Name:	Class: Sec 4	Class Register No: _____ O Level Index No: ____/0434

100

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Parent's Signature

PRELIMINARY EXAMINATION 2006
SECONDARY 4

Mathematics
4017/2

Monday 18th September 2006
2 hours 30 minutes

Instructions to Candidates:

Additional Materials:

6 sheets of answer paper
1 sheet of graph paper

Instructions To Candidates:

Write your name, centre number and index number in the space provided on the answer paper.
Write your answers and working on the separate answer paper provided.
Omission of essential working will result in the loss of marks.
Show all working on the same page as the rest of the answer.
If more than one sheet of paper is used, fasten the sheets together.

Section A (88 marks)
Answer all questions

Section B (12 marks)
Answer any one question.

Information For Candidates:

The marks are given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.
You are expected to use an electronic calculator to evaluate explicit numerical expressions.
You may use mathematical tables as well, if necessary.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to 3 significant figures, or 1 decimal place in the case of angles in degrees.
You are reminded of the need for clear presentation in your answers.

This paper comprises 9 printed pages

DO NOT TURN OVER THE PAGE UNTIL YOU ARE TOLD TO DO SO

Section A [88 marks]

Answer all the questions in this section.

1.

a) Find the value of $x^3 + 3x^2 - 2(x+3)$ when $x = -2$ [2]

b) Express $\frac{1}{x+y} - \frac{2}{x-y} + \frac{2x+4y}{x^2-y^2}$ as a single fraction in its simplest form. [3]

c) Given that $3x = \sqrt{\frac{y+5}{3y-4}}$, express y in terms of x. [2]

2. The cost of printing a school magazine is charged according to either of two schemes,

Scheme A

\$3.50 each for first 500 copies.
\$3.40 each for the 501st copy to the 1500th copy,
\$3.20 each for 1501st copy onwards.

On top of the above, the printing company charges a flat fee of \$125 for each order, irrespective of the quantities printed.

Scheme B

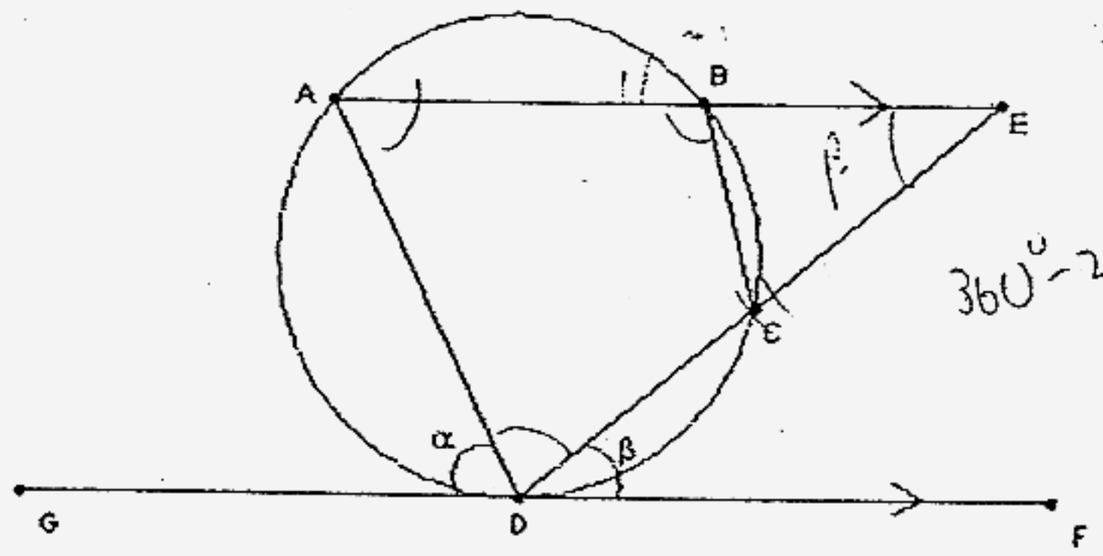
A flat rate of \$3.60 for each copy with no extra fees chargeable.

a) Calculate the total cost of producing 400 copies of magazine for each of the payment schemes. Hence deduce which scheme will be cheaper. [3]

b) Calculate the total cost of producing 1800 copies of magazine for each of the payment schemes. Hence deduce which scheme will be cheaper. [3]

c) How many copies need to be printed such that both schemes cost the same? [4]

3a) The diagram shows a cyclic quadrilateral $ABCD$ with AB and DC extended to meet at E such that AB and GF are parallel lines. GF is a tangent of the circle $ABCD$ at D.



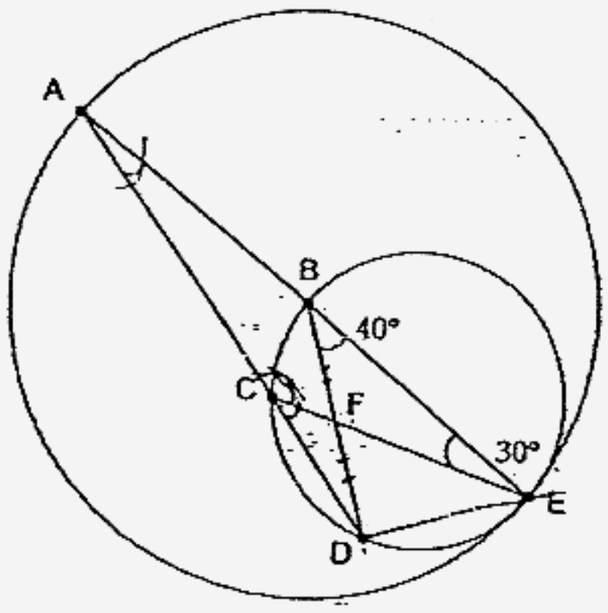
Calculate, leaving your answer in terms of α and β,

i) $\angle ABC$ [2]

ii) Prove that ΔADE and ΔCBE are similar. [2]

iii) Write down an equation in terms of α and β, such that ΔADE is an isosceles triangle where $AE = DE$. [2]

3b) The diagram shows two circles intersecting at the point E. The larger circle has centre B. $\angle EBD$ is 40° and $\angle BEC$ is 30° and F bisects the line BD.



