

Name	Index Number	Class
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**CRESCENT GIRLS' SCHOOL  
SECONDARY FOUR  
PRELIMINARY EXAMINATION 2006**

**MATHEMATICS**

**4017/1**

Paper 1

11 SEP 2006

Candidates answer on the Question Paper.  
Additional materials: Geometrical instruments

**2 hours**

**READ THESE INSTRUCTIONS FIRST**

Write your name, index number and class in the spaces at the top of this page.  
Write in dark blue or black pen in the spaces provided on the Question paper.  
You may use a pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters or correction fluid.

Answer all questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below that question.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 80.

**NEITHER ELECTRONIC CALCULATORS NOR MATHEMATICAL TABLES MAY BE USED  
IN THIS PAPER.**

For Examiner's Use

This question paper consists of 13 printed pages, including the cover page.

NEITHER ELECTRONIC CALCULATORS NOR MATHEMATICAL TABLES  
MAY BE USED IN THIS PAPER

1 Given that  $\sqrt{4.89} = 2.211$  and  $\sqrt{48.9} = 6.993$ , evaluate,

(a)  $\sqrt{48900}$ ,

(b)  $\sqrt{0.489}$ .

Answer (a).....[1]

(b).....[1]

2 A bus is supposed to start at 12 25 and to reach its destination at 13 50. It starts 3 minutes late and arrives 10 minutes late. How long did it take to reach its destination?

Answer ..... h ..... min[2]

3 At a winery, one pipe can fill a tank with wine in 3 hours and another pipe can empty the tank in 5 hours. If the valves to both pipes are open, it takes  $t$  hours to fill the empty tank. Find  $t$ .

Answer  $t =$ .....[2]

- 4 (a) Expressed as the product of prime factors,  
 $630 = 2 \times 3^2 \times 5 \times 7$  and  $495 = 3^2 \times 5 \times 11$ .  
 Use these results to find the smallest integer  $m$ , such that  $630m$  is a perfect square.
- (b) If  $2^9 \times 3^6 \times 5^3 = N$ , find the value of  $\sqrt[3]{N}$ .

Answer (a)  $m = \dots\dots\dots[1]$

(b)  $\sqrt[3]{N} = \dots\dots\dots[1]$

- 5 One angle of a polygon is  $140^\circ$  and the rest are each  $40^\circ$  smaller. Find the number of sides of the polygon.

Answer  $\dots\dots\dots[2]$

- 6 If  $\frac{1}{x} + \frac{1}{y} = \frac{1}{a}$  and  $x + y = b$ , find the following in terms of  $a$  and  $b$ .

(a)  $xy$ , (b)  $x^2 + y^2$ .

Answer (a)  $xy = \dots\dots\dots[1]$

(b)  $x^2 + y^2 = \dots\dots\dots[2]$

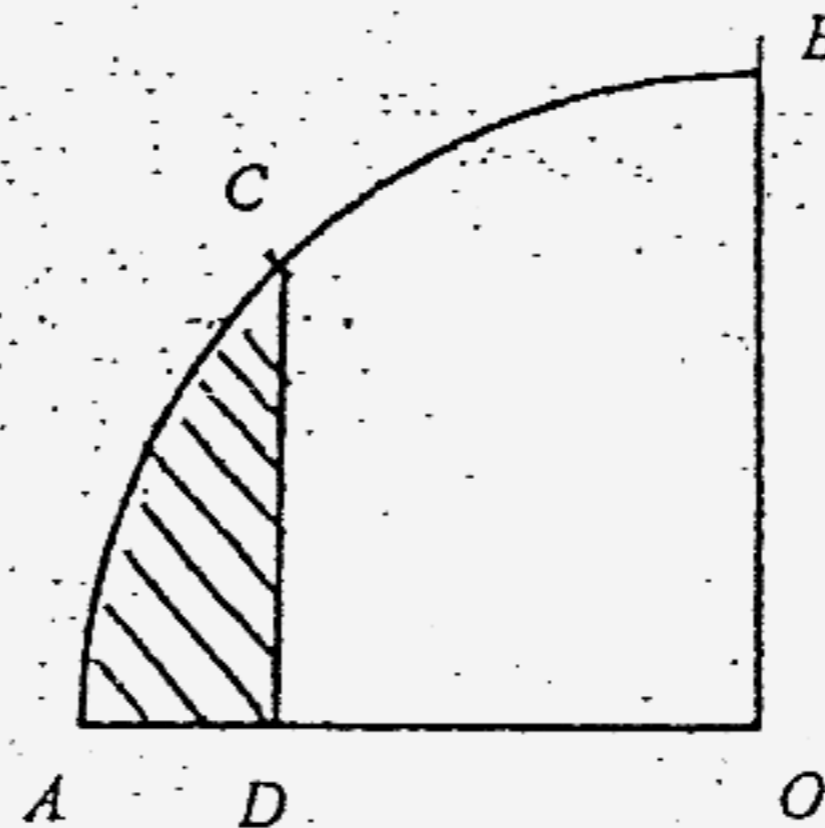
7 (a) Solve for  $x$  given that  $x^{\frac{2}{3}} = \frac{1}{16}$ .

(b) Express  $\frac{3}{1+x-2x^2} + \frac{x}{x-1}$  as a single fraction in its simplest form.

Answer (a)  $x = \dots\dots\dots$  [1]

(b)  $\dots\dots\dots$  [3]

- 8 In the figure,  $AOB$  is a quadrant of radius 7 cm,  $C$  is the midpoint of arc  $AB$ , and  $CD$  is perpendicular to  $AD$ . Find the area of the shaded part. (Take  $\pi = \frac{22}{7}$ )



Answer  $\dots\dots\dots$  cm<sup>2</sup> [3]

9

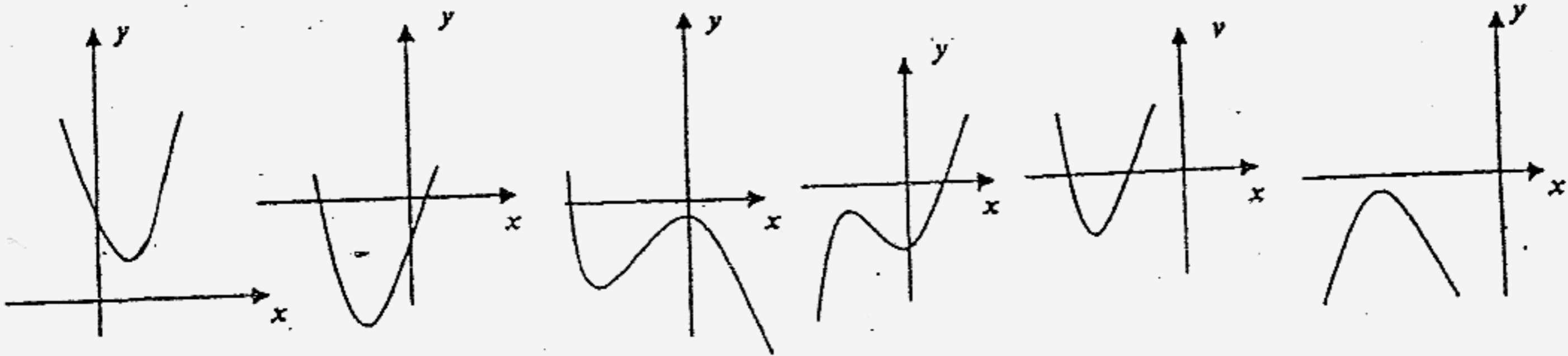


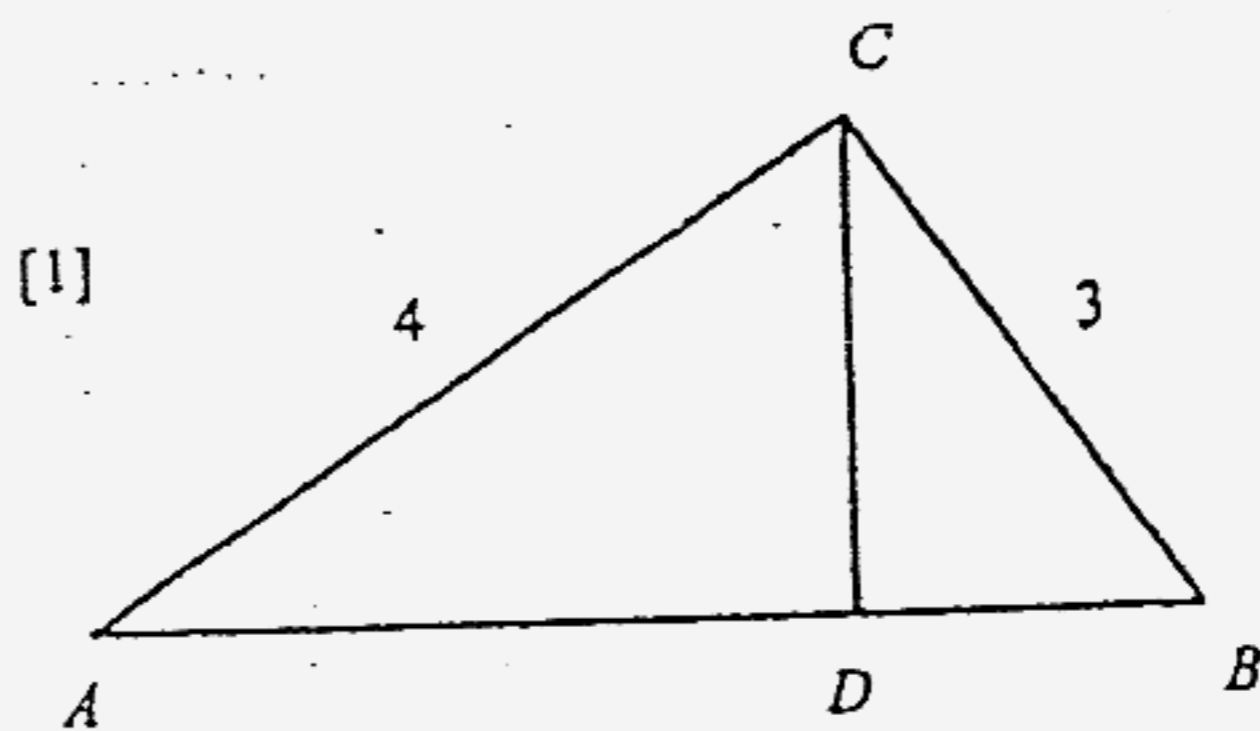
Figure 1      Figure 2      Figure 3      Figure 4      Figure 5      Figure 6

Which of the graphs shown above could be the graph of

- (a)  $y = x^2 + 3x - 4$ ,
- (b)  $y = x^3 + 2x^2 - 4$ ,
- (c)  $y = 2(x + 3)^2 - 1$ .

