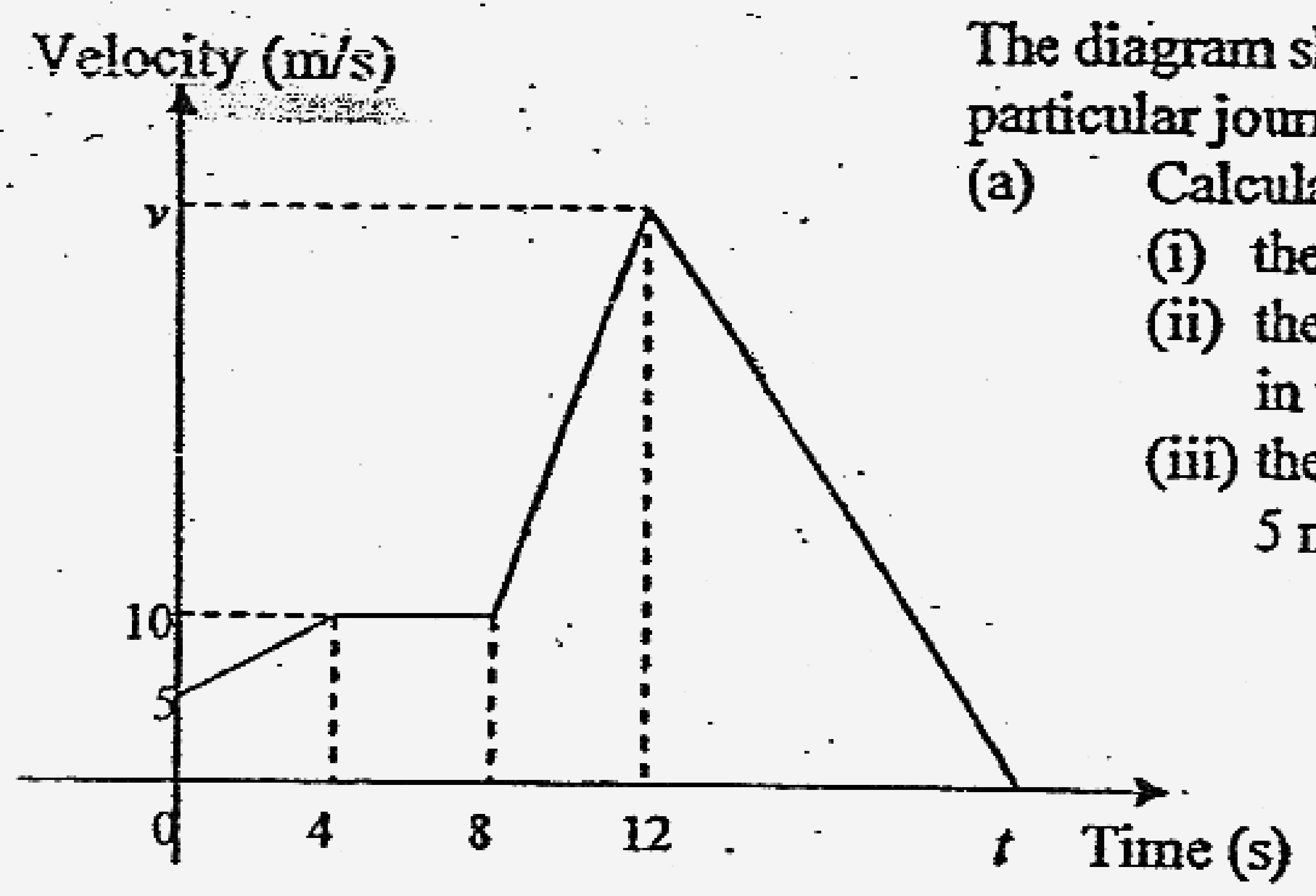
Answer (a) $p =$ 21 [1]

(b) $Q =$ $\frac{1}{2}(q+q^2)$ [1]

(c) 66 tins [1]

(d) When $n = 6$, $N = 21$ and when $n = 7$, $N = 28$
⇒ 25 tins are not sufficient to build the 7th pattern [2]

23.



The diagram shows the velocity-time graph for a particular journey over a period of t seconds.

(a) Calculate

(i) the acceleration during the first 4 seconds,

(ii) the value of v if the total distance travelled in the first 12 seconds is 150m,

(iii) the value of t, given that the retardation is $5\ m/s^2$.