Find

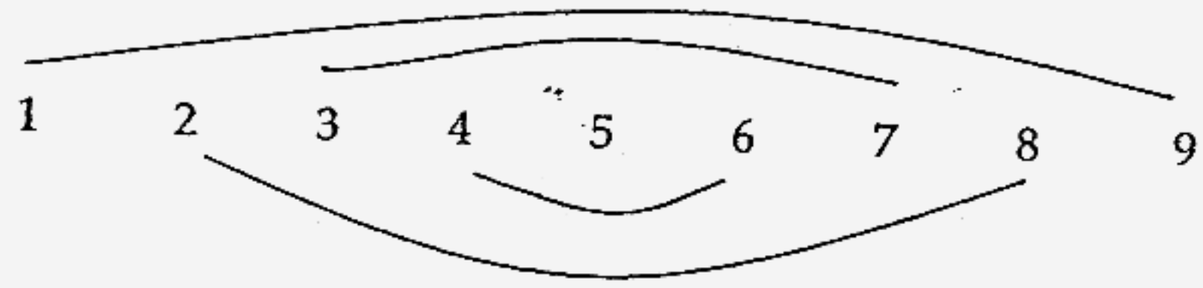
i) $\angle DCE$ [1]

ii) $\angle EAD$ [1]

iii) Prove that ΔBFE and ΔCFD are similar [2]

iv) Given that $CF = kFB$, where k is a constant, and ΔCFD has an area of $2cm^2$, find in terms of k, area of ΔBFE [2]

4. One way of adding numbers 1 to 9 is by pairing the numbers as shown in the diagram.



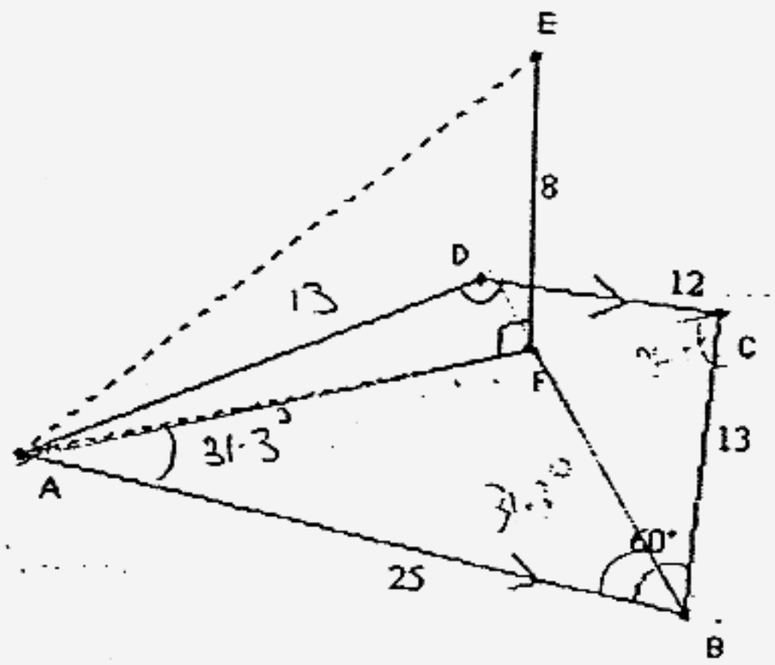
As we can see, numbers on each side of the number 5 is paired up and the pairs all sum to 10.

Thus the sum of numbers from 1 to 9 = 5 + 4 x 10 = 45.

Using the same procedure, copy and complete the following table. [6]

Instruction	Middle number	Number of pairs	Sum of a pair	Total Sum of all numbers
Sum from 1 to 9	5	4	10	5 + 4 x 10 = 45
Sum from 3 to 7				
Sum from 16 to 22				
Sum from 123 to 321				

5. The figure shows a grass patch shaped like a trapezium with an 8m tall lamp post standing at F, the intersection of the two diagonals AC and BD. B is 25m east of A, C is 12m east of D, B and C are 13m apart and $\angle ABC = 60°$.



i) Find the length of the diagonal AC. [2]

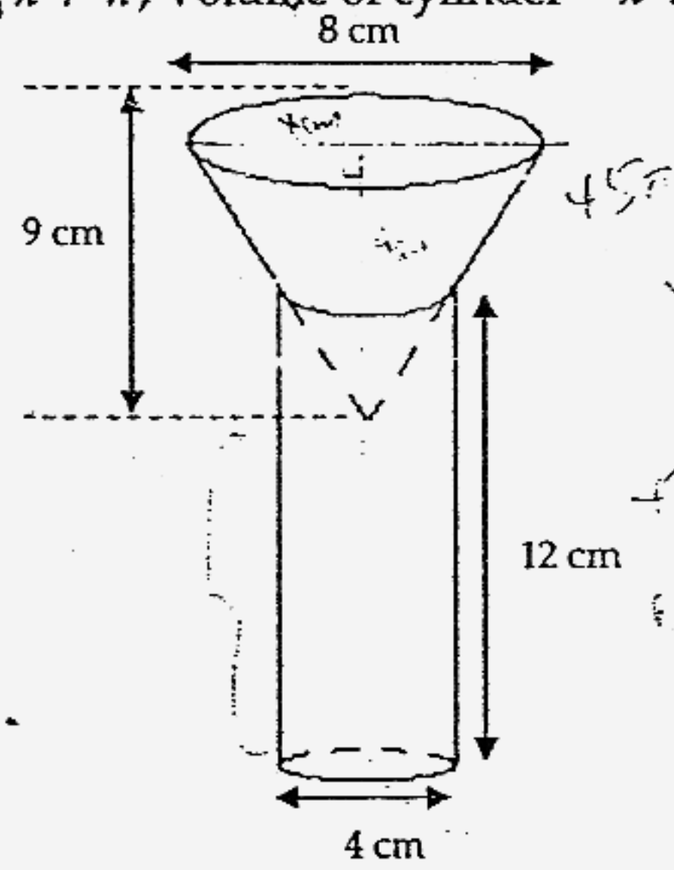
ii) Find the area of triangle ABC. [2]

iii) Find the bearing of F from A. [2]

iv) Find the distance AF [5]

v) Find the angle of elevation of E from A. [1]

6. A tin cone is initially fully filled with water. It is placed in a cylindrical glass and a hole is made at the tip of the cone such that water starts to flow down into the glass.
[Volume of cone = $\frac{1}{3}\pi r^2 h$, Volume of cylinder = $\pi r^2 h$]

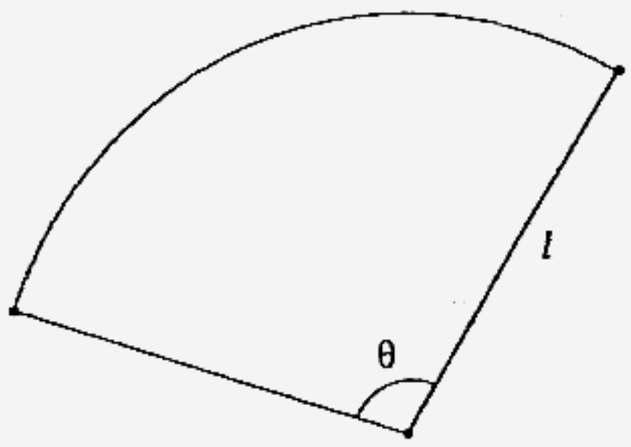


i) Given that it takes 1 minute for all the water in the cone to be drained at a constant rate, find the rate r cm³/s at which water is being drained from the cone. Leave your answer in terms of π. [2]

ii) Find the rate, h cm/s, at which the water level in the cylinder is rising. Leave your answer in terms of π. [2]

iii) After all the water in the cone has been drained, the water in the cylinder is also poured away. With both cone and cylinder now empty, water is now poured into the cone at a rate of 2π cm³/s. How long does it take before water starts to spill from the cone? [2]

