Name:	Class: Sec	Class Register No:

/ 80	Parent's Signature

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

MID YEAR EXAMINATION 2007
SECONDARY 3

Additional Mathematics Paper
4038

4th May 2007
2 hours

Instructions to Candidates:

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Given non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of 4 printed pages (including this cover page)

Mathematical Formulae

1. Algebra

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$, where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$.

2. Trigonometry

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\cos A - \cos B = 2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \tfrac{1}{2}bc\sin A$$

1. Find the value of k for which the line $x+3y=k$ is a tangent to the curve $y^2=2x+3$ and find the x-coordinate of the point at which this tangent touches the curve. [7]

2. (a) The remainder of $2x^3+5x^2+4x+k$ when divided by $(x-1)$ is equal to the remainder of x^3-kx+7 when divided by $(x+2)$. Find the value of k. [4]

   (b) Factorise the expression x^3-7x-6 completely. Hence, or otherwise, solve the equation $x^3-7x-6=0$. [8]

3. Find the range of values of k for which $y=-kx^2-6kx+18$ is always positive. [4]

4. The roots of the quadratic equation $x^2-4x-5=0$ are α and β.

   (i) State the values of $\alpha+\beta$, $\alpha\beta$ and $\alpha^2+\beta^2$ [3]

   (ii) Hence, find the quadratic equation in x whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [5]

5. (i) By expressing in the form of $a(x+b)^2+c$, show that $y=2x^2-4x+7$ is always positive for all real values of x. [3]

   (ii) Sketch the graph of $y=2x^2-4x+7$ for $-3\le x\le 2$. [2]

   (iii) By adding a line $y=|10x|$ or otherwise, find the number of solutions for $2x^2-4x+7=|10x|$ for $-3\le x\le 2$. [2]

6. Express $\frac{4x^3-3x+2}{2x^2+3x-2}$ into its partial fractions. [7]

7. Solve the following equations:

   (a) $4^{x+1}+7(2^x)=2$ [4]

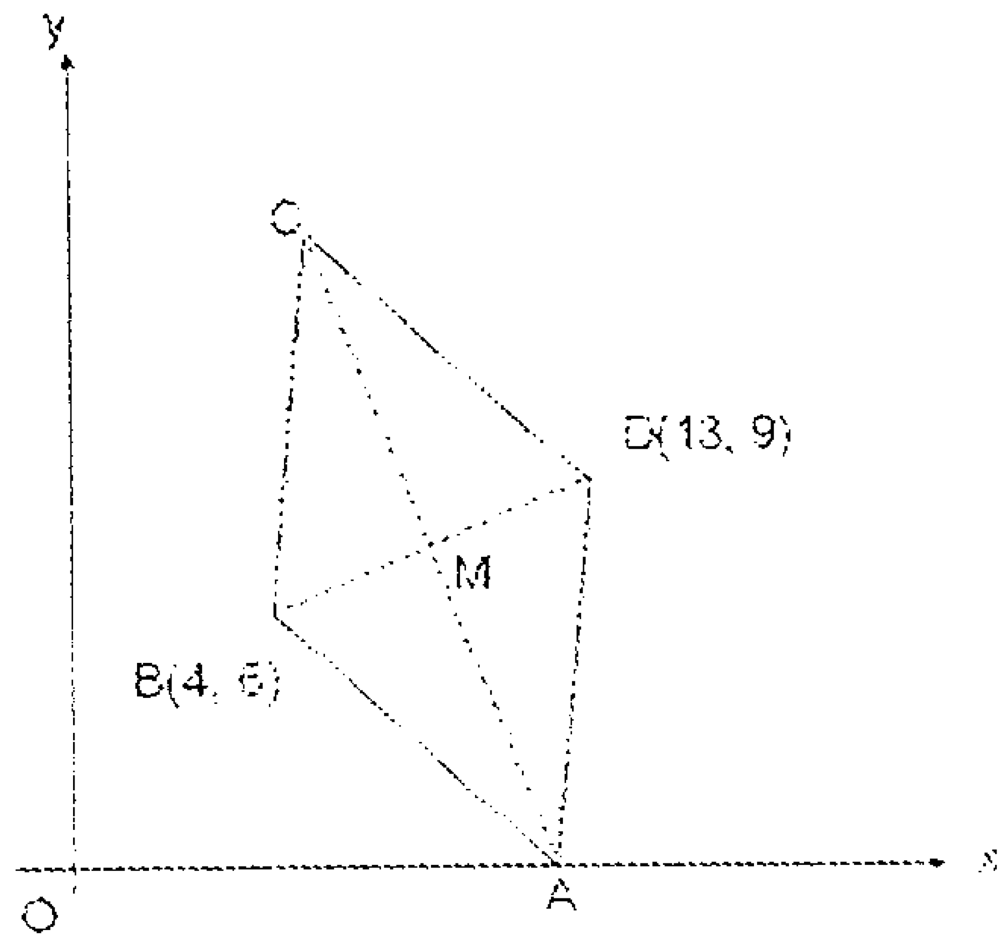
   (b) $\log_5 x-\log_{25}(x+10)=\frac{1}{2}$ [4]