Answer (a) Figure .....[1]  
 (b) Figure .....[1]  
 (c) Figure .....[1]

10 In the diagram,  $AC$  is perpendicular to  $BC$  and  $CD$  is perpendicular to  $AB$ .  
 Show that triangle  $ACD$  is similar to triangle  $CBD$ .  
 Hence, find the length of  $BD$ , given that  $AC$  is 4 cm and  $BC$  is 3 cm.



[1]

Answer  $BD =$  ..... cm [2]

- 11 Given that  $p = 1.2 \times 10^{-6}$  and  $q = 4.32 \times 10^{-4}$ .  
 Calculate the following, expressing each answer in standard form.
- (a)  $p + q$ , giving your answer correct to 3 significant figures,
- (b)  $\frac{q}{4p}$ .

Answer (a) .....[2]

(b) .....[2]

- 12 Factorise the following completely.

(a)  $2p + 6 - pq - 3q$ ,

(b)  $12x^4 - 27x^2$ ,

(c)  $7 - 2x^2 - 13x$ .

Answer (a) .....[1]

(b) .....[2]

(c) .....[1]

13 A nature reserve of area  $225 \text{ km}^2$  is represented on a map by an area of  $36 \text{ cm}^2$ . Find

- (a) the scale of the map, in the form  $1 : n$ ,
- (b) the area of the nature reserve on a map, in  $\text{cm}^2$ , whose scale is 1 cm to 5 km.

Answer (a) ..... [2]

(b) .....  $\text{cm}^2$  [2]

14 Given that  $5 \leq x \leq 10$ ,  $y = \frac{k}{x^3}$ , where  $k$  is a constant and the smallest value of  $y$  is 2, find

- (a) the value of  $k$ ,
- (b) the largest value of  $y$ ,
- (c) the largest value of  $\frac{x-y}{xy}$ .

Answer (a)  $k =$  ..... [1]

(b)  $y =$  ..... [1]

(c) ..... [2]

15  $y$  is inversely proportional to the square root of  $x$ .

Given that  $x = 32$  for a particular value of  $y$ ,

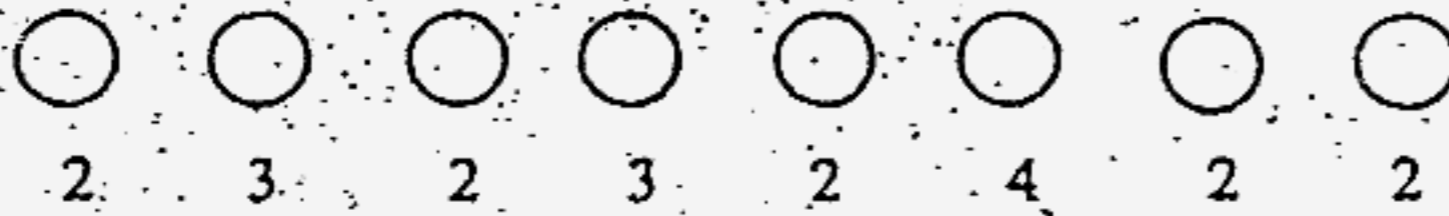
- (a) describe the change in  $y$  when  $x$  increases by four times,
- (b) find the value of  $x$  when the particular value of  $y$  is doubled.

Answer (a) ..... [2]

(b)  $x =$  ..... [2]

16 A ball is dropped at random into one of the eight holes, shown as  $\bigcirc$  in the diagram.

The number under each hole gives the score obtained when the ball drops into that hole.



- (a) State the probability of scoring 2.  
If two identical balls are dropped into the holes one after another, find the probability
- (b) that one ball scores a 3 and another ball scores a 4,
- (c) of scoring a total of 6.

Answer (a) ..... [1]

(b) ..... [2]

(c) ..... [2]

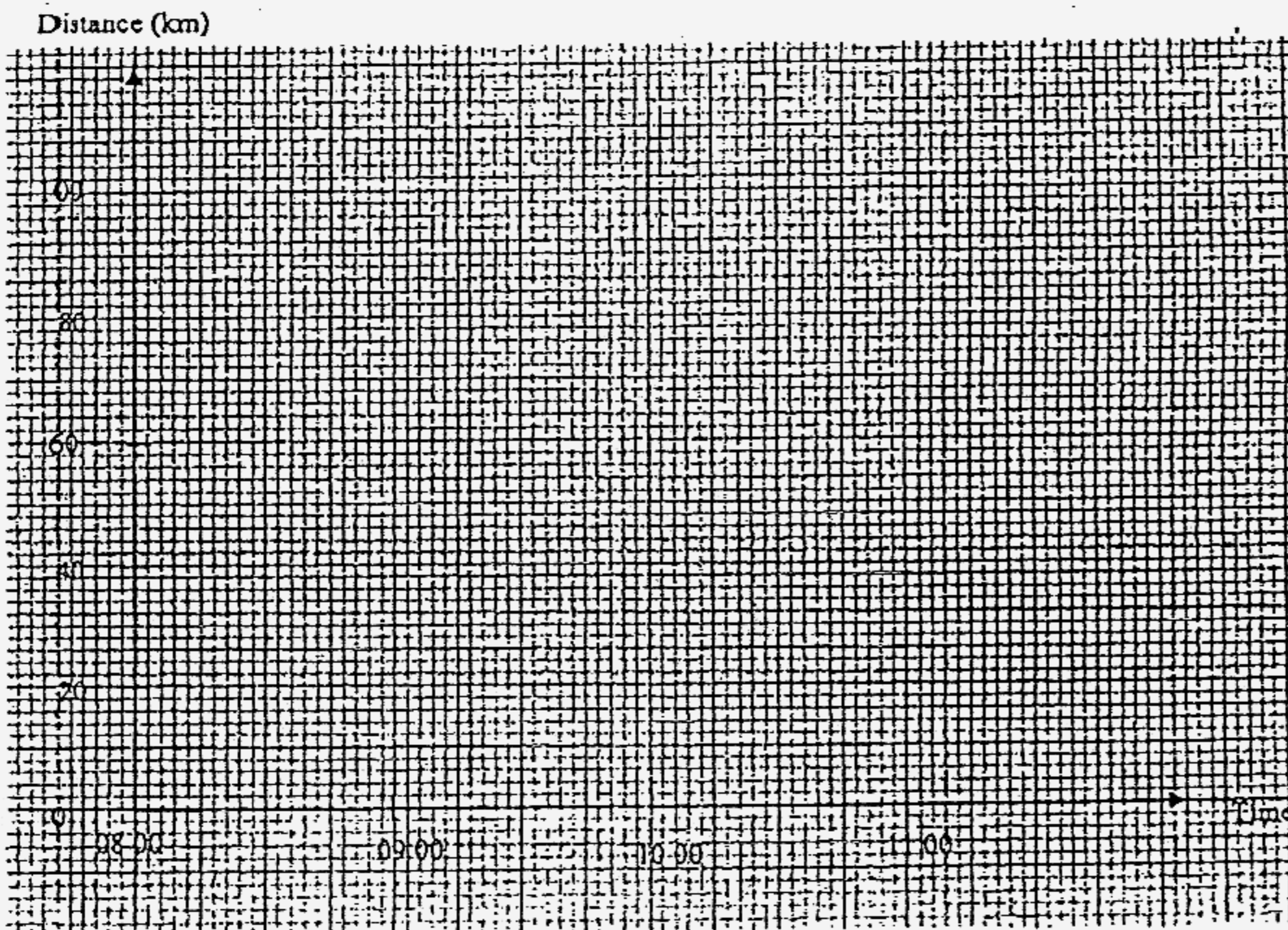


17 Mr Johnson leaves  $X$  at 08 00 for  $Y$  which is 90 km away. For the first 1.5 h, he travels at a steady speed of 40 km/h. Then he stops for half an hour, after which he completes his journey at a steady speed of 45 km/h.

- (a) Draw a distance-time graph.
- (b) Use the graph to state the time at which Mr Johnson reaches  $Y$ .

At 08 45, Mr Thomas leaves  $Y$ , traveling at a steady speed and passes Mr Johnson at 09 15.

- (c) Draw the distance-time graph of Mr Thomas.
- (d) Hence find the speed at which Mr Thomas travels.