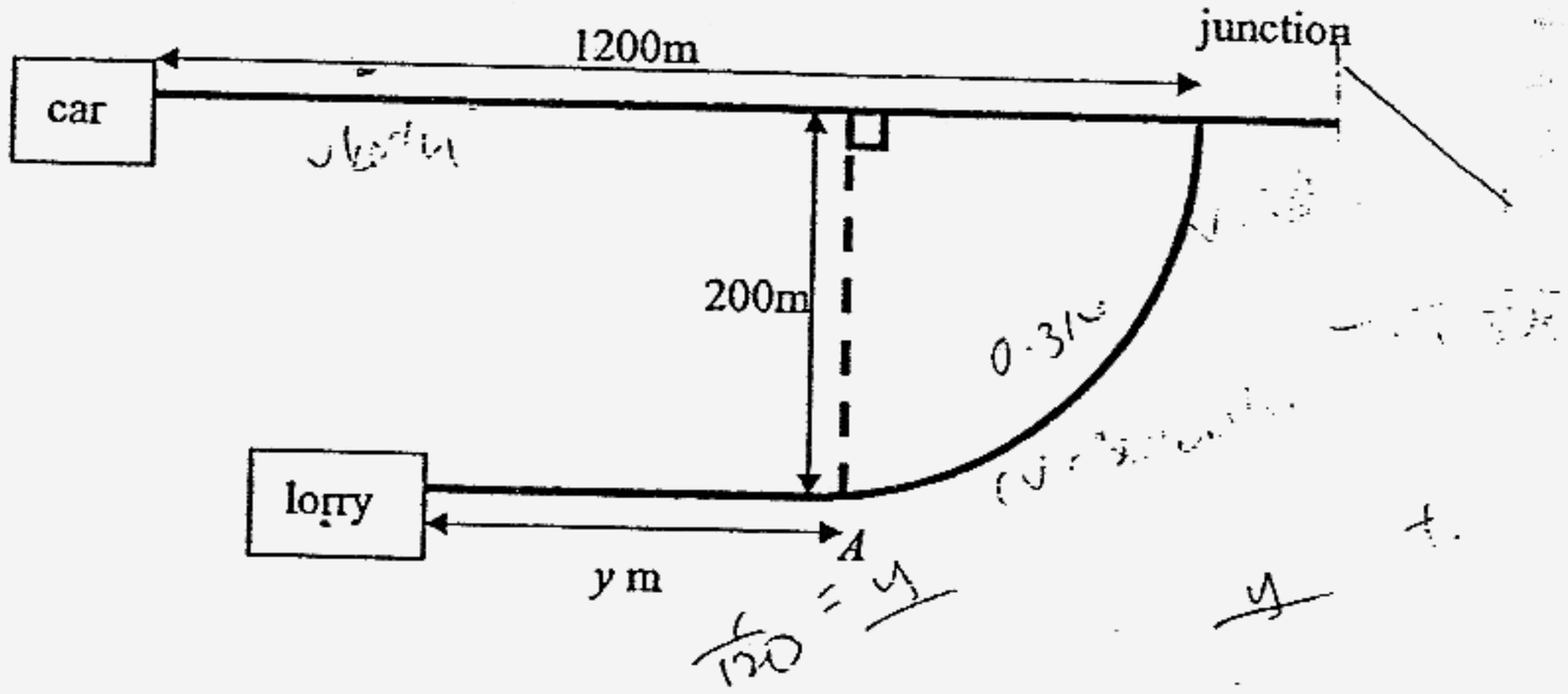
iv) The tin cone is made up of a tin sheet that when opened up is a sector of a circle, as shown in the diagram.



Find the radius, l, of the sector used to make the tin cone. Give your answer to 3 significant figures. [1]

v) Find the angle θ in degrees. [3]

7. A car and a lorry are moving towards the same junction from two different roads as shown in the diagram. The car is initially 1200m from the junction and traveling at a constant speed of v km/h. The lorry, initially y m from a circular bend that leads to the junction, takes 30 seconds to reduce its speed uniformly such that it enters the bend at A at a constant speed of $(v-30)$ km/h.



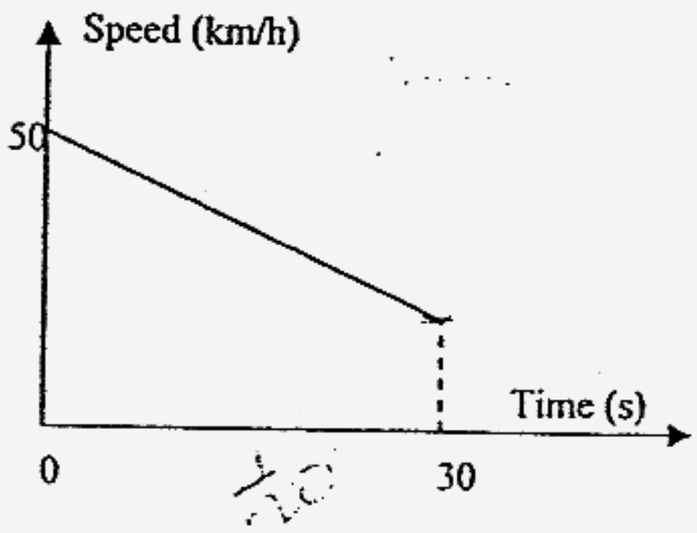
i) Find in terms of v, the time taken for the car to reach the junction. [1]

ii) Show that the time taken by the lorry to reach the junction is [2]
$\left(\frac{1}{120}+\frac{\pi}{10(v-30)}\right)$ h.

iii) Write down a condition for a collision to occur at the junction. Take $\pi=\frac{22}{7}$ [3]
and show that it reduces to $7v^2-954v+30240=0$

iv) Hence solve the equation, giving your answers to three significant figures. [3]

v) The lorry's speed time graph when approaching the bend is as shown in the graph below.