(a)(iii) Given retardation $= 5\ m/s^2$

$\Rightarrow \frac{0-30}{t-12} = -5$

$\Rightarrow t - 12 = 6$

$\therefore t = 18$

(a)(i) accel $= \frac{10-5}{4-0}$

$= 1\frac{1}{4}\ m/s^2$

Answer (a) (i) $1\frac{1}{4}$ m/s^2 [1]

(ii) $V = 30$ [1]

(iii) $t =$ 18 [1]

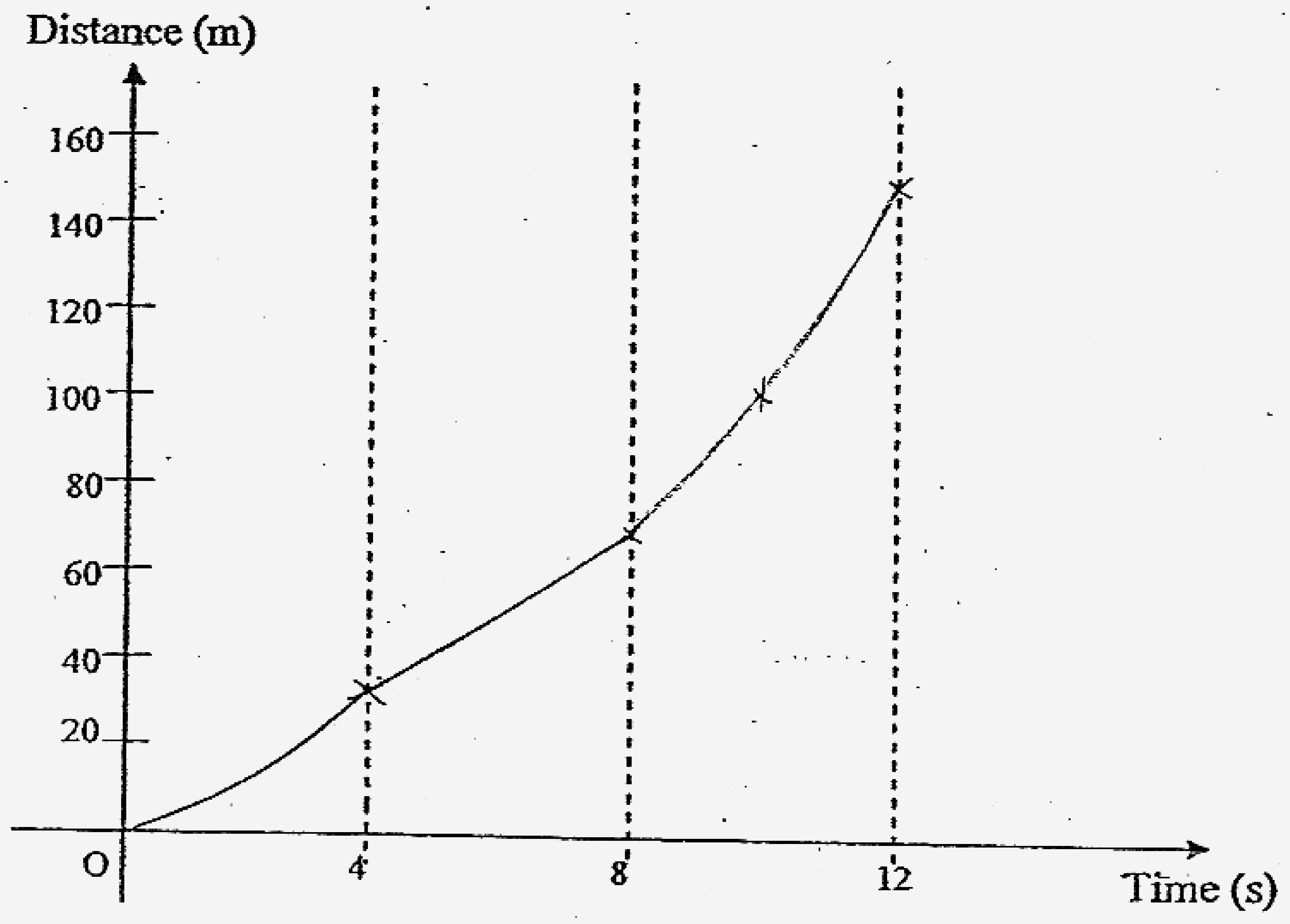
(a)(ii) Given Dist $= 150m$

$\Rightarrow \frac{1}{2}(5+10)(4) + (10)(4) + \frac{1}{2}(10+v)(4) = 150$

$\Rightarrow 2(10+v) = 150 - 40 - 30$

$\therefore v = 30$

(b) Complete the sketch of the distance-time graph given below, for the first 12 seconds of the journey.