[2], [1]

Answer (b) ..... [1]

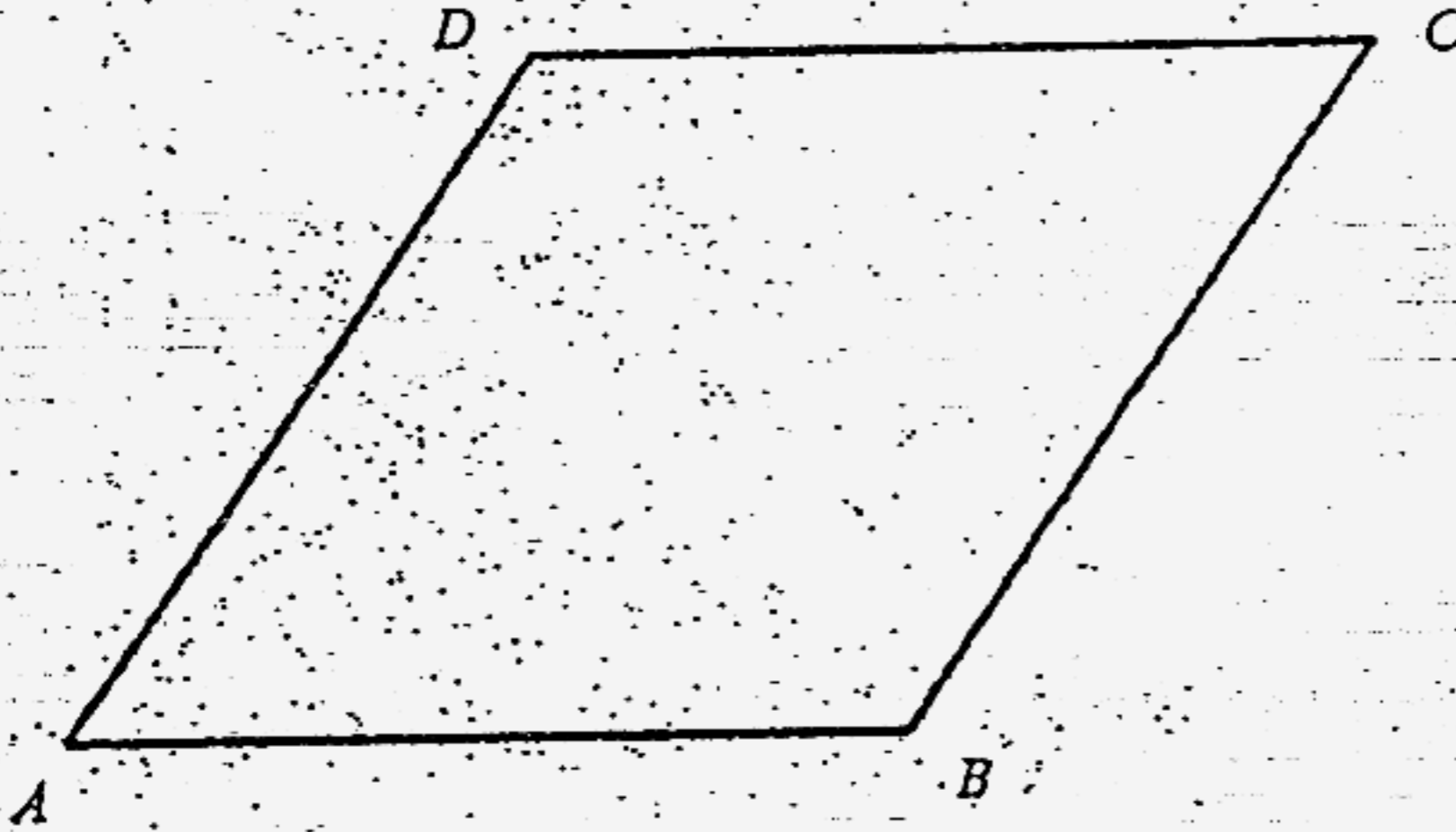
(d) ..... km/h [1]

18 (a) Using the rhombus provided, construct the locus of points that are equidistant.

- (i) from  $AB$  and  $AD$ ,
- (ii) from  $A$  and  $B$ ,
- (iii) from  $A$ , as the point  $B$  from  $D$ .

(b) The position of point  $X$ , which lies inside the rhombus, is such that  $AX \leq BD$ ,  $AX \geq XB$  and  $X$  is nearer to  $AB$  than to  $AD$ . Shade the region representing the set of possible positions of  $X$ .

[5]

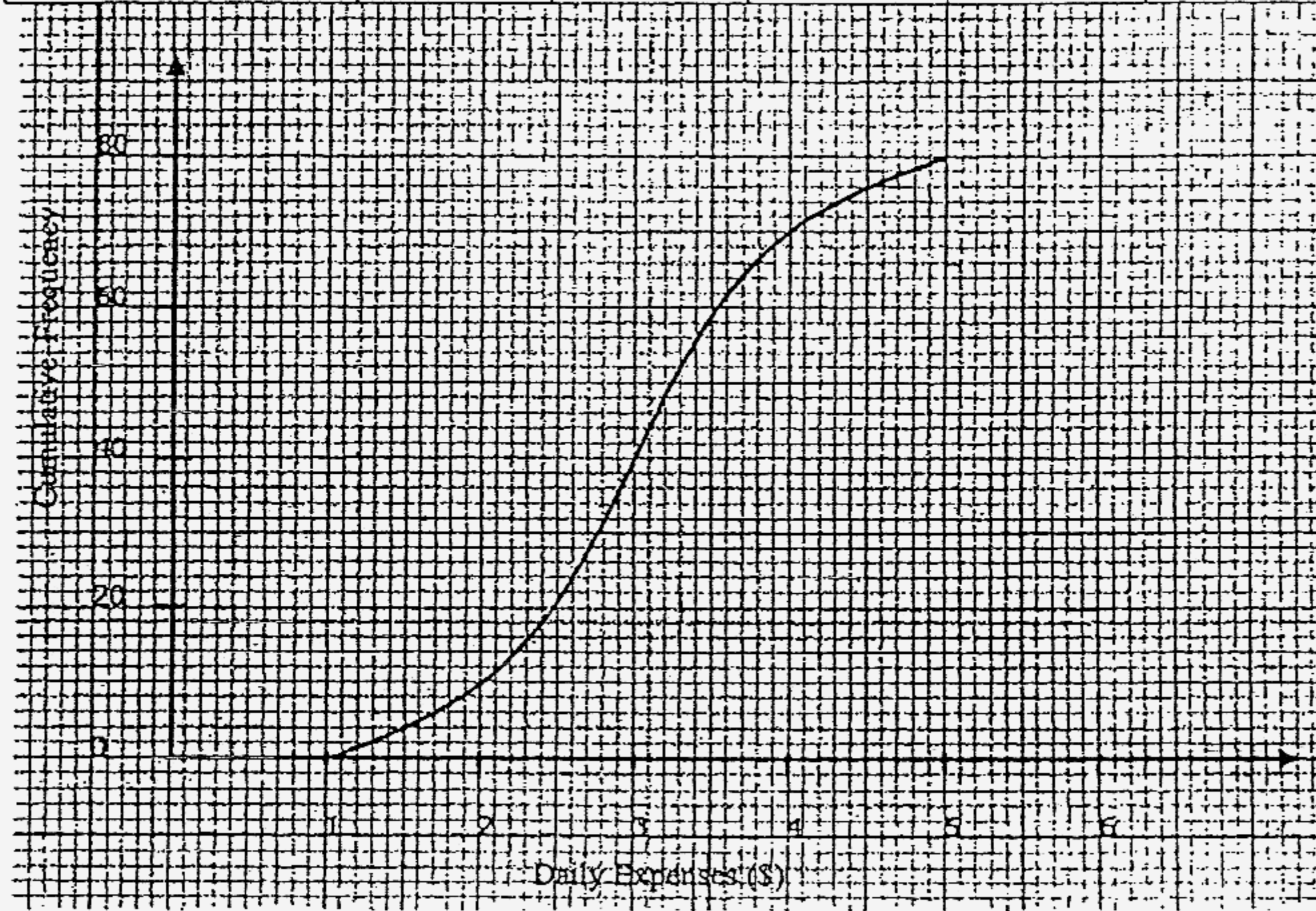


19 The graph is a cumulative frequency curve showing the daily expenses of 80 students. Use the graph to find, as accurately as possible,

- (a) the median,
- (b) the interquartile range,
- (c) the number of students who spend more than \$3.60,
- (d) the values of  $a$ ,  $b$  and  $c$  in the table below.

If 20% of the students spend less than \$ $p$ , find  $p$ .

Daily Expenses \$ $x$	$x < 1$	$1 \leq x < 2$	$2 \leq x < 3$	$3 \leq x < 4$	$4 \leq x < 5$
No. of students	0	10	$a$	$b$	$c$



Answer (a) ..... [1]

(b) ..... [1]

