Geylang Methodist Secondary School
Preliminary Examinations 2005

MATHEMATICS
PAPER 2

4017/2
Dollar version

Secondary 4 Express / 5 Normal

2 hour 30 minutes

Additional materials:
Writing paper
Graph paper (2 sheets)

TIME 2 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, class and register number in the spaces provided on the writing paper.

Write your answers and working on the separate writing paper provided.

Show all your working on the same page as the rest of the answer.

Omission of essential working will result in loss of marks.

Section A

Answer all questions.

Section B

Answer only ONE question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 100.

You are expected to use an electronic calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

This question paper consists of 9 printed pages and 1 blank page.

Section A (88 marks)
Answer ALL the questions in this section

1a). Amy and Betty opened bank accounts with United bank.

    i) Amy deposited $800 in her account. This account pays simple interest at the rate
       of 5% per annum. Calculate the total amount in her account after 3 years. [1]

    ii) Betty deposited $800 in her account. This account pays compound interest at the
       rated of 5% per annum.
       Who will have more money in her account after 3 years? How much more? [3]

  b) Carol borrowed a sum of money at 10% per annum compound interest. After 3
     years, she owed a total of $532.40. Calculate how much she borrowed. [2]

2. (a) The total cost of water per month supplied to a household is calculated as follows :

    Fixed charge per household $= \$7.50$

    Cost of each unit of water $= \$0.86$

    Water conservation tax $= 10\%$ for each unit of water used

    i) Write down in its simplest form the formula connecting the total cost, $C, and

      n, the number of units of water used. [2]

    ii) Calculate the total cost for a household using 14.3 units of water in the month of

      August. [1]

    iii) Calculate the number of units used when the total cost is $26.42 . [1]

  (b) Andrew paid $9.50 for some petrol.

    i) If the petrol costs x cents per litre, write down an expression in terms of x for

      the number of litres of petrol Andrew bought. [1]

    ii) If the price per litre is increased by 5 cents, write down an expression in terms of x for
      the number of litres of petrol he obtained for the same amount of money.

                                                 [1]

    iii) If the amount of petrol obtained is half a litre less, form an equation in x and show that
      this reduces to

$$x^2 + 5x - 9500 = 0 \qquad [3]$$

   iv) Calculate the original price of petrol per litre. [2]

3 a). Given that $\dfrac{2xy}{15} - v = \dfrac{vy}{5} + \dfrac{2x}{3}$, make y the subject of the formula,

leaving your answer as a single fraction in its lowest term. [4]

b). Simplify $\dfrac{x^2 - y^2}{(x+y)^2} \div \dfrac{(x-y)^2}{3y}$ [2]

4. The diagram shows two horizontal triangular fields. ABC and ACD. which are surrounded by fences. It is given that DAB is a straight line, AC = 65 m, AD = 84 m $\angle CAB = 60°$ and $\angle ABC = 72°$. Calculate

   (a)   the length of the fence BC,   [2]
   (b)   the area of the field ACD,   [2]
   (c)   the length of the fence CD,   [2]
   (d)   the shortest distance from C to DB.   [2]