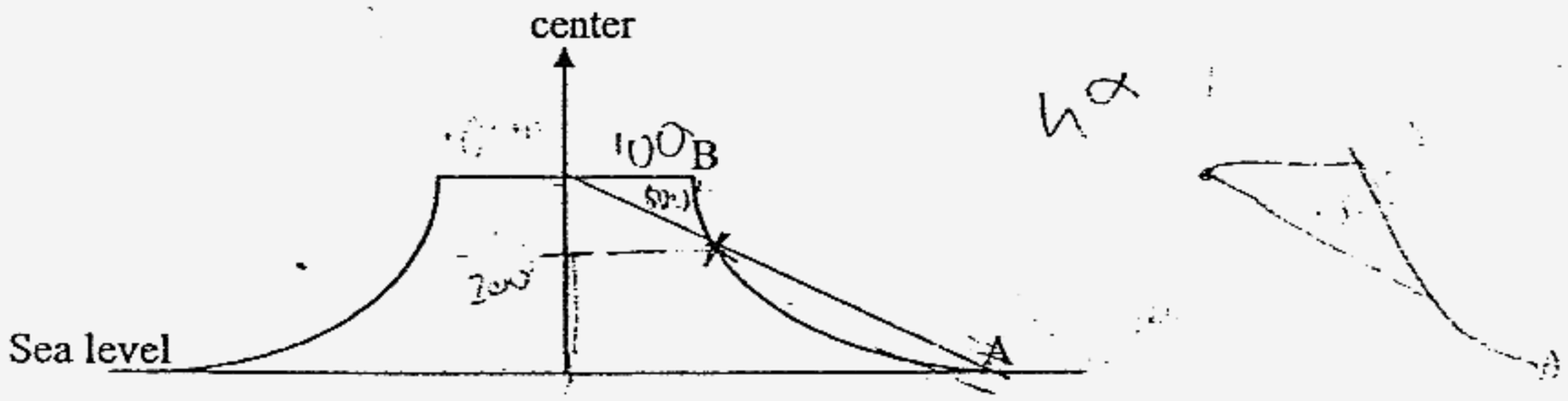


Find the distance traveled by the lorry prior to reaching the bend. [3]

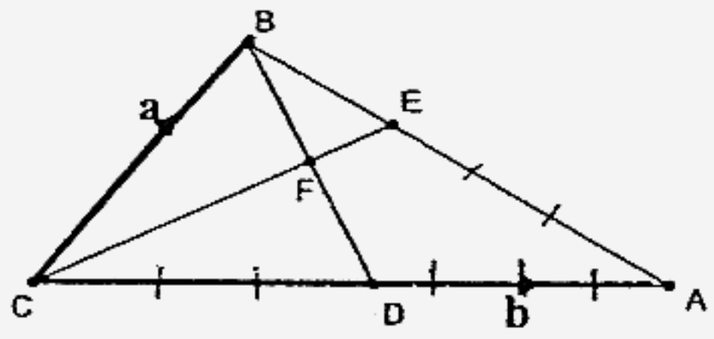
8.

a) If 8 men can make 80 chairs in 6 days, how long will 12 men take to make 300 chairs? [2]

b) A volcanic crater has a flattened circular top of radius 100m. As a man walks up the side of a volcanic crater from A to B, the height above sea level is inversely proportional to the square of the distance from the center. Given that 500m from the centre, the man is 200m above sea level, find the height of the volcanic crater. [4]



9. In the diagram, $\overrightarrow{CB}$ = a and $\overrightarrow{CA}$ = b. The points E and D lie on AB and CA respectively such that $3BE = EA$ and $4CD = 3DA$.



a) Express $\overrightarrow{BD}$ in terms of a and b. [2]

b) Show that $\overrightarrow{CE} = \frac{1}{4}(3\mathbf{a} + \mathbf{b})$. [2]

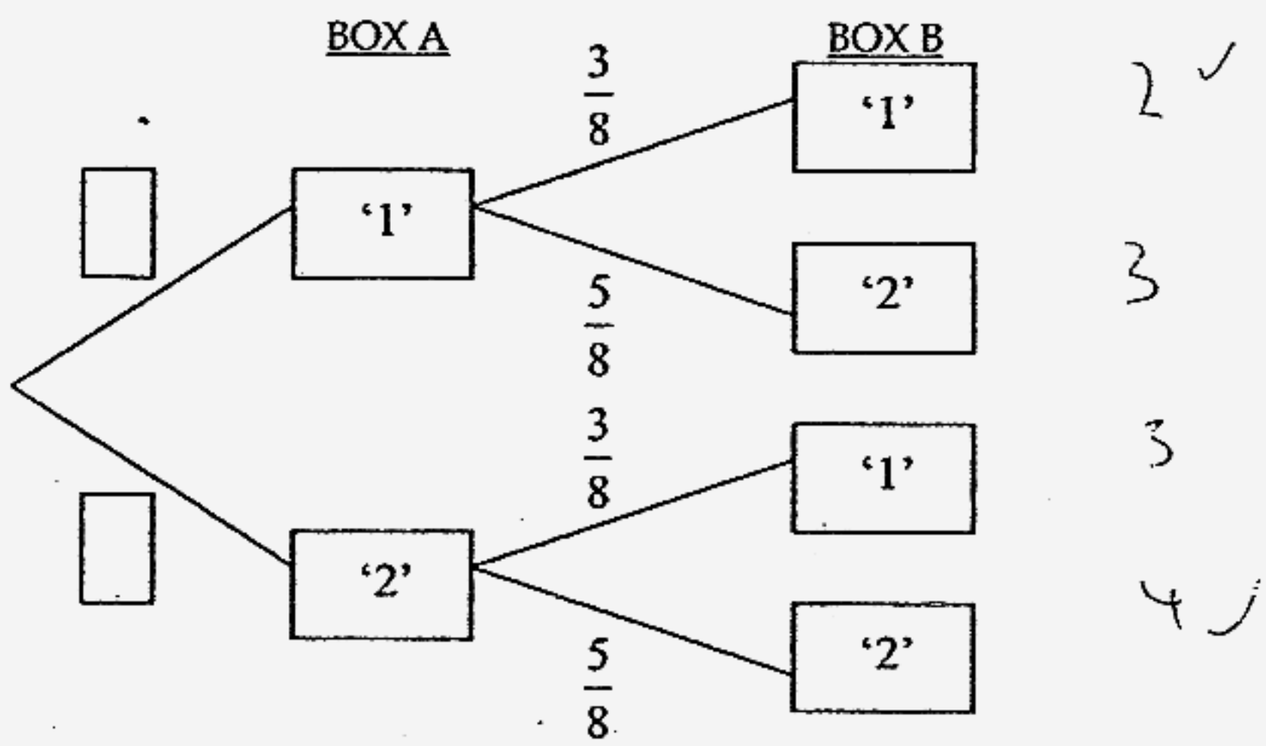
c) The lines CE and BD intersect at the point F.
Given that $\overrightarrow{BF} = k\,\overrightarrow{BD}$, show that $\overrightarrow{CF} = (1-k)\mathbf{a} + \frac{3}{7}k\mathbf{b}$. [2]

10. In a game, one card is drawn from Box *A* and another card is drawn from Box *B*. The number of cards in each box is shown in the table below.

Box	Card labeled '1'	Card labeled '2'
A	2	7
B	3	5

The player's score is recorded by taking the sum of the numbers shown on the two cards.

i) Copy and complete the probability tree diagram. [1]



ii) Find the probability of getting a card labeled '1' from Box *B*. [1]

iii) Find the probability of drawing two cards such that the sum of the two cards is an even number. [1]

iv) The table shows the scores recorded by Tom after playing a total of 18 games.