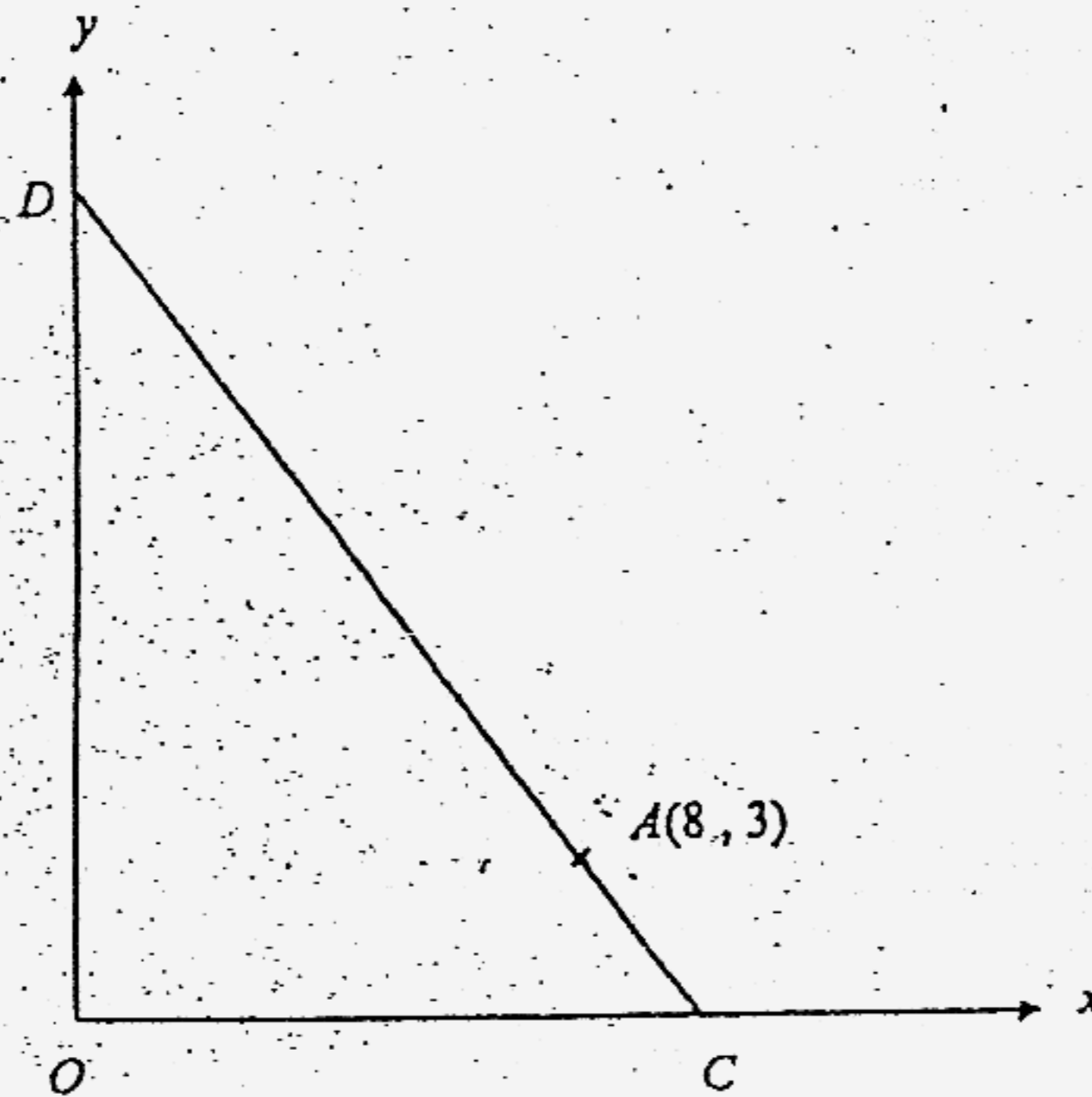
(c) ..... [1]

(d)  $a =$  .....,  $b =$  .....,  $c =$  ..... [2]

$p =$  ..... [1]

20 In the figure,  $O$  is the origin and  $A$  is the point  $(8, 3)$ .

- $B$  is a point on the  $x$ -axis such that gradient of  $AB$  is 2. Find the coordinates of  $B$ .
- $C$  is another point on the  $x$ -axis such that the length of  $AB$  is equal to the length of  $AC$ . Find the equation of  $AC$ .
- Write down the coordinates of  $D$ .
- Find the equation of the straight line passing through the midpoint of  $AD$  and parallel to the line  $x = 0$ .
- Find the area of  $\triangle ABC$ .



Answer (a)  $B$  (....., .....) [1]

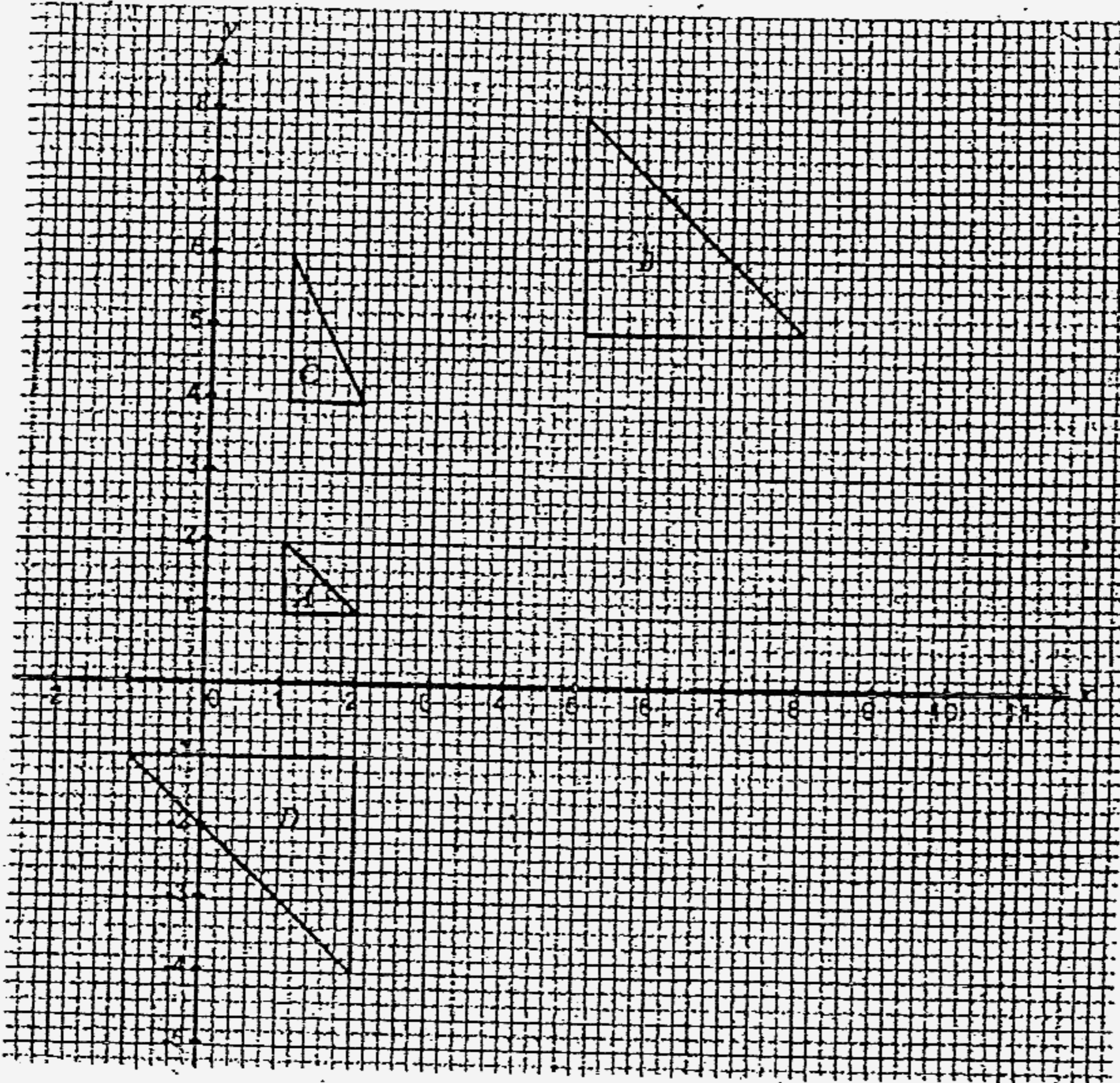
(b) ..... [2]

(c)  $D$  (....., .....) [1]

(d) ..... [1]

(e) ..... units<sup>2</sup> [1]

- 21 (a) Describe the transformation that maps triangle  $A$  onto  
 (i) triangle  $B$ ,  
 (ii) triangle  $C$ .
- (b) Triangle  $B$  is mapped onto triangle  $D$  by a vertical translation followed by a reflection. Find (i) the translation vector,  
 (ii) the equation of the line of reflection.
- (c) The transformation  $H$  is a shear parallel to the  $x$ -axis, with  $x$ -axis invariant and scale factor  $-2$ . On the diagram, draw and label  $H(D)$ .



[1]

Answer (a)(i) ..... [2]

(ii) ..... [2]

(b)(i)  $\left( \begin{array}{c} \phantom{0} \\ \phantom{0} \end{array} \right)$  [1] (ii) ..... [1]

Section A [88 marks]

Answer all the questions in this section.

1. The list price of a bottle of omega 3 cost \$63.50 and a bottle of vitamin E cost \$39.20.
- (a) Mrs. Fong bought a bottle of omega 3 and a bottle of Vitamin E at the list price. In addition, she had to pay 5% GST. Calculate the total amount she paid for her purchases. [1]
- (b) During the Chinese New Year sale, the list price of omega 3 was \$76.20 per bottle and a discount of 20% was given. Assuming the GST remains the same, calculate the actual percentage discount. [3]
- (c) During the Singapore Sale, a discount of  $p\%$  was given. Mrs. Fong paid \$212.24, inclusive of 5% GST, for three bottles of omega 3 and one bottle of vitamin E. Calculate the value of  $p$ . [3]
- (d) Nancy the sales representative, gets 5% commission on the list price of all the products she sold. On a particular Sunday, she sold twice as many bottles of omega 3 as vitamin E. That day, she collected \$116.34 commission. Calculate
- (i) the total sales for that day, [1]
- (ii) the number of bottles of omega 3 sold. [2]

2. The following stem and leaf diagram shows the masses, to the nearest gram, of 34 eggs.

Stem	Leaf
6	3 4 9 0 5 4 6 8
7	5 4 3 7 4 3 0 4 5 1 2 6 2 7 6 9
8	0 3 1 5 7
9	1 2 1 2 4

(a) State the mode. [1]

(b) State the median. [2]

The table below gives the information in a different form.

Mass	$60 \leq x < 70$	$70 \leq x < 75$	$75 \leq x < 80$	$80 \leq x < 95$
No. of eggs	8	$p$	7	10

(c) Find the value of  $p$ . [1]

(d) Draw a histogram to represent this information, given that the class interval  $75 \leq x < 80$  has a width of 1 cm and a height of 7 cm. [3]

(e) Two eggs are picked. What is the probability that both belong to the modal class? [2]

3(a) Given that  $s = \frac{3r}{\sqrt{4t^2 - 9}}$ , express  $t$  in terms of  $r$  and  $s$  in its simplest form. [3]

(b) Mr. Gomes bought a basket of durians for \$600. He paid \$ $x$  for each durian.