A coconut tree of height 16.2 m is grown at point C.
   (e)   Find the smallest possible angle of elevation of the top of the tree from a point on DB. [2]
   (f)   A boy has climbed exactly half way up the tree when he sees a girl walking along DB. Calculate the biggest angle of depression of the girl when viewed by the boy. [2]
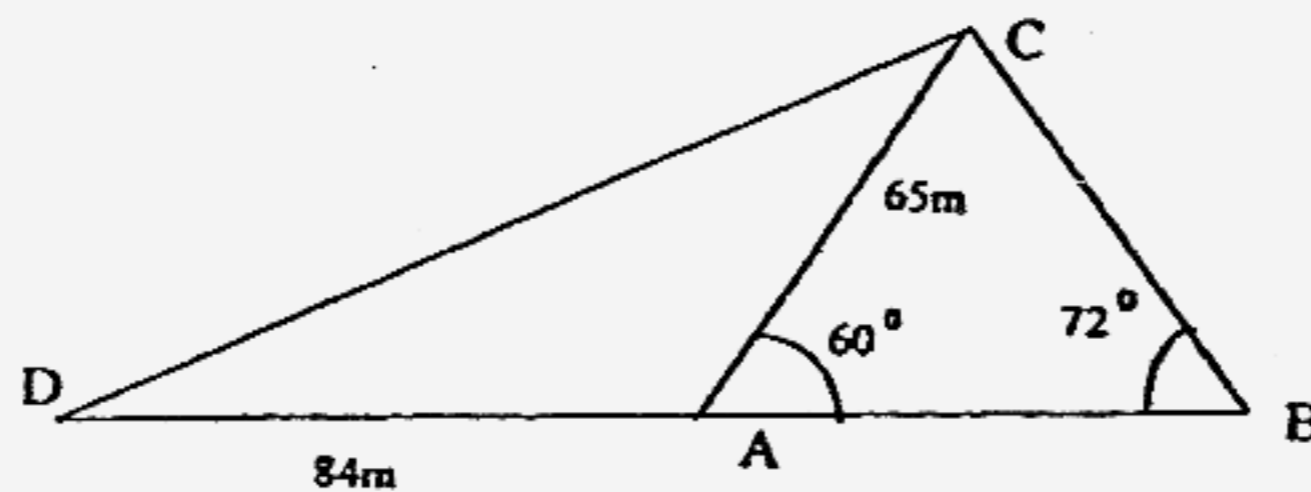
5. Figure 1 shows a trough, constructed by taking a slice of a cylinder of radius 1.5m and length 3.5m. The cross-section ACB of the trough is a segment of a circle. O is the center of this circle and $\angle AOB = 90°$.
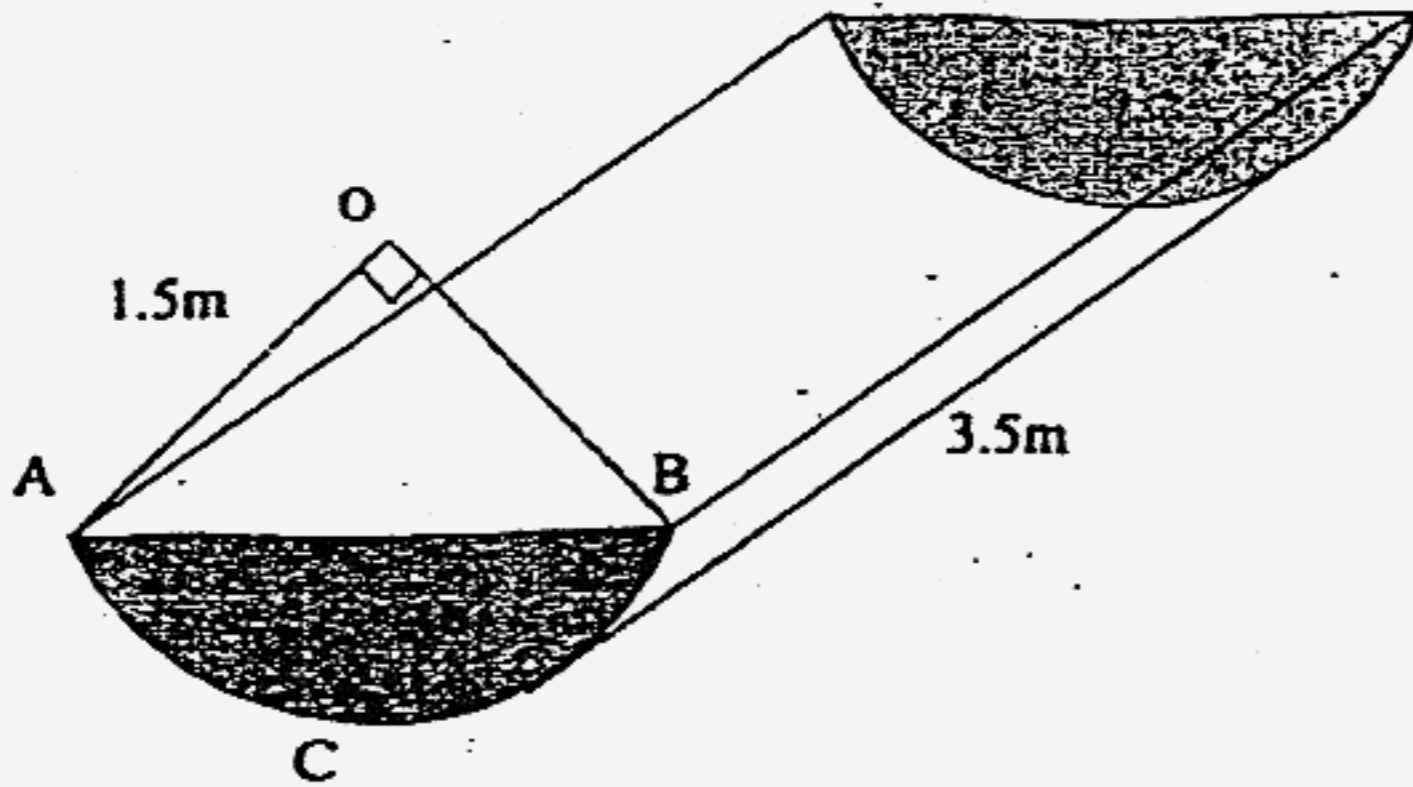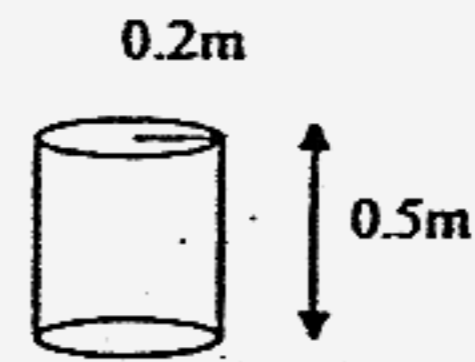