Score	3	3	2	2	3	4	4	3	4	2	2	3	2	3	2	2	4	2

a) What is the modal score? [1]

b) What is the mean score? [1]

c) What is the median score? [1]

v) Suppose all the cards in Box *B* are now placed in Box *A* and two cards are drawn without replacement, what is the probability of getting two cards, both labeled '1'? [1]

Section B [12 marks]
Answer one question in this section.

11. **Answer the whole of this question on a sheet of graph paper.**
Given that $y = (2+x)(x-3)$, copy and complete the following table. [1]

x	-3	-2	-1	0	1	2	3	4
y		0		-6	-6		0	6

a) Using a scale of 2 cm to 1 unit on the x-axis and 1 cm to 1 unit on the y-axis, draw the graph of $y = (2+x)(x-3)$ for $-3 \le x \le 4$. [3]

b) Write down the range of values of x for which $(2+x)(x-3) > 1$. [2]

c) Write down the equation of the axis of symmetry of the graph. [1]

d) By drawing a suitable straight line, estimate the solution of the equation $x^2 - 2x = 7$ [3]

e) By drawing a tangent, estimate the gradient of the curve $y = x^2 - x - 6$ at the point where $x = 2$. [2]

12. **Answer the whole of this question on a sheet of graph paper.**

a) Using a scale of 2 cm to represent 1 unit on both axes, draw axes for $-4 \le x \le 5$ and $-4 \le y \le 5$. Draw and label the triangles ABC, PQR and LMC whose vertices are $A(2,2)$, $B(2,1)$, $C(0,1)$, $P(-2,-2)$, $Q(-1,-2)$, $R(-1,0)$, $L(2,-1)$, $M(2,-2)$ [3]

b) A single transformation G maps ΔABC to ΔPQR. Describe the transformation G completely. Hence find the point X such that $G^6(X)=A$ [2]

c) A single transformation H maps ΔABC to ΔLMC. Describe the transformation H completely. [2]

d) ΔABC is mapped onto ΔSTU by a rotation about C, 90° anticlockwise followed by an enlargement, center (-4, 1) scale factor 2. Draw and label ΔSTU on the graph. [3]

e) ΔABC can also be mapped onto ΔSTU by first performing an enlargement center C, scale factor 2 followed by a rotation F. Describe the transformation F completely. [2]

CCHMS 2006 PRELIM E MATH P2 Date No.

1(a) $x^3 + 3x^2 - 2(x+3)$

$= (-2)^3 + 3(-2)^2 - 2(-2+3)$ when $x = -2$

$= -8 + 12 - 2$

$= 2$ # ✓

(b) $\frac{1}{x+y} - \frac{2}{x-y} + \frac{2x+4y}{x^2-y^2}$

$= \frac{(x-y) - 2(x+y) + (2x+4y)}{(x+y)(x-y)}$

$= \frac{x-y-2x-2y+2x+4y}{(x+y)(x-y)}$

$= \frac{x+y}{(x+y)(x-y)}$

$= \frac{1}{x-y}$ # ✓

(c) $3x = \sqrt{\frac{y+5}{3y-4}}$

$9x^2 = \frac{y+5}{3y-4}$

$27x^2y - 36x^2 = y+5$

$27x^2y - y = 36x^2 + 5$

$y(27x^2 - 1) = 36x^2 + 5$

$y = \frac{36x^2+5}{27x^2-1}$ # ✓

Date No.

2(a) Scheme A = 125 + 400 (3.50)

= $1525 #

Scheme B = 400 (3.60)

= $1440 #

Hence Scheme B will be cheaper #

(b) Scheme A = 125 + 500 (3.50) + 1000(3.40) + 300 (3.20)

= $6235

Scheme B = 1800 (3.60)

= $6480

Hence Scheme A will be cheaper #

(c) let x be the no. of copies needed (x must be between 400 & 1800 coz it's as stated in Q)

If $(3.60)x = 125 + 500(3.50) + 1000(3.40) + (x-1500)(3.20)$

$3.6x = 5275 + 3.2x - 4800$

$0.4x = 475$

$x = 1187.5$

∴ no. of copies is 1187.5 #

3(a) (i) $\angle ABC = 180° - \angle ADC$ (opp. ∠s of cyclic quad)

$= 180° - (180° - \alpha - \beta)$ (∠s on str line GDF)

$= \alpha + \beta$ #

(ii) $A\hat{E}D = C\hat{E}B$ (shared / common ∠)

$A\hat{D}E = 180° - \alpha - \beta$

$= 180° - A\hat{B}C$ (supp ∠s)

$= C\hat{B}E$

$D\hat{A}E = B\hat{C}E$ (Remaining ∠)

∴ By AAA ppty, ΔADE and ΔCBE are similar

(iii) $\angle EAD = \angle ADG = \alpha$ (alt ∠s)

$\angle AED = \angle EDF = \beta$ (alt ∠s)

If ΔADE isosceles where $AE = DE$

$\Rightarrow \angle EDA = \angle EAD = \alpha$

Hence $\alpha + \alpha + \beta = 180°$

∴ Eqn is $2\alpha + \beta = 180°$ #

(b) (i) $\angle DCE = \angle DBE$ (∠s in same seg)

$= 40°$

(ii) $\angle EAD = \angle DCE - \angle AEC$ (ext ∠ of Δ)

$= 40° - 30°$

$= 10°$

(iii) $B\hat{F}E = C\hat{F}D$ (vert opp ∠s)

$B\hat{E}F = C\hat{D}F$ (∠s in same seg)

$F\hat{B}E = F\hat{C}D$ (∠s in same seg)

∴ By AAA ppty, ΔBFE and ΔCFD are similar

(iv) $\frac{\text{Area } \Delta BFE}{\text{Area } \Delta CFD} = \left(\frac{BF}{CF}\right)^2$

$\Rightarrow \frac{\text{Area } \Delta BFE}{2} = \left(\frac{BF}{kBF}\right)^2$

$= \left(\frac{1}{k}\right)^2$

∴ Area $\Delta BFE = 2 \times \left(\frac{1}{k}\right)^2$

$= \frac{2}{k^2}$ #

Date No.

4.