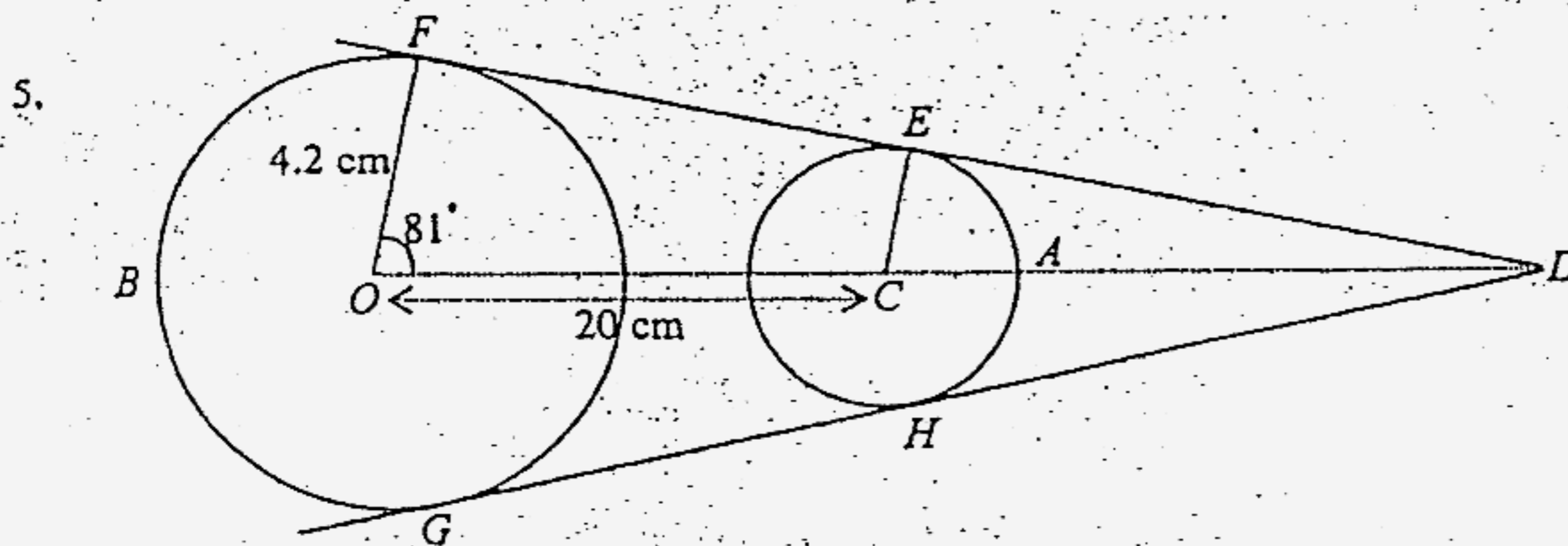
(i) Write down an expression in terms of  $x$  for the number of durians he bought. [1]

(ii) As 10 durians were rotten, he decided to sell each of the remaining durians at a profit of \$1.20. Write down an expression in terms of  $x$  for the sum of money he would receive if all the remaining durians were sold. [2]

(iii) He sold all the durians and made a profit of \$90. Write down an equation in  $x$  to represent this information, and show that it reduces to  $5x^2 + 51x - 360 = 0$ . [2]

(iv) Solve the equation and find the cost price of each durian. [2]

- 4(a) Mr. Lim intends to deposit \$6000 for 8 years. Bank B pays simple interest of 4.5% per annum while Bank C pays compound interest of 4.3% per annum. Which bank will yield him more interest and by how much? [5]
- (b) John borrowed \$2500 from a money lender who charged him  $r\%$  compound interest per year. After 3 years, the amount of money he owed the money lender was \$3327.50. Find the value of  $r$ . [2]



The diagram shows the cross section of a toy game. A red tape  $AEFBGHA$  is wrapped tightly round the two circles with centres  $O$  and  $C$  respectively.

$FE$  and  $GH$  are common tangents to the circles.  $FE$  and  $GH$  produced intersect at  $D$ . Given that  $OC = 20$  cm,  $\angle FOC = 81^\circ$  and the radius of the big circle is  $4.2$  cm. Calculate

- (a) the length of the arc  $FBG$ , [2]
- (b) the length  $GD$ , [1]
- (c) the radius of the small circle, [2]
- (d) the length of the red tape. [3]



6.