Figure I (trough)

Figure II (bucket)

Water is poured into the trough by using a cylindrical bucket shown in Fig II, which has radius 0.2 m and height 0.5 m.

Using $\pi = 3.142$, find

(a) the area of the segment ACB, [3]
(b) the volume of the trough, [1]
(c) the total internal surface area of the trough, [3]
(d) the volume of the bucket. [1]

The trough is to be filled completely with water from the cylindrical bucket.

(e) What is the minimum number of buckets that must be used? [2]

6 (a) Two points P and Q have position vectors p and q respectively, relative to the origin O.
   Given that $\mathbf{p} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ and $\overrightarrow{PQ} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$.

   (i) Find q. [1]
   (ii) Calculate the length of PQ. [1]
   (iii) Given that $2\overrightarrow{PX} = \overrightarrow{QP}$, find the position vector of X. [2]

   (b) ABCD is a parallelogram. X is a point on BC such that $BX : XC = 2:3$. Given that $\overrightarrow{AB} = 6p + 3q$ and $\overrightarrow{AD} = 10p - 5q$.

   (i) Express $\overrightarrow{AX}$ and $\overrightarrow{DX}$ in terms of p and q. [2]

(ii)   Given that area of $\triangle AXB = 4$ cm^2, calculate the numerical value of

$$\frac{\text{Area of } \triangle AXB}{\text{Area of } ABCD}$$

and hence the area of $ABCD$.

[2]

(iii)   Given that Y is on AB produced such that DXY is a straight line. Express $\overrightarrow{AY}$ in terms of p and q.