Instruction	Middle number	Number of pairs	Sum of a pair	Total Sum of all numbers
Sum from 1 to 9	5	4	10	$5+4\times10=45$
Sum from 3 to 7	5	2	10	$5+2\times10=25$
Sum from 16 to 22	19	3	38	$19+3\times38=133$
Sum from 123 to 321	222	99	444	$222+99\times444=44178$

5 (i) $AC = \sqrt{13^2+25^2-2(13)(25)\cos 60^\circ}$ (cos rule)

$= 21.656\ldots$

$= 21.7$ m (3 s.f.) #

(ii) Area $\Delta ABC = \frac{1}{2}(13)(25)\sin 60^\circ$

$= 140.729\ldots$

$= 141\ m^2$ (3 s.f.) #

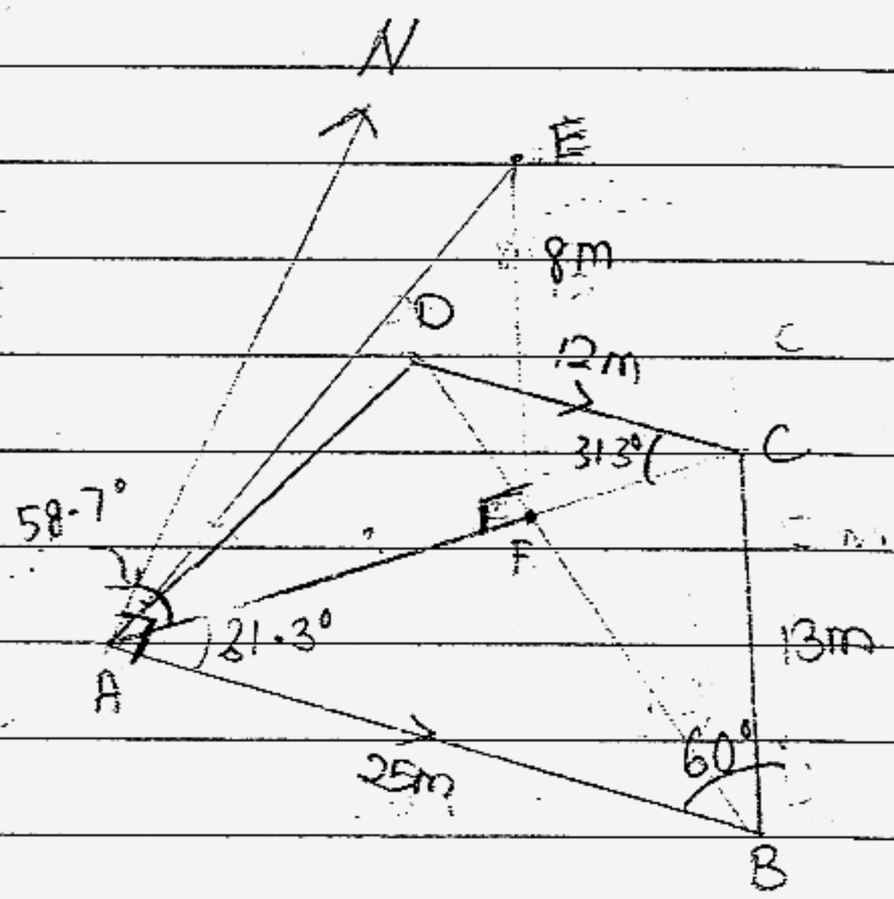
(iii) $\frac{\sin \hat{CAB}}{13} = \frac{\sin 60^\circ}{AC}$

$\Rightarrow \hat{CAB} = \sin^{-1}\left(\frac{\sin 60^\circ}{AC}\times 13\right)$

Bearing of F from A $= 90^\circ - \hat{CAB}$

$= 58.677\ldots$

$= 058.7^\circ$ (1 d.p.) #



iv)

v)

6(i) $\text{Rate} = \frac{\text{Vol. of water}}{\text{time}}$

$\Rightarrow r = \frac{\frac{1}{3}\pi(4)^2(9)\ cm^3}{60\ s}$

$= \frac{4}{5}\pi\ cm^3/s$ #

(ii) Vol water in cylinder = Vol water leaked from cone

$\Rightarrow \pi(2)^2 h\ cm^3 = \frac{4}{5}\pi\ cm^3/s$

$h = \frac{4}{5}\pi \times \frac{1}{4\pi}$

$= \frac{1}{5}$ //

iii) In $\Rightarrow 2\pi\ cm^3/s$

Out $\Rightarrow \frac{4}{5}\pi\ cm^3/s$

$2\pi - \frac{4}{5} = 1\frac{1}{5}\ cm^3/s$

Vol cone $= \frac{1}{3}\pi(4)^2(9) = 48\pi$

$48\pi \div 1\frac{1}{5} = 40\pi s = 125\frac{5}{7} s$ //

iv) $l = \sqrt{4^2+9^2} = \sqrt{97} = 9.85\ cm$ //

v) $9.85\theta = 8\pi$

$\Rightarrow \theta = \frac{8\pi}{9.85} = 146.2°$ //

Date No.

7(i) Time taken by car = $\frac{1200m}{v\ km/h}$

$= \left(\frac{1.2}{v}\right) h$

(ii) Circumference of sector = $\frac{1}{4} \times 2 \times \pi \times 0.2 = \frac{\pi}{10}$ km

Time taken to travel = 30sec + $\frac{\frac{\pi}{10}}{v-30} h$

$= \left(\frac{1}{120} + \frac{\pi}{10(v-30)}\right) h$ // (shown)

(iii) let $\frac{1.2}{v} = \frac{1}{120} + \frac{\pi}{10(v-30)}$

$\Rightarrow 1.2(120)(10)(v-30) = v(10)(v-30) + \pi(v)(120)$

$\Rightarrow 144(v-30) = v(v-30) + 12\pi v$

$144v - 4320 = v^2 - 30v + 12\left(\frac{22}{7}\right)v$ (Take $\pi = \frac{22}{7}$)

$7v^2 - 210v + 264v - 1008v + 30240 = 0$

$7v^2 - 954v + 30240 = 0$ (SHOWN)

(iv) $v = \frac{-(-954) \pm \sqrt{(-954)^2 - 4(7)(30240)}}{2(7)}$

$= \frac{954 \pm \sqrt{63396}}{14}$

$= 50.2$ or 86.1 (3 s.f.)

(v) If $v = 50.2$, $v - 30 = 50.2 - 30 = 20.2$ km/h

$\Rightarrow$ dis travelled $= \frac{1}{2} \times \frac{1}{120} \times (50 + 20.2)$

$= 0.2925$ km

$= 292.5$ km //

If $v = 86.1$, $v - 30 = 86.1 - 30 = 56.1$ km/h

$\Rightarrow$ dis travelled $= \frac{1}{2} \times \frac{1}{120} \times (50 + 56.1)$

$= 0.442$ km

$= 442$ km //

Date No.

8(a)

Men	Chairs	Days
8	80	6
12	300	d

$$d = 6 \times \frac{300}{80} \div \frac{12}{8}$$

$$= 6 \times \frac{300}{80} \times \frac{8}{12}$$

$\therefore$ no. of days = 15 #

(b) Given $h \propto \frac{1}{d^2}$

Let $h = \frac{k}{d^2}$

$\Rightarrow 200 = \frac{k}{(500)^2}$

$\therefore k = 50\,000\,000$

Hence $h = \frac{50\,000\,000}{d^2}$

Since radius of crater = 100 m

Let $h = \frac{50\,000\,000}{(100)^2}$

$\therefore$ height of crater = 5000 m #

Date No.