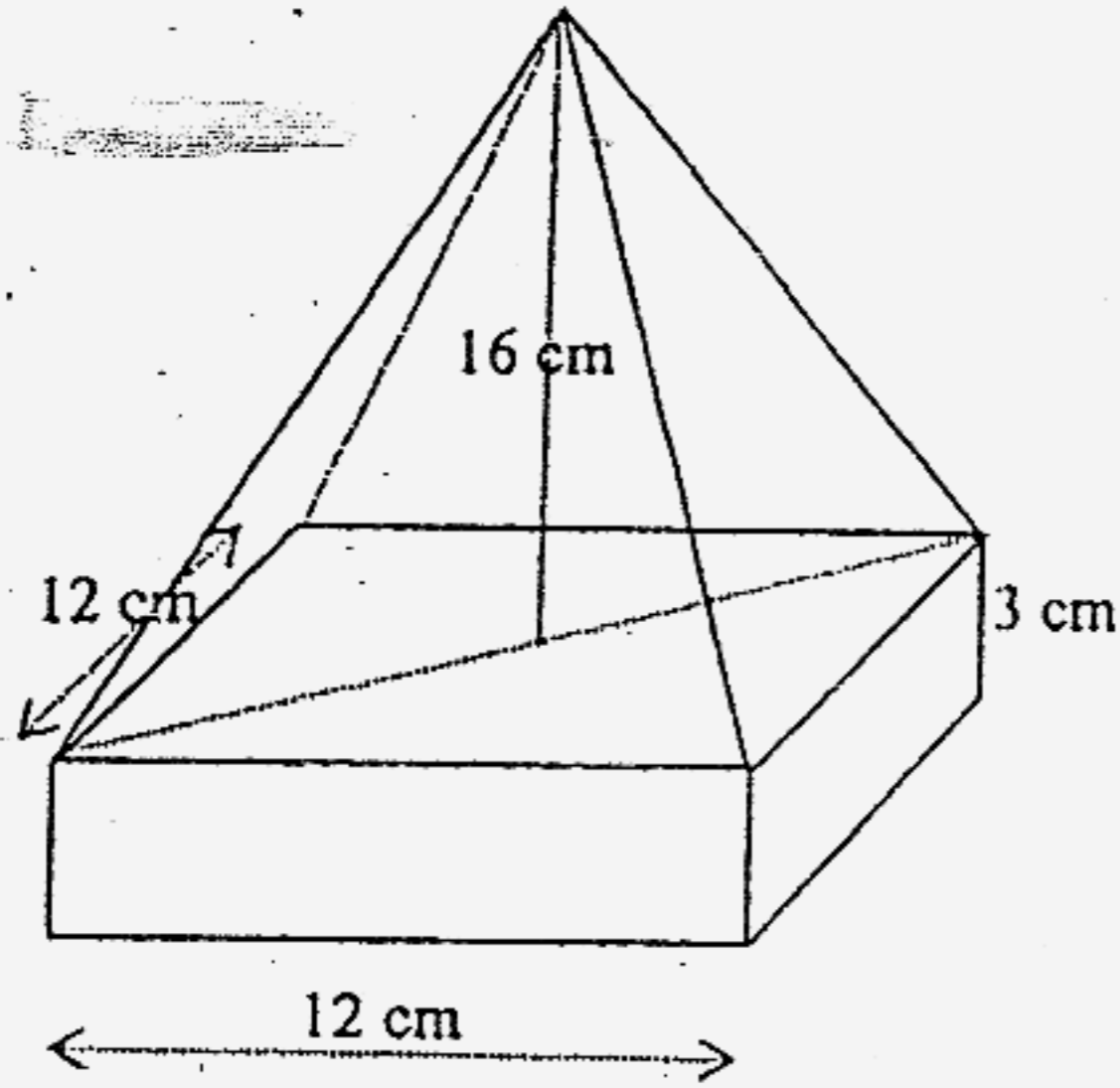


Diagram I

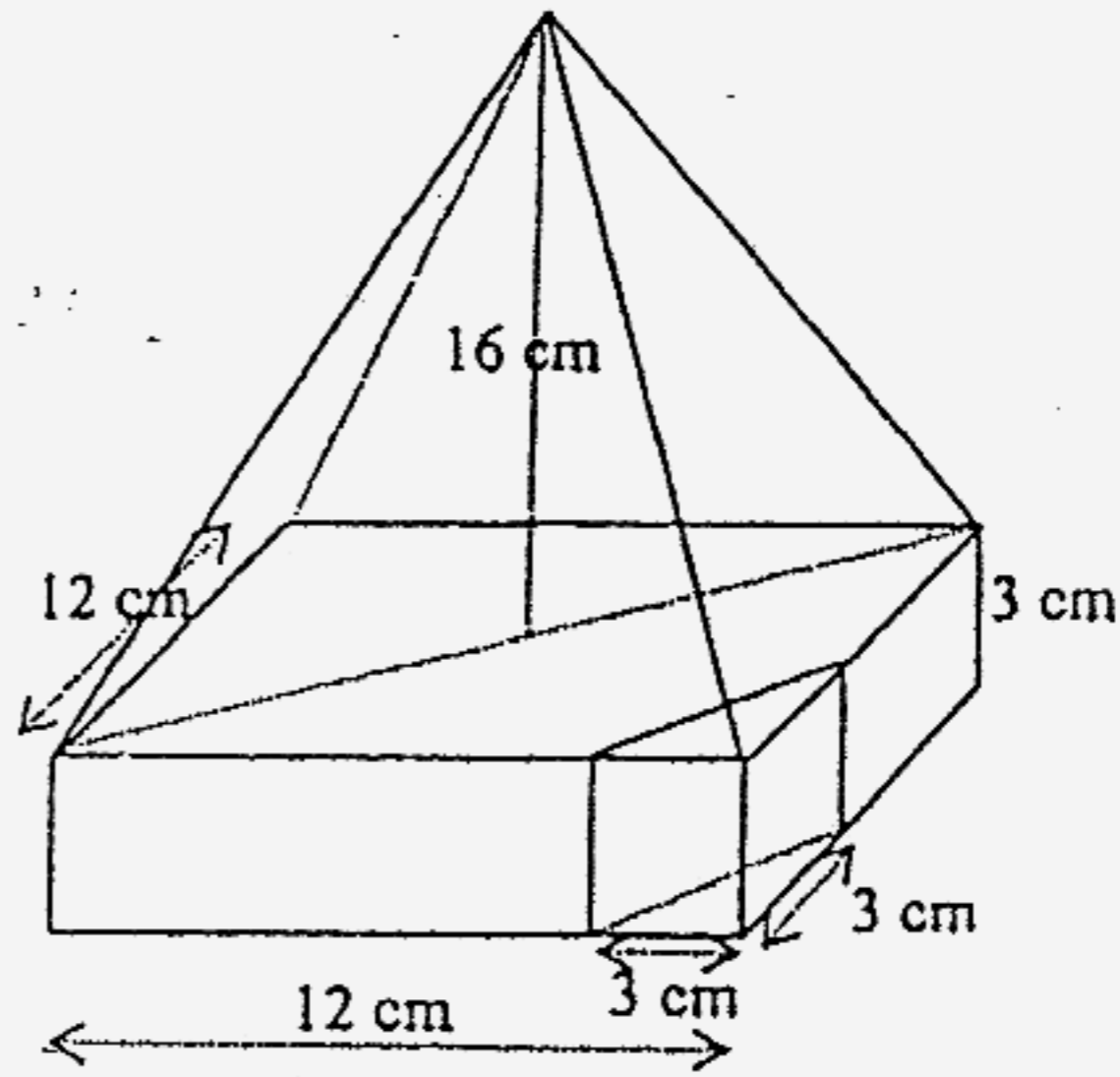
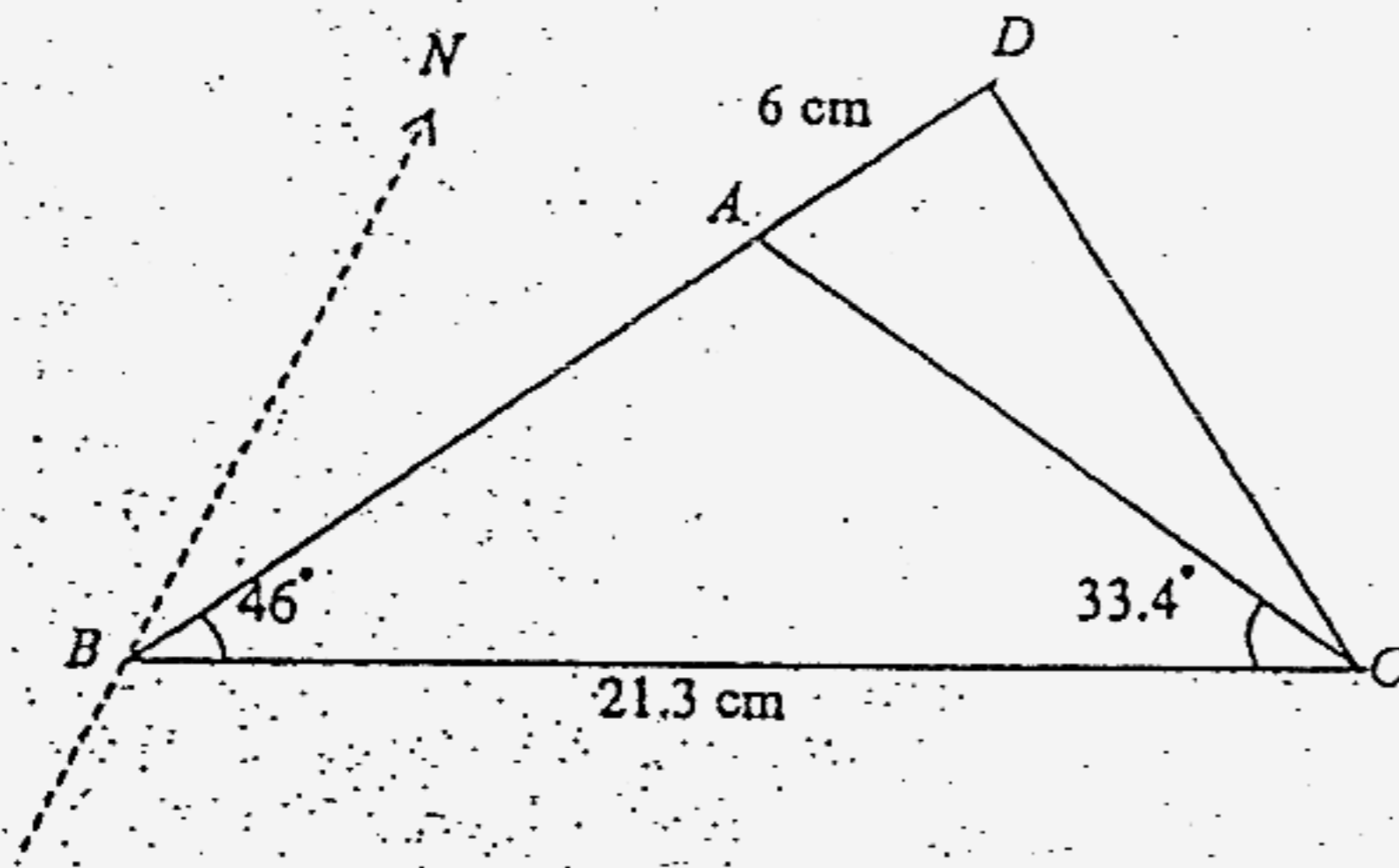


Diagram II

Diagram I shows a wooden block consisting of a solid right pyramid on top of a cuboid. The pyramid has a height of 16 cm. The cuboid has a square base of length 12 cm and a height of 3 cm. Calculate

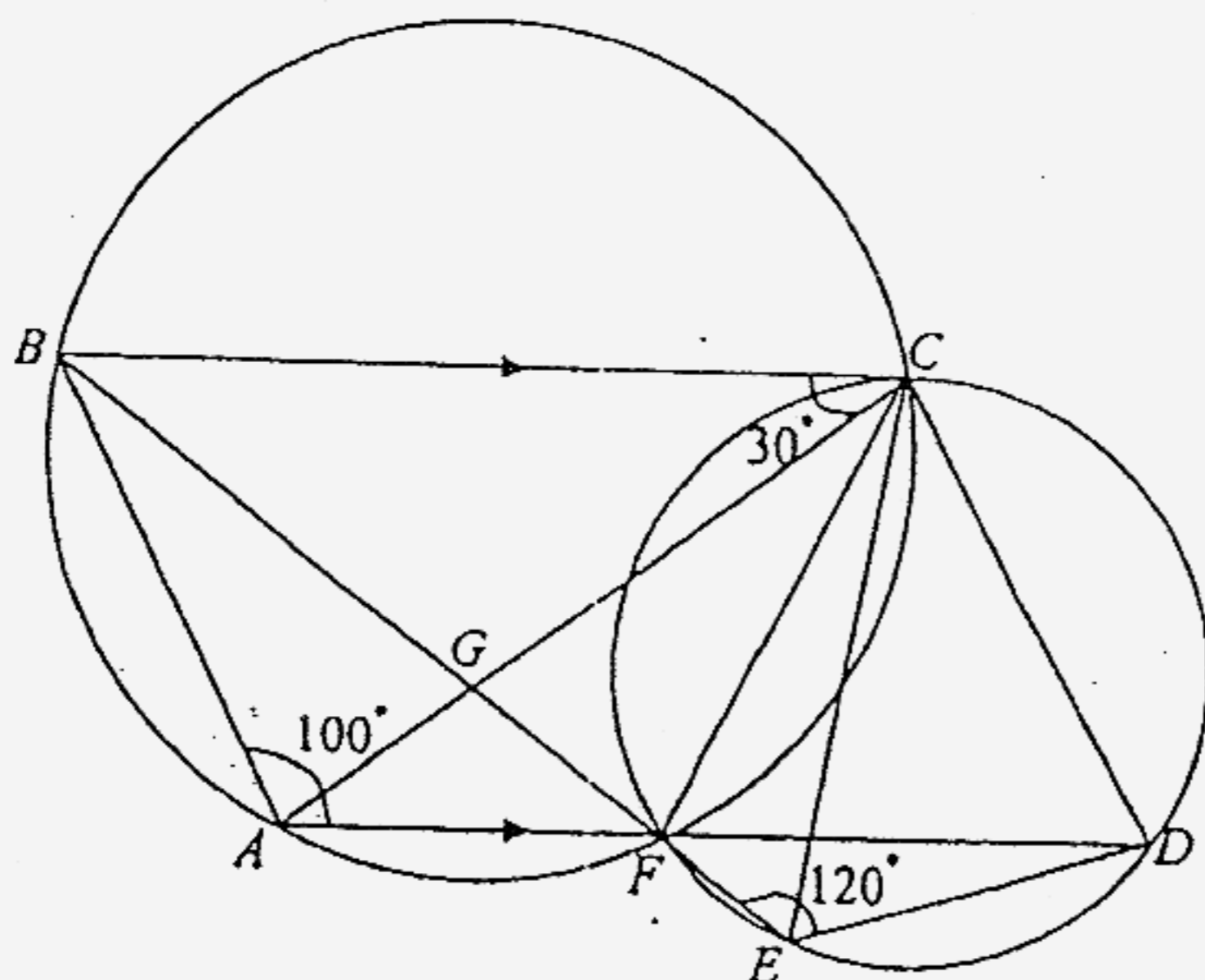
- (a) (i) the volume of the pyramid, [1]  
 (ii) the volume of the wooden block, [1]  
 (iii) the surface area of the pyramid, [2]  
 (iv) the surface area of the wooden block. [2]
- (b) Another smaller wooden block is geometrically similar to the one above, and its base has a length of 5 cm. Calculate the surface area of the smaller wooden block, giving your answer to the nearest  $\text{cm}^2$ . [2]
- (c) If a triangular prism is removed from each of the four corners of the cuboid as shown in diagram II, calculate
- (i) the cross-sectional area of the octagonal base, [1]  
 (ii) the volume of the remaining wooden block. [2]  
 [volume of the pyramid =  $\frac{1}{3}$  base area  $\times$  height]



*ABC* is a triangle in which *C* is due east of *B*,  $BC = 21.3$  cm,  $\angle ABC = 46^\circ$  and  $\angle ACB = 33.4^\circ$ ; *D* is on *BA* produced such that  $AD = 6$  cm. Calculate

- (a) the length of *BA*, [2]
- (b) the length of *CD*, [2]
- (c)  $\angle ACD$ , [2]
- (d) the bearing of *C* from *D*, [2]
- (e) how far east is *C* of *D*. [2]

8.