[2]

---End of Paper---

Maris Stella High Prelim Paper 2

Section A [88 marks]

Answer all the questions in this section

1. a) Express $\frac{3}{x-1}-\frac{2}{5x+3}$ as a single fraction in its simplest form. [3]

b) Given that $x=\frac{-b-\sqrt{b^2-4ac}}{2a}$, express c in terms of a, b and x. [3]

c) Simplify the expression $\frac{2a^2-5a+3}{2a^2+5a-12}$. [3]

2. [Volume of a cone $= \frac{1}{3} \times$ base area $\times$ height]

a) A beverage company sells tonic drink canned in cylindrical containers as shown in Diagram A. The thickness of the material used for the can is negligible. The radius of the container is 3 cm and the height is 7.5 cm.

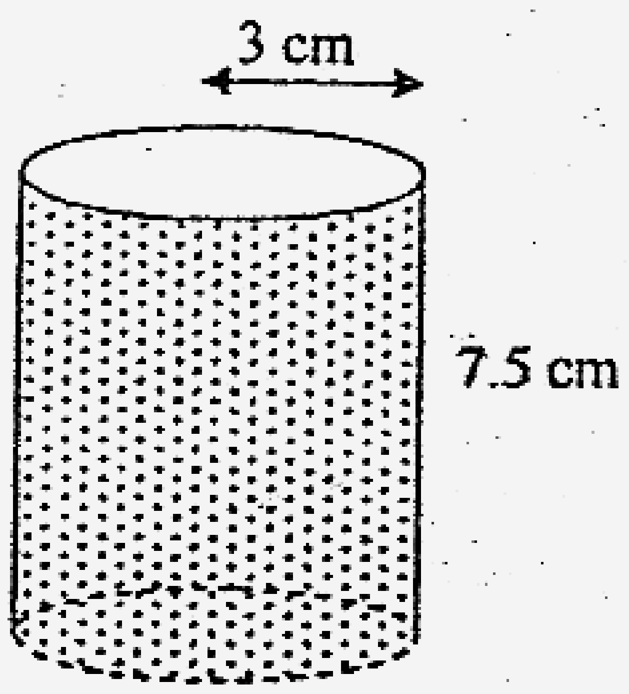


Diagram A

i) Calculate, correct to the nearest cubic centimetre, the volume of tonic drink which will fill the container. [2]

ii) A label is used to completely cover the curved surface of the container. Calculate, correct to the nearest square centimetre, the area of the label. [2]

b) As part of a re-branding campaign, the company decides to use containers in which the bottom part of the inside is a right cone, as shown in Diagram B.

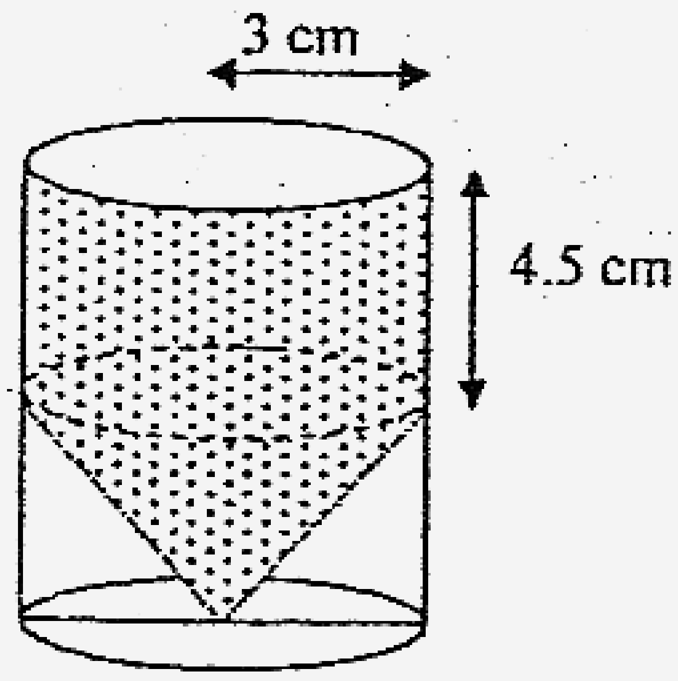


Diagram B

i) Calculate, correct to the nearest cubic centimetre, the volume of tonic drink which will fill the new container. [3]

ii) The price of tonic drink using the original container (as in Diagram A) is $6.40 and the price of tonic drink using the new container (as in Diagram B) is $5.60. Showing your reasoning clearly, state which container is the better value for money. [2]

iii) The beverage company wishes to make a container which is geometrically similar to the one shown in Diagram B and can hold eight times the volume of tonic drink. Find the height of this container. [2]