[3]

10p - 5q

6p + 3q
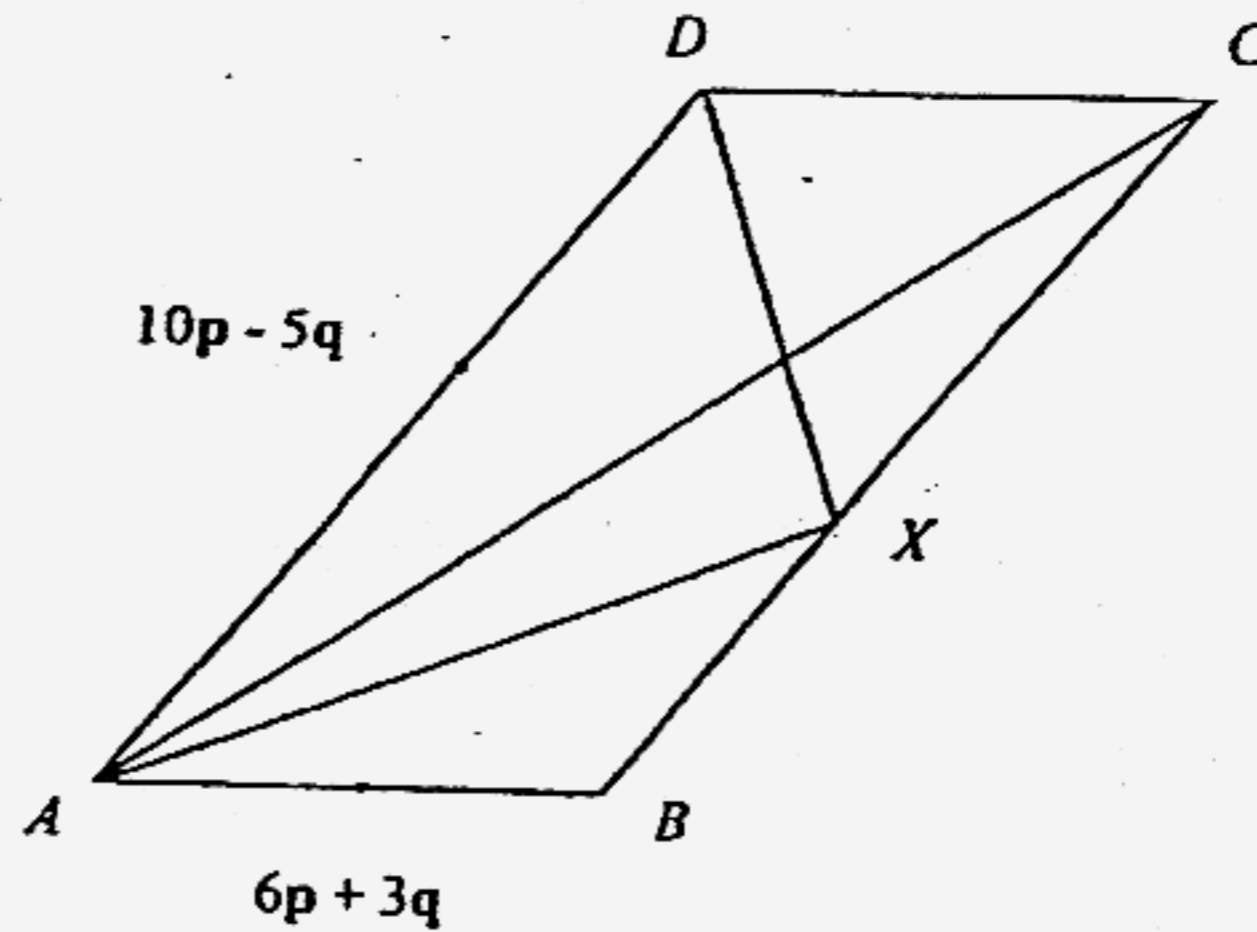
7.
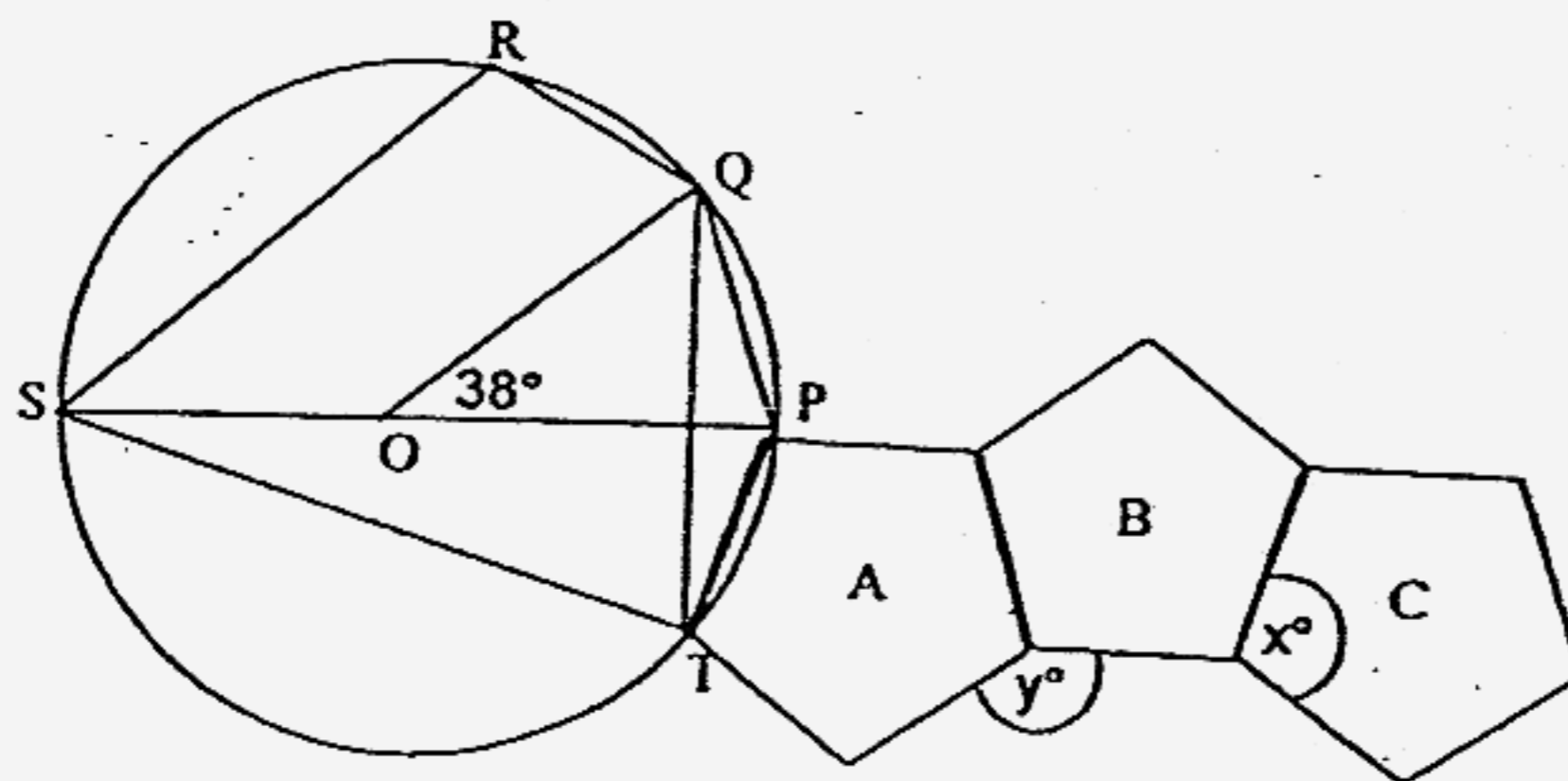
In the diagram, P, Q, R, S and T lie on the circumference of a circle center O. OQ is parallel to SR. SP is the diameter of the circle. $\angle POQ = 38°$. A, B and C are three regular pentagons.