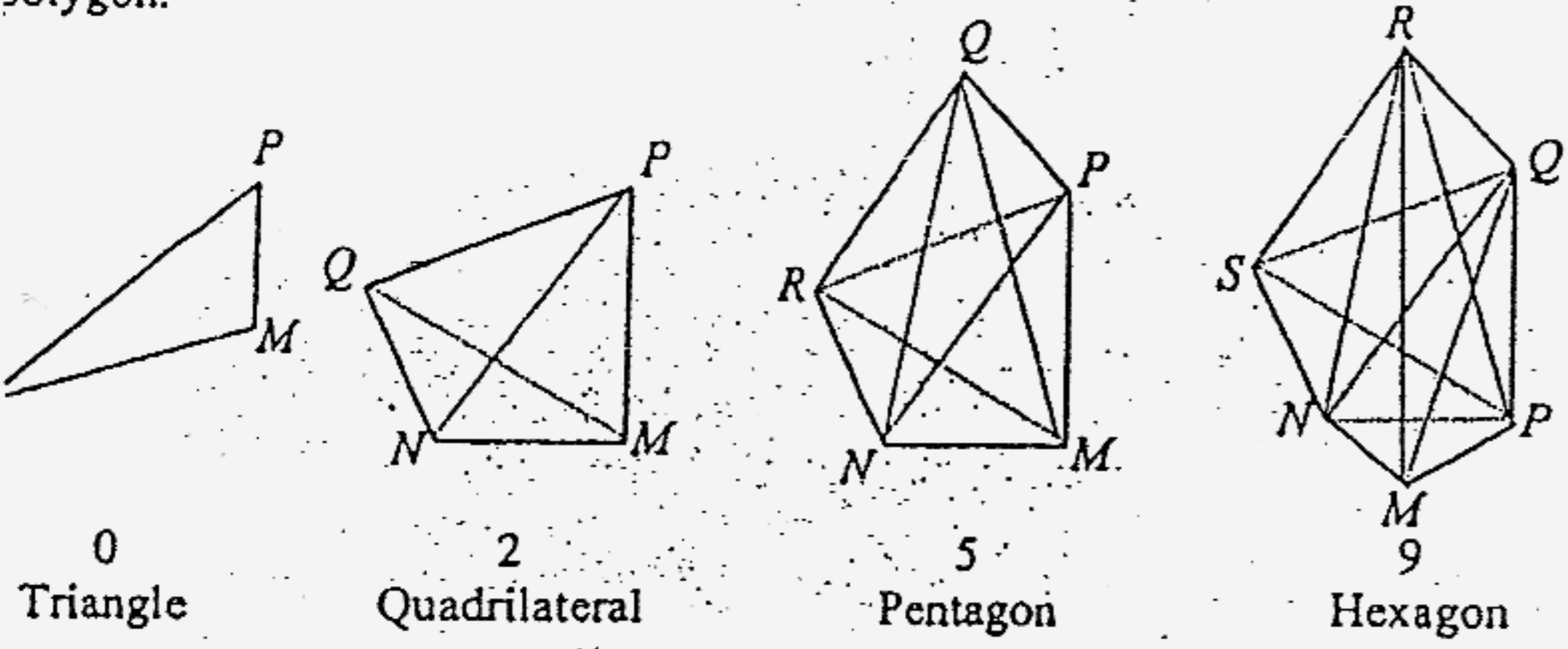


In the figure, circle  $ABCF$  and circle  $CDEF$  intersect at  $C$  and  $F$ .  $AFD$  and  $BFE$  are straight lines,  $BC$  and  $AD$  are parallel.

Given that  $\angle ACB = 30^\circ$  and  $\angle BAF = 100^\circ$  and  $\angle FED = 120^\circ$ ,

- (a) Is  $ABCD$  a cyclic quad? Explain. [1]
- (b) Stating your reasons, find
- (i)  $\angle ACD$ , [3]
  - (ii)  $\angle CDA$ , [1]
  - (iii)  $\angle EDF$ , [2]
  - (iv)  $\angle ACE$ . [2]
- (c) Prove that  $\triangle ABG$  and  $\triangle FCG$  are congruent. [3]

Study the following polygons and note the maximum number of diagonals in each polygon.



Information is tabulated in the table below.

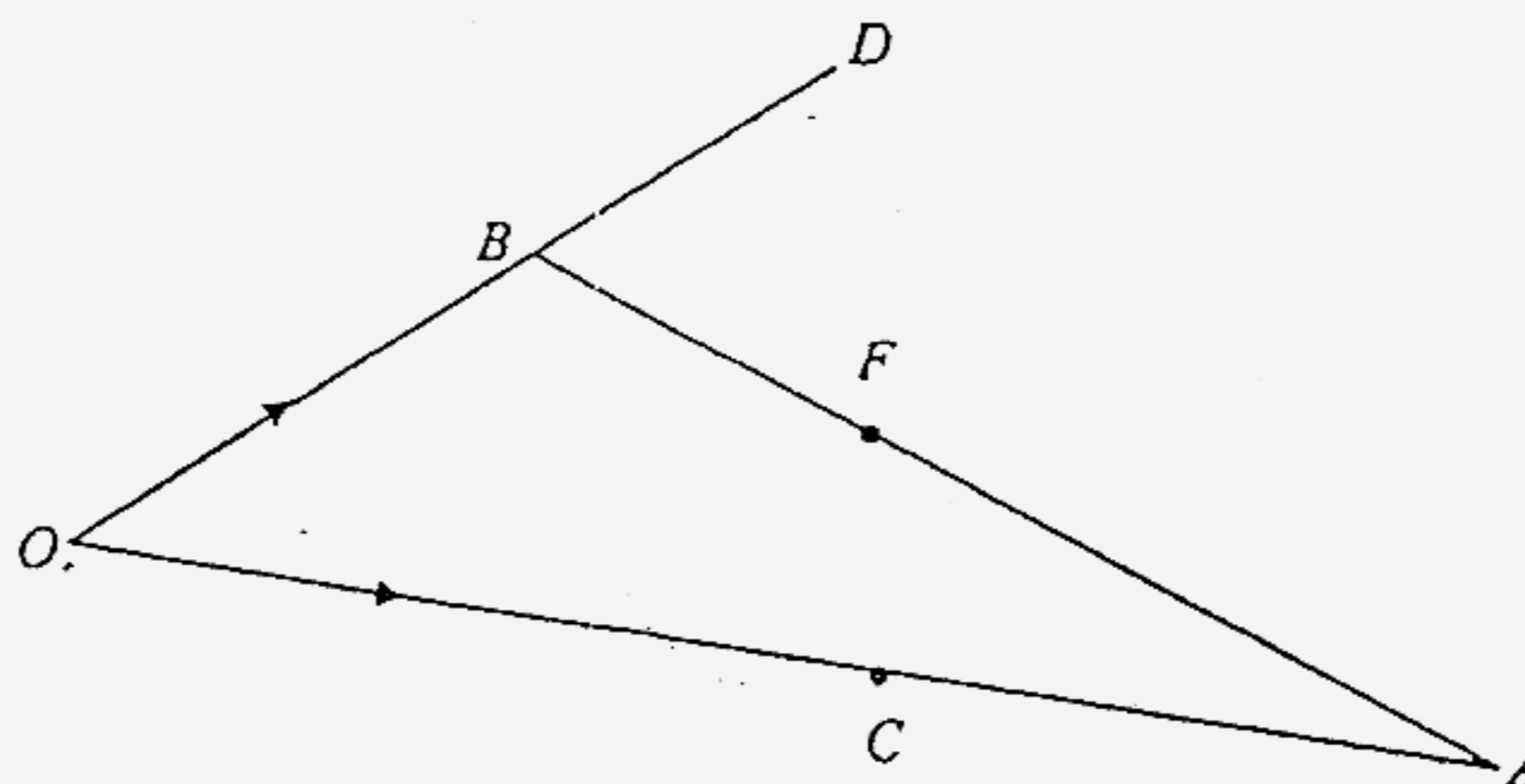
Polygon	No. of Vertices	No. of diagonals from each vertex starting with <i>N</i> in chronological order (diagonals must not be double counted)								No. of diagonals	
		<i>V</i>	<i>N</i>	<i>M</i>	<i>P</i>	<i>Q</i>	<i>R</i>	<i>S</i>	<i>T</i>		<i>U</i>
Triangle	3	0	0	0	0	0	0	0	0	0	0
Quadrilateral	4	1	1	0	0	0	0	0	0	0	2
Pentagon	5	2	2	1	0	0	0	0	0	0	5
Hexagon	6	3	3	2	1	0	0	0	0	0	9
Heptagon	7	4	4	3	2	1	0	0	0	0	14
Octagon											

- a) Write down the numbers in the row for the octagon under the same headings. [2]
- b) Find a relationship between *V* and *N*. [1]
- c) By studying the numbers under columns *V*, *N* and *D*, form an equation relating *D* and *V*. [3]
- d) Using your formula, or otherwise, find the value of *V* if *D* = 275. [2]
- e) Give a simple reason why the number 103 could not appear under the column *D*. [3]

Section B [12 marks]

Answer one question in this section.