8. Express $\left(\frac{13}{4+\sqrt{3}}\right)^2$ in the form $p+q\sqrt{3}$, where p and q are integers. [4]

9. Given that $\frac{a^x}{b^{x-3}} \times \frac{b^y}{(a^{y-1})^2} = ab^5$, find the value of x and of y. [6]

10. Given that $\log_2 x = a$ and $\log_8 y = b$, express $\log_2 xy$ in terms of a and b. [4]

11. **Solutions to this question by accurate drawing will not be accepted.**
The diagram shows a rhombus ABCD with AB = BC = CD = DA. The point A lies on the x-axis. B and D are the points (4, 6) and (13, 9) respectively.



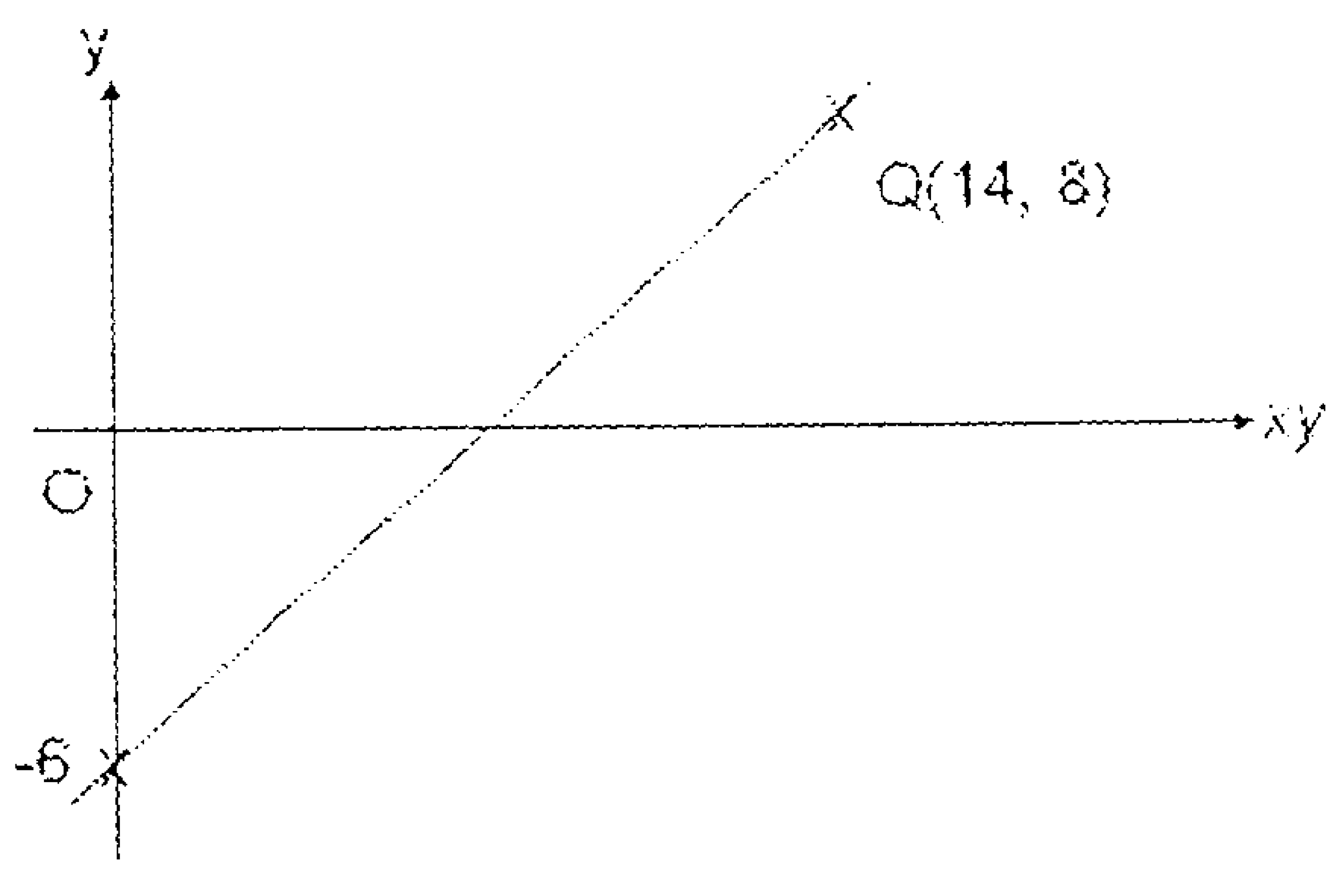
Find

(a) the coordinates of M, the midpoint of BD, [2]

(b) the equation of AC [3]

(c) the coordinates of A and of C, [4]

12. The equation $y = \frac{a}{b-x}$, where a and b are constants, can be represented by a straight line when y is plotted against xy as shown in the diagram. Calculate the value of a and of b. [4]



End of Paper

2007 S3 Additional Mathematics MYE Solution

1. Find the value of k for which the line $x+3y=k$ is a tangent to the curve $y^2=2x+3$ and find the x-coordinate of the point at which this tangent touches the curve. [7]

Solution:

$y^2 = 2x+3$ --------- (1)

$x+3y=k$ ---------- (2)

$x=k-3y$ ---------- (3)

Substitute (3) into (1):

$$y^2 = 2(k-3y)+3$$
$$y^2+6y-(2k+3)=0$$

For tangent to the curve, $b^2-4ac=0$

$$36-4(1)(-(2k+3))=0$$
$$36+4(1)(2k+3)=0$$
$$36+8k+12=0$$
$$48+8k=0$$
$$k=-6$$

Ans: $k = -6$ for line to be tangent to curve

$y^2=2x+3$ --------- (1)

$x=-3y-6$ ---------- (3)