9(a) $\vec{BD} = \vec{BC} + \vec{CD}$

$= -\underset{\sim}{a} + \frac{3}{7}\vec{CA}$ $\left(\because \frac{CD}{DA} = \frac{3}{4}\right)$

$= -\underset{\sim}{a} + \frac{3}{7}\underset{\sim}{b}$ #

(b) $\vec{CE} = \vec{CB} + \vec{BE}$

$= \underset{\sim}{a} + \frac{1}{4}\vec{BA}$ $\left(\because \frac{BE}{EA} = \frac{1}{3}\right)$

$= \underset{\sim}{a} + \frac{1}{4}(\vec{BC} + \vec{CA})$

$= \underset{\sim}{a} + \frac{1}{4}(-\underset{\sim}{a} + \underset{\sim}{b})$

$= \frac{1}{4}(3\underset{\sim}{a} + \underset{\sim}{b})$ # (SHOWN)

(c) Given $\vec{BF} = k\vec{BD}$

$\Rightarrow \vec{BC} + \vec{CF} = k\vec{BD}$

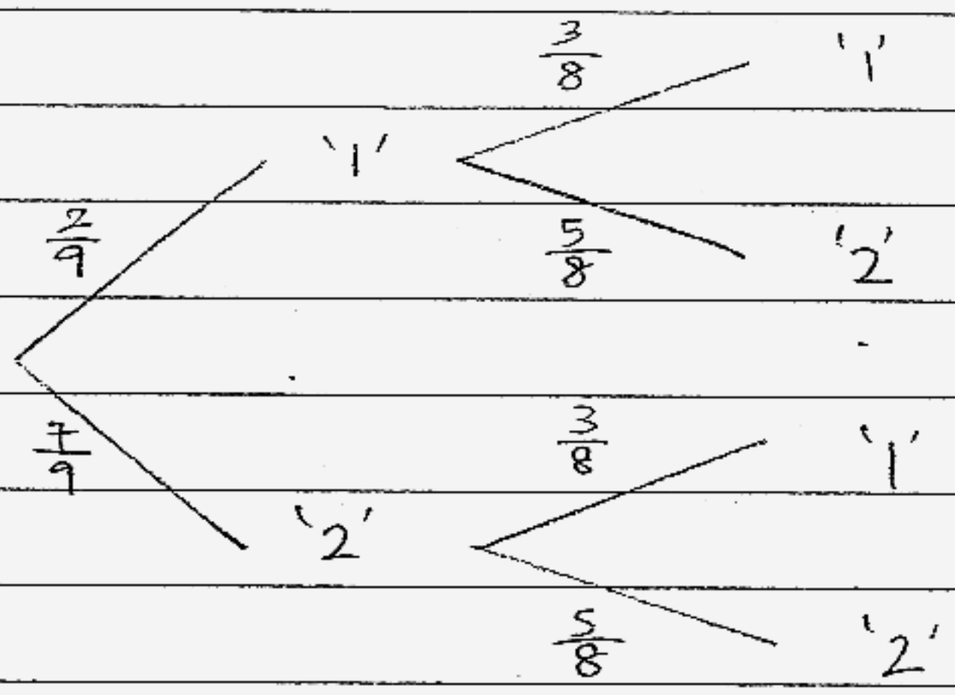
$\Rightarrow \vec{CF} = k(-\underset{\sim}{a} + \frac{3}{7}\underset{\sim}{b}) - \underset{\sim}{a}$

$= -k\underset{\sim}{a} + \frac{3}{7}k\underset{\sim}{b} - \underset{\sim}{a}$

$= (1-k)\underset{\sim}{a} + \frac{3}{7}k\underset{\sim}{b}$ # (SHOWN)

Date No.

10 (i) Box A Box A



(ii) $P(\text{'1' from Box B}) = P(\text{'1', '1'}) + P(\text{'2', '1'})$

$= \left(\frac{2}{9} \times \frac{3}{8}\right) + \left(\frac{7}{9} \times \frac{3}{8}\right)$

$= \frac{17}{72}$ #

(iii) $P(\text{sum is even}) = P(\text{sum} = 2) + P(\text{sum} = 4)$

$= \left(\frac{2}{9} \times \frac{3}{8}\right) + \left(\frac{7}{9} \times \frac{5}{8}\right)$