10.

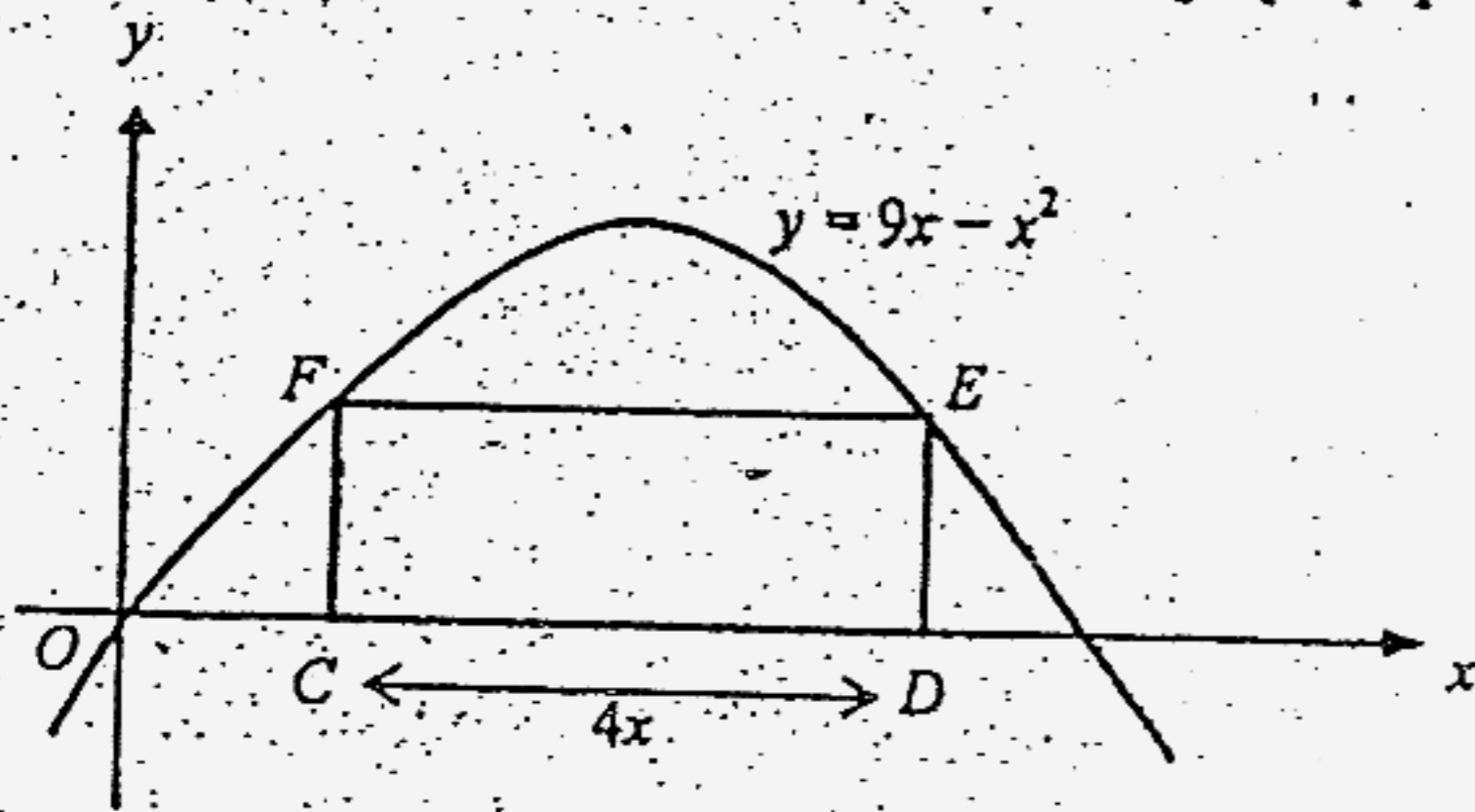


In the diagram,  $OAB$  is a triangle.  $C$  is a point on  $OA$  such that  $OC : OA = 5 : 9$ ,  $F$  is a point on  $AB$  such that  $AF : FB = 2 : 1$ . The side  $OB$  is produced to the point  $D$  such that  $OB : BD = 3 : 2$ .

It is given that  $\vec{OA} = \mathbf{a}$  and  $\vec{OB} = \mathbf{b}$ .

- (a) Express as simply as possible, in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$ ,
- (i)  $\vec{AB}$ , [1]
  - (ii)  $\vec{AF}$ , [1]
  - (iii)  $\vec{OF}$ , [1]
  - (iv)  $\vec{OD}$ , [1]
- (b) What can you tell about the points  $C$ ,  $F$  and  $D$ ? [5]
- (c) If  $O$  is the origin,  $\mathbf{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$  and  $\vec{BA} = \begin{pmatrix} -10 \\ -7 \end{pmatrix}$ , find  $|\vec{OA}|$ . [3]

11 Answer the whole of this question on a sheet of graph paper.



The figure shows part of the parabola  $y = 9x - x^2$  with a rectangle  $CDEF$  which fits between the curve and the  $x$ -axis where  $CD$  equals  $4x$ . Express  $CF$  in terms of  $x$ .

[3]

(a) Show that the area  $A$  of  $CDEF$  is given by  $A = 81x - 16x^3$ .

[1]

(b) The table below shows some values of  $x$  and the corresponding values of

$A$  correct to one decimal place, find the values of  $p$  and  $q$ .

[2]

$x$	0.2	0.4	0.8	1	1.4	1.6	1.8	2	2.2	2.4
$A$	16.1	31.4	$p$	$q$	69.5	64	52.5	34	7.8	-26.8

(c) Taking 1 cm to represent 0.2 unit on the  $x$ -axis and 2 cm to represent 10 units on the  $y$ -axis, draw the graph of  $A = 81x - 16x^3$ , in the range

$$0.2 \leq x \leq 2.4.$$

[2]

(d) Find the value of  $x$  for which the area is the greatest.

[1]

(e) By drawing another line on the same axes, find the solution of the equation  $\frac{1}{x} + 19 - 4x^2 = 0$ .

[2]

(f) Use your graph to estimate the  $x$ -ordinate of the point on  $A = 81x - 16x^3$  where the gradient of the tangent is 35.

[1]

End of Paper

2006 Prelim Exam Math P1  
CGS

- 1(a) 221.1  
 (b) 0.6993  
 2. 1h 32 min  
 3. 7.5  
 4(a) 70  
 (b) 360  
 5. 5  
 6(a) ab  
 (b)  $b^2 - 2ab$   
 7(a)  $1/64$   
 (b)  $(2x+3)/(2x+1)$   
 8. 7  
 9(a) Fig 2  
 (b) Fig 4  
 (c) Fig 5  
 10. 1.8  
 11(a)  $4.33 \times 10^{-4}$   
 (b)  $9 \times 10^1$   
 12(a)  $(2 - q)(p + 3)$   
 (b)  $3x^2(2x - 3)(2x + 3)$   
 (c)  $(1 - 2x)(x + 7)$   
 13(a) 1:250 000  
 (b) 9  
 14(a) 2000  
 (b) 16  
 (c)  $2/5$  or 0.4  
 15(a) y is halved when x increases by four times  
 (b) 8  
 16(a)  $5/8$   
 (b)  $1/16$   
 (c)  $7/32$   
 17(b) 10 38 - 10 40 (d) 80  
 18(a) \$3  
 (b)  $\$1 \pm 0.05$   
 (c) 18  
 (d)  $a=30, b=30, c=10$   
 $p=2.30$   
 20(a) B(6.5, 0)  
 (b)  $y = -2x + 19$   
 (c) D(0, 19)  
 (d)  $x = 4$   
 (e) 4.5  
 21(a)(i) An enlargement, centre(-1, -1), scale factor 3  
 (ii) A stretch, parallel to y-axis, invariant line  $y = -2$ , factor 2.  
 (b)(i)  $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$  (ii)  $y = -x + 4$

2006 E. Maths II Prelim Exam Answer

1. \$107.835    4%     $p=12\%$     \$2326.80    28
2. mode = 74    median = 74.5     $p=9$     36/561
3.  $t = \pm \frac{3}{2} \sqrt{\frac{r^2}{s^2} + 1}$      $\frac{600}{x}$      $(\frac{600}{x} - 10)(x + 1.2)$     \$4.80
4. \$2160    \$2402.8    Bank C ;    10%
5. 14.5 cm    26.5 cm     $r = 1.07$  cm    57.0 cm
6. 768 cm<sup>3</sup>    17.1 cm    410 cm<sup>2</sup>    698 cm<sup>2</sup>    121.2 cm<sup>2</sup>    126 cm<sup>2</sup>    1146 cm<sup>3</sup>
7. 11.9 cm    15.6 cm    22.2°    145.6°    8.84 cm
8. ABCD is not a cyclic quad, the point D is not on the same circle as A, B and C.  
     110°    40°    30°    80°    AAS
9.  $V = N+3$      $D = V(V-3) \div 2$      $V = 25$     For D ) 5, D is always multiple of 2  
     factors greater than 1. 103 is a prime number, it could appear under column D.
- 10(a)  $b - a$      $\frac{2}{3}(b - a)$      $\frac{1}{3}(a + 2b)$      $\frac{5}{3}b$   
     (c) 13.1 units
11.  $\frac{81}{4} - 4x^2$      $p = 56.6$      $q = 65$      $x = 1.3$      $x = 2.2$      $x = 1$