ı one of the theatre in the Esplanade, there are 30 rows with 40 seats in each row. Seats in ıe first ten rows are priced at $85 each and those in the other rows are priced at $60 each.

) Show that the maximum possible takings are $ 82 000. [1]

) At one performance, there were 140 people sitting at the first ten rows and 350 people sitting in the back twenty rows.

i) Express the attendance as a fraction of the maximum possible attendance, giving your answer in its lowest terms. [1]

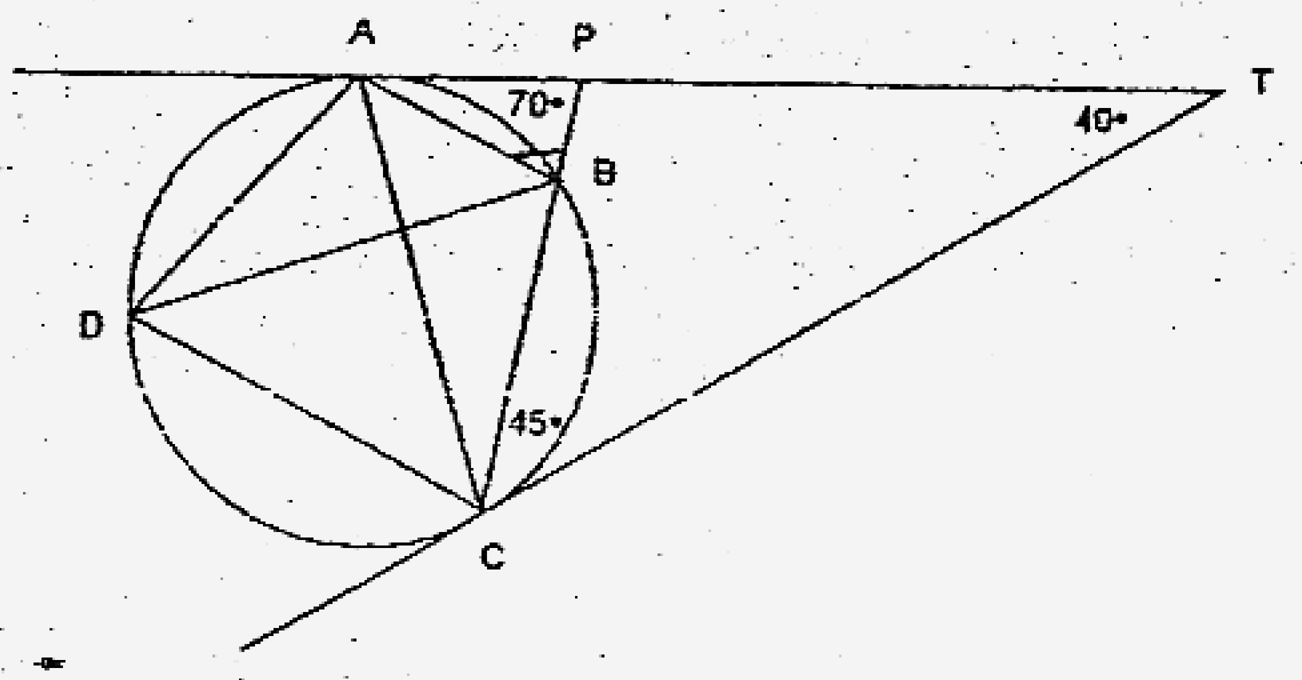
ii) Calculate the amount of money received for the seats at this performance. [1]

iii) Express this amount as a percentage of the maximum possible takings. [1]

At another performance, children under the age of 12 are allowed in the $60-seats only and were charged half price. The theatre was full and the takings were $ 65 500. Calculate the number of children present. [2]

It was decided that for future performances, the price of the seats in the first ten rows would be increased by 25%. Assuming that the theatre is filled with adults, calculate, correct to 2 decimal places, the percentage increase in the maximum possible taking. [2]