$= \frac{41}{72}$ #

(iv) (a) Modal score = 2 #

(b) Mean score $= \frac{3}{8}(2) + 6(3) + 4(4)$

18

$= 2\frac{7}{9}$ #

(c) Median position $= \frac{18+1}{2} = 9.5^{th}$

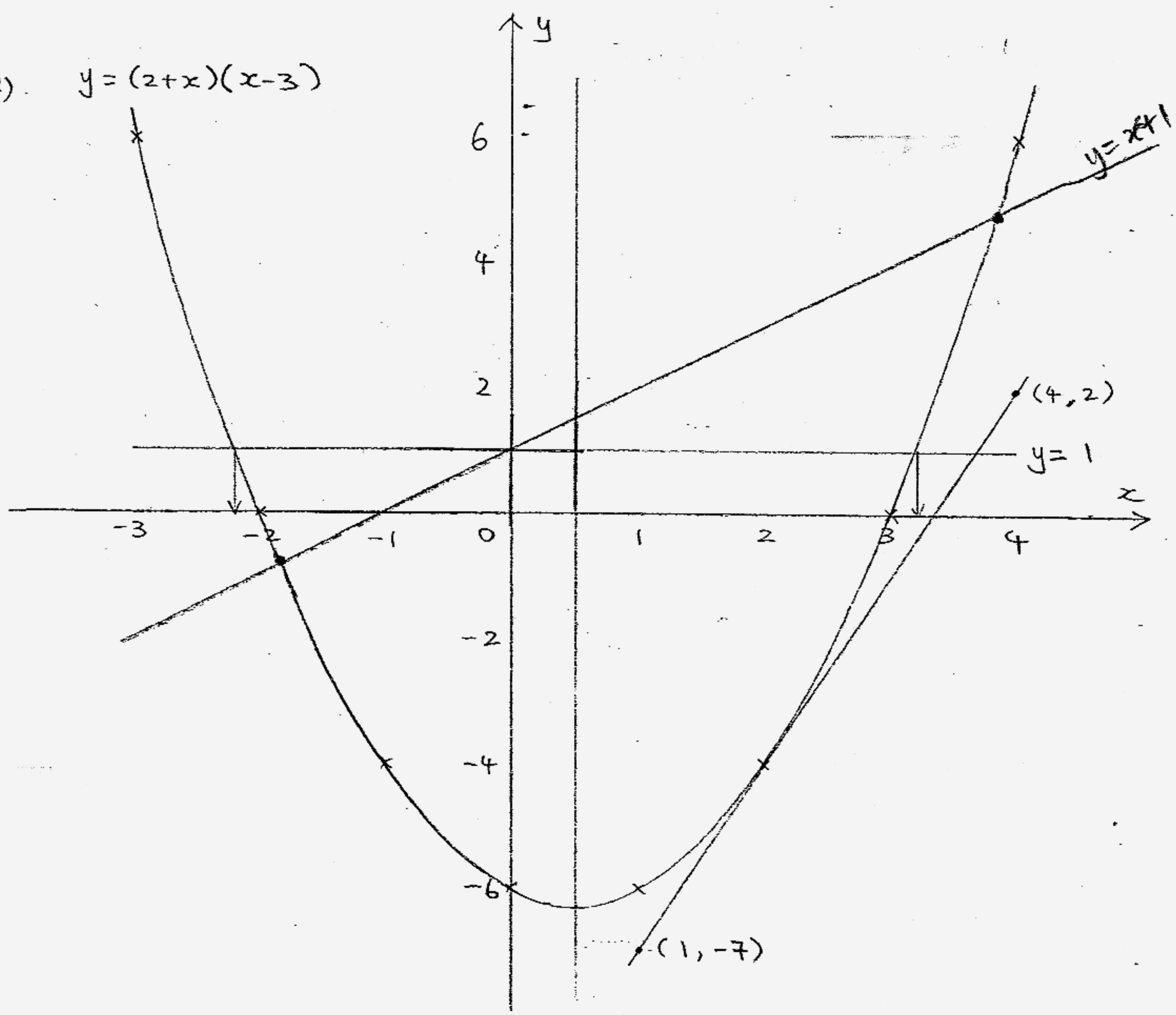
$\therefore$ Median Score = 3 #

(v) $P(\text{both labelled '1'}) = \frac{5}{17} \times \frac{4}{16}$ (w/o replacement)

$= \frac{5}{68}$ #

Qn #11

(a) $y = (2+x)(x-3)$



x	-3	-2	-1	0	1	2	3	4
y	6	0	-4	-6	-6	-4	0	6

(b) $x < -2.2$ or $x > 3.2$ #

(c) $x = \frac{1}{2}$ #

(d) Given $x^2 - 2x = 7$

$\Rightarrow x^2 - 2x - 6 = 1$

$\Rightarrow x^2 - x - 6 = x + 1$

$\Rightarrow y = x + 1$

From the graph, soln is

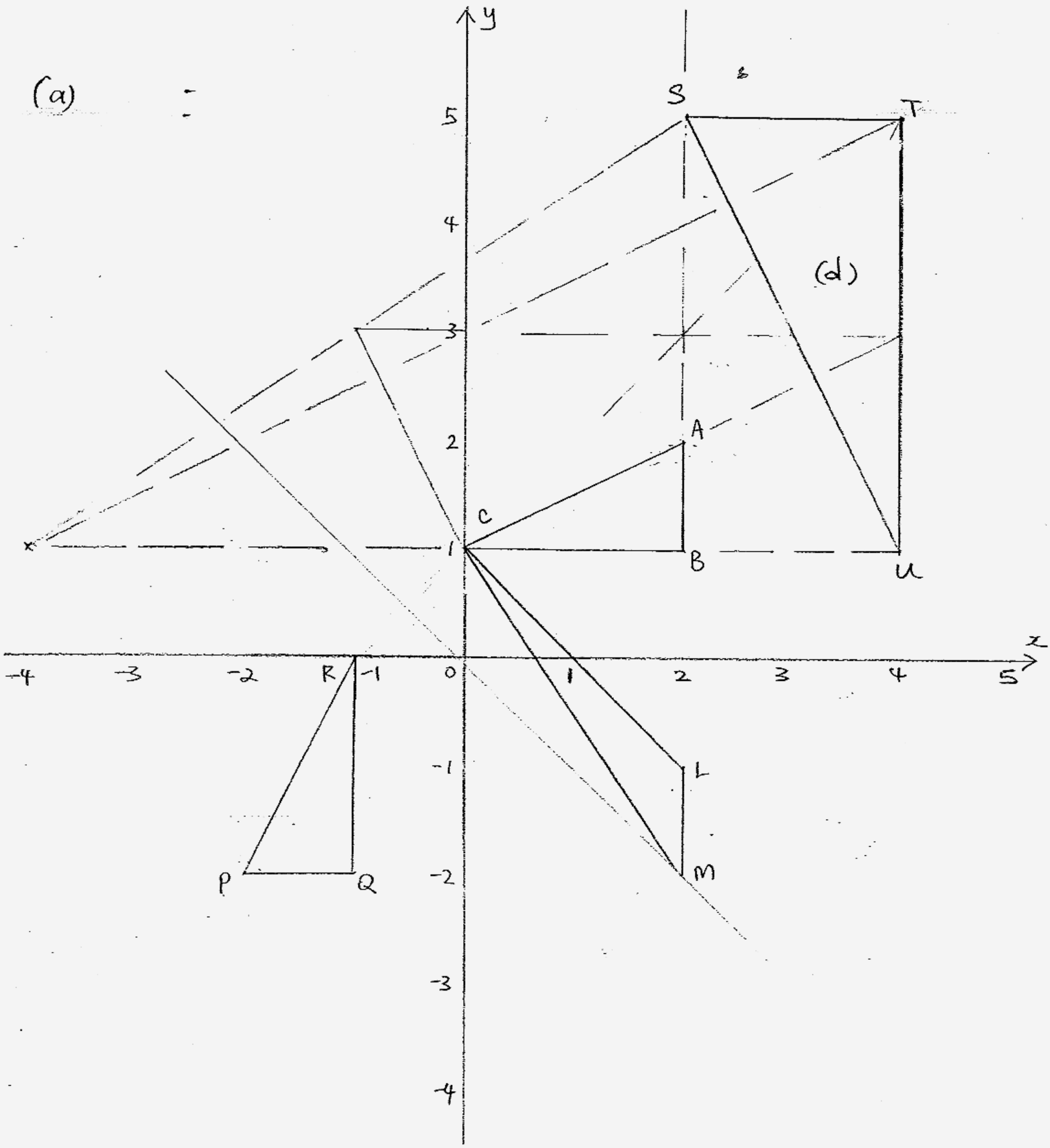
$x = -1.85$ or $x = 3.85$ #

(e) Gradient $= \frac{2-(-7)}{4-(1)}$

$= 3$ #

Qn #12

(a)



(b) G is a reflection in the line $y=-x$

$\therefore$ coord of X = coord of A = (2, 2) #

(c) H is a shear with y-axis as invariant line and shear factor $-\frac{3}{2}$ #

(e) F is a 90° anticlockwise rotation about point (2, 3) #