(a) Find
    (i) $\angle PTS$,   [1]
    (ii) $\angle OPQ$,   [1]
    (iii) $\angle OSR$,   [1]
    (iv) $\angle QRS$,   [1]
    (v) $\angle PTQ$,   [1]
    (vi) $\angle QTS$.   [1]

(b) Find
    (i) x ,   [2]
    (ii) y .   [1]

(c) Additional pentagons are added to these three pentagons to form a closed ring surrounding a regular polygon. What is the total number of pentagons which form the ring?
                                                    [2]

8   **Answer the whole of this question on a sheet of graph paper.**

(a) The distribution of marks scored by a group of 360 students in a test is given in the table below:

Mark	20	30	40	50	60	70	80	90	100
Number of students who scored less than or equal to this mark.	0	6	16	30	54	132	288	350	360

Using a vertical scale of 1 cm to represent 20 students and a horizontal scale of 2 cm to represent 10 marks, plot these values and draw a smooth curve through your points.   [3]

(b) Use your graph to find
    (i)   median mark,   [1]
    (ii)   the interquartile range,   [1]
    (iii)   the percentage of students who scored more than 75 marks.   [2]

(c) Based on the students' performance in the test, the students are being classified according to 3 categories as follows:
        T refers to top 15% of the student population,
        M refers to the next 70% and
        B refers to the bottom 15% of student population.
Find the minimum mark obtained by students in the T category and the maximum mark obtained by students in the B category.   [2]

(d) From the result of (c), do you think this is a good test? Explain your reason(s).   [1]

(e) Find the probability that 2 students chosen at random will each have a mark greater than 70.
                                                   [2]

9. Consecutive even numbers are arranged in the following pattern,

Row	Pattern	Sum of row (S)	Number of even numbers (N)	Average of row (A)
1	2	2	1	2
2	4 6	10	2	5
3	8 10 12	30	3	10
4	14 16 18 20	68	4	17
5	22 24 26 28 30	130	5	26
6				
7				

(a) Copy and complete the sixth and seventh rows of the table. [2]
(b) Write down a formula connecting A and N. [1]
(c) Write down a formula connecting S and N. [2]
(d) Write down the value of S and of A for the 12th row. [2]
(e) Give a simple reason why the number 8100 could not be the average of any row. [2]

Section B (12 marks)
Answer ONLY ONE question

10. **Answer the whole of this question on a single sheet of graph paper.**

The vertices of $\triangle ABC$ are $A(1, 1)$, $B(2, 2)$ and $C(0, 3)$. The vertices of triangle $\triangle A_1B_1C_1$ are $A_1(-2, 2)$, $B_1(-3, 3)$ and $C_1(-4, 1)$.

(a) Using a scale of 2 cm to 1 unit on each axis, draw, x- and y-axes for $-4 \leq x \leq 4$ and $-2 \leq y \leq 7$. Draw and label $\triangle ABC$ and $\triangle A_1B_1C_1$. [1]

(b) (i) $\triangle ABC$ is mapped onto $\triangle A_1B_1C_1$ by a single transformation. Describe this single transformation clearly. [2]

    (ii) $\triangle ABC$ can also be mapped onto $\triangle A_1B_1C_1$ by two successive transformations, the second being a translation. Write down the translation vector. [2]

(c) $\triangle A_2B_2C_2$ are such that its vertices are $A_2(2,0)$, $B_2(3, 1)$ and $C_2(4, -1)$. ABC is mapped onto $\triangle A_2B_2C_2$ by a reflection. Write down the equation of this line of reflection. [1]

(d) A shear with shear factor 3 parallel to the y-axis maps $\triangle ABC$ onto $\triangle AB_3C_3$. Draw and label clearly $\triangle AB_3C_3$. [2]

(e) The enlargement with factor 2 and center $(1, 2)$ maps $\triangle ABC$ onto $\triangle A_4B_4C_4$. Draw and label clearly $\triangle A_4B_4C_4$. Hence or otherwise, state the ratio of the area $\triangle ABC$: area $\triangle A_4B_4C_4$. [2]

(f) A stretch parallel to the x-axis with stretch factor 5 maps the point A onto $(-3, 1)$. Determine the equation of the invariant line and hence, write down the coordinates of the image of B. [2]

8

11 Answer the whole of this question on a piece of graph paper

The variables x and y are connected by the equation $y = 5 - \dfrac{x^2}{10} - \dfrac{4}{x}$.