4. a) In the diagram on the right, TA and TC are tangents to the circle through A, B, C and D and CB produced meet AT at P. Given that $\angle ABP = 70°$, $\angle ATC = 40°$ and $\angle TCP = 45°$, calculate, stating clearly your reasons,

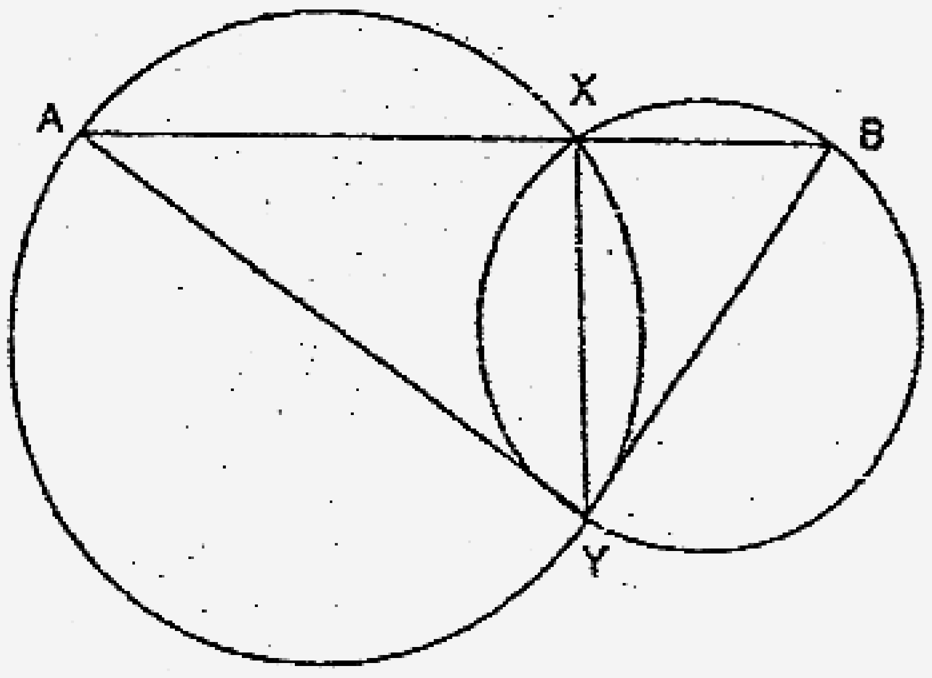


i) $\angle ACB$, [2]

ii) $\angle ADC$, [2]

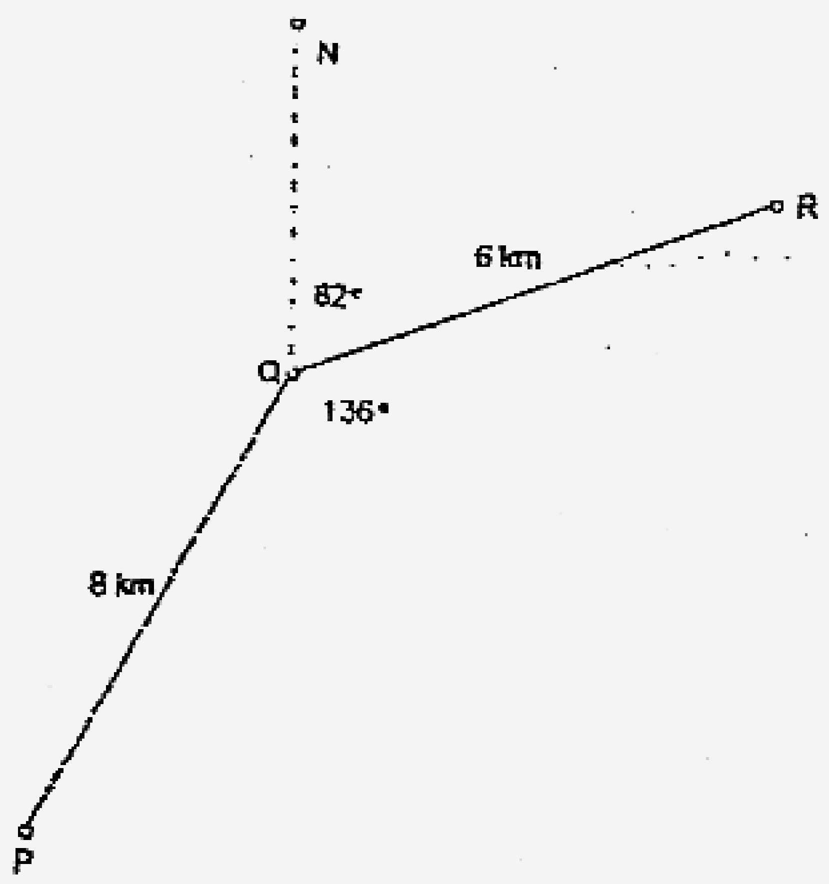
iii) $\angle BDC$. [2]

b) In the diagram below, AXB is a straight line and AY is a diameter of the circle AXY.



Prove that BY is a diameter of the circle BXY. [3]

5. A ship sails 8 km from port, P toward a lighthouse, Q. It then sails 6 km from Q to an island, R on a bearing of 082°



a) Given that $\angle PQR = 136°$, calculate

i) the bearing of Q from P, [2]

ii) how far is Q east of P, [2]

iii) the distance PR. [2]

b) The ship finally sails from R to a position T, which is due north of Q. Given that $\angle QTR = 40°$, calculate

i) the distance RT, [2]

ii) the largest angle of elevation of the top of the lighthouse from the ship as it sails from R to T, given that the height of the lighthouse is 220 m. [3]

Town P and Q are 280 km apart. Rex drove from Town P to Town Q at an average speed of x km/h.

a) Write down an expression, in terms of x, for the time, in hours taken by Rex to travel from Town P to Town Q. [1]

b) On his return journey, his average speed increased by 10 km/h. Write down an expression, in terms of x, for the time, in hours that he took for the journey from Town Q to Town P. [1]

c) Given also that the difference between the two journeys was 30 minutes, form an equation in x, and show that it reduces to $x^2 + 10x - 5600 = 0$ [2]

d) Solve the equation $x^2 + 10x - 5600 = 0$ and hence find the total time that Rex spent travelling. [3]

7. **Answer the whole of this question on a sheet of graph paper.**

A box containing 250 peaches was opened and each peach was weighed. The distribution of the masses of the peaches is given in the following table.

Mass (m grams)	Frequency
$60 < m \leq 100$	20
$100 < m \leq 120$	60
$120 < m \leq 140$	70
$140 < m \leq 160$	40
$160 < m \leq 220$	60

a) When a histogram is drawn to illustrate this information, the height of the column representing peaches with mass m in the interval $60 < m \leq 100$ is 10 cm.
Calculate the height of the column that represents values of m in $160 < m \leq 220$. [2]

b) Calculate an estimate of the mean mass of the peaches in the box. [2]

c) By using a scale of 2 cm to 20 grams on the horizontal axis, construct a histogram to represent the above distribution. Label your axes clearly. [4]

8. **Answer the whole of this question on a sheet of plain paper.**

a) Draw a quadrilateral $ABCD$ in which $AB = 12$ cm, $\angle BAD = 78°$, $AD = 8.2$ cm, $\angle ABC = 105°$ and $\angle ADC = 80°$. Measure and write down the length of AC. [4]

b) On your diagram, draw the locus of all the points within the quadrilateral which are

i) equidistant from A and B, [1]

ii) equidistant from A and D. [1]

Hence draw a circle which passes through the points A, B and D. [1]

c) On your diagram, draw the locus of [1]

i) the points which is equidistant from BC and CD, [2]

ii) the point Z such that area of $\Delta AZB = 24\ \text{cm}^2$.

d) A point F lies within the quadrilateral and is such that

$$\angle BCF \geq \angle DCF, \quad AF \geq BF, \quad \text{area of } \Delta AFB < 24\ \text{cm}^2.$$

On your diagram, shade the region in the quadrilateral in which F must lie. [2]

\nswer the whole of this question on a sheet of graph paper.

\ triangle A has vertices (2, 4), (3, 1), and (5, 1) while triangle B has vertices (8, 4), 6, 10) and (2, 10).

) Using a scale of 1 cm to represent 1 unit on each axis, draw axes for the values of x and y in the ranges $-8 \leq x \leq 10$ and $-10 \leq y \leq 12$ respectively.