Some corresponding values of x and y are given in the following table.

x	0.5	0.7	1	2	3	4	5	6	7	8
y	-3.0	-0.8	0.9	2.6	2.8	2.4	1.7	0.7	-0.5	-1.9

a) Taking 2 cm to represent 1 unit on each axis, draw the graph of $y = 5 - \dfrac{x^2}{10} - \dfrac{4}{x}$ for values of x in the range $0.5 \le x \le 8$.　　　　　　　　　　　[2]

b)　i) Find the x-coordinates of the points on the curve when $y = 2.5$　　[2]

　　ii) Write down but do not simplify the equation in x which has these values as its solutions.　　　　　　　　　　　　　　　[1]

c) By drawing a tangent, find the coordinates of the point P on the curve
　　$y = 5 - \dfrac{x^2}{10} - \dfrac{4}{x}$　such that the gradient at P is $-\dfrac{1}{3}$.　　　　[2]

d) By drawing suitable straight lines, use your graph to solve

　　i)　$4 - \dfrac{x^2}{10} - \dfrac{4}{x} \le 0$　,　　　　　　　　　　　[2]

　　ii)　$x^3 + 5x^2 - 50x + 40 = 0$.　　　　　　　　　　[3]

End of Paper

Answers

1ai) $920 ii) Betty will have $6.10 more b) $400	7ai) 90° ii) 71° iii) 38° iv) 109° v) 19° vi) 71° bi) 108° ii) 144° c) 10

2ai) $C = 0.946\,n + 7.5$
  ii) $21.03
 iii)   20
 	
bi) $\dfrac{950}{x}$
 	
ii) $\dfrac{950}{x+5}$
iv)   95 cents
	8bi) 73
	  ii) 13
	 iii) 40%
	  c) min mark = 82
	       max mark = 60
	  d) Not a good test. It is too easy. Many
	      students got high marks.
	  e) $\dfrac{4313}{10770}$

3a) $y = \dfrac{5(3v + 2x)}{(2x - 3v)}$
 	
b) $\dfrac{3y}{x^2 - y^2}$
	9a) 32 34 36 38 40 42, S = 222, N = 6, A = 37
	     44 46 48 50 52 54 56, S = 350, N = 7, A = 50
	  b) $A = N^2 + 1$
	  c) $S = N(N^2 + 1)$
	  d) $S = 1740$, $A = 145$
	  e) If $A = 8100$, then $N = 89.994$ (but N
	     must be a whole number)

4a) 59.2 m
 b) 2360 m²
 c) 129 m
 d) 56.3 m
 e) 7.1°
 f) 8.2°
	10bi) 90° anticlockwise rotation about (-1,0)
	  ii) $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$
	  c) $y = x - 1$
	  d) $B_3\,(2,5)$, $C_3\,(0,0)$
	  e) 1 : 4
	  f) $x = 2$ Image of B = (2,2)

5a) 0.642 m²
 b) 2.25 m³
 c) 9.53 m³
 d) 0.06284 m³
 e) 36
	11bi) 1.85 or 3.85
	  ii) $5 - \dfrac{x^2}{10} - \dfrac{4}{x} = 2.5$
	  c) (3.3 , 2.7)
	  di) $1.05 \le x \le 5.7$
	  ii) 0.9 or 4.35

6ai) $\begin{pmatrix} 3 \\ 8 \end{pmatrix}$ ii) 10 iii) $\begin{pmatrix} -9 \\ -1 \end{pmatrix}$ bi) 10p + q , 6q ii) 20 cm³ iii) 5(2p + q)	

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