Draw and label the triangles A and B. [2]

) The triangle B can be mapped onto triangle A by an enlargement. Find

i) the coordinates of the centre of the enlargement, [1]

ii) the scale factor of the enlargement. [1]

R is a clockwise rotation of 90° about the point (3, -1). Draw and label the triangle $R(A)$. [1]

P is a reflection in the line $y = -2$ and Q is a translation with column vector $\begin{pmatrix} 0 \\ -9 \end{pmatrix}$.

i) Draw and label the triangles $Q(A)$ and $PQ(A)$. [2]

ii) Describe completely the single transformation, which is equivalent to PQ. [2]

Triangle C has vertices (8, 3), (6, 6) and (2, 6). Triangle C can be mapped onto triangle B by a stretch. Find

i) the equation of the invariant line, [1]

ii) the stretch factor. [1]

Triangle D has vertices (-1, 1), (1, 1) and (-8, 4). Describe completely the single transformation, which maps triangle A into triangle D. [2]

Section B [12 marks]

Answer one of the two questions in this section

10. **Answer the whole of this question on a sheet of graph paper**

The amount of weight loss, measured to the nearest kg, of 180 women in a slimming centre were recorded and grouped as shown in the table below.

Weight loss (kg)	6 – 10	11 – 15	16 – 20	21 – 25	26 – 30	31 – 35
No. of women	16	24	a	62	22	8

a) Calculate the value of a. [1]

The corresponding cumulative frequency table for this distribution is as shown below.

Weight loss (kg)	≤ 10	≤ 15	≤ 20	≤ 25	≤ 30	≤ 35
Cumulative frequency	16	40	b	150	172	180

b) Calculate the value of b. [1]

c) Using a scale of 2 cm to represent 5 kg on the x–axis and 2 cm to represent 20 women on the y–axis, draw the cumulative frequency curve. [3]

d) Use the curve to estimate

i) the median, [1]

ii) inter-quartile range. [2]

e) Free gifts are given to women who have lost more than 28 kg.

Calculate the probability that when two women were picked from the group, both will be given the free gifts. [2]

f) The centre charges the women according to the following schemes:

For weight loss that is $\leq$ 10 kg the charge is $ 2 960,

For weight loss that is > 28 kg, the charge is $ 3 050,

All other weight loss is charged at $ 3 000.

Use the curve to estimate the total earnings of the centre from these women, showing relevant and clear workings/markings on how you arrive at your answer. [2]

8. a) AC = 14 cm

c) ii) h = 4 cm

9 b i) COE (4, 4)

ii) -2

d) ii) reflection, y = 2.5

e i) invariant y = 2

SF = 2

f) Shear, y = -1, SF = -2

10 i) a = 48

ii) b = 88

10 iv) a) 20.1 – 20.5 kg

b) 7 – 8 kg

v) $\frac{4}{537}$

vi) $54000 – 54300

Answer the whole of this question on a sheet of graph paper.

Copy and complete the following table of values for the graph of $y = 3x + \frac{60}{x} - 35$. [2]

x	1.5	2	2.5	3	4	5	6	7	8
y	9.5		-3.5	-6			-7	-5.4	

Using a scale of 2 cm to represent 1 unit on the horizontal x-axis, and 1 cm to represent 1 unit on the vertical y-axis, draw the graph of $y = 3x + \frac{60}{x} - 35$ for $1.5 \le x \le 8$. [3]

(a) Use your graph to find

(i) the least value of y, [1]

(ii) the range of values of x for which y is less than -4. [2]

(b) By drawing a suitable tangent to your curve, find the coordinates of the point A at which the gradient of the tangent at A is $-\frac{3}{4}$. [2]

(c) By drawing a suitable straight line, find the solutions to the equation $4x + \frac{60}{x} = 40$. [2]

Answer Key for Preliminary 2 E Math Paper 2 2006

1a	$\frac{13x+11}{(x-1)(5x+3)}$	4a	25°
1b	$c = -ax^2 - bx$	4b	70°
1c	$\frac{a-1}{a+4}$	4c	45°
2ai)	212 cm^3	5ai)	038°
2aii)	141 cm^2	5aii)	4.93 km
2bi)	156 cm^3	5aiii)	13.0 km
2bii)	Original bottle	5bi)	9.24 km
2biii)	15 cm	5bii)	2.5°
3bi)	$\frac{49}{120}$	6a	$\frac{280}{x}$ h
3bii)	$32900	6b	$\frac{280}{x+10}$ h
3biii)	40.1%	6d	$x = -80$ or 70, 7.5 h
3c	550		
3d	10.37%	7i)	20 cm
		7ii)	138.8 g