Substitute (3) into (1):

$y^2+6y-(2(-6)+3)=0$

$y^2+6y-(-12+3)=0$

$y^2+6y+9=0$

$(y+3)^2=0$

$y=-3$

When $y=-3$, $x=-3(-3)-6=3$

Ans: $x=3$

Comment [S1]: M1

Comment [S2]: M1

Comment [S3]: M1

Comment [S4]: A1

Comment [S5]: Deduct if not written

Comment [S6]: M1 for substitution

Comment [S7]: M1

Comment [S8]: A1

Comment [S9]: Deduct if not written

2. (a) The remainder of $2x^3 + 5x^2 + 4x + k$ when divided by $(x-1)$ is equal to the remainder of $x^3 - kx + 7$ when divided by $(x+2)$. Find the value of k. [4]

(b) Factorise the expression $x^3 - 7x - 6$ completely. Hence, or otherwise, solve the equation $x^3 - 7x - 6 = 0$. [8]

(a) Let $f(x) = 2x^3 + 5x^2 + 4x + k$

When divided by $(x-1)$, remainder,

$$f(1) = 2(1)^3 + 5(1)^2 + 4(1) + k$$

Comment [S10]: M1:2

$$f(1) = 11 + k$$

Comment [S11]: A1

Let $g(x) = x^3 - kx + 7$

When divided by $(x+2)$ remainder,

$$g(-2) = (-2)^3 - k(-2) + 7$$

Comment [S12]: M1:2

$$g(-2) = 2k - 1$$

Comment [S13]: A1

Since the remainders for both are the same,

$$11 + k = 2k - 1$$

Comment [S14]: M

$$k - 2k = -11 - 1$$

$$k = 12$$

Comment [S15]: M

(b) Let $h(x) = x^3 - 7x - 6$

$h(3) = (3)^3 - 7(3) - 6 = 0$

Comment [S16]: M1 for trial method

$(x-3)$ is a factor of $h(x) = x^3 - 7x - 6$

Comment [S17]: or (x+2) or (x+1) A1

$h(x) = x^3 - 7x - 6$

By either comparing coefficients, long division or synthetic method,

Comment [S18]: M1

Thus, $h(x) = (x-3)(x^2 + 3x + 2)$

Comment [S19]: M1

$h(x) = (x-3)(x+2)(x+1)$

Comment [S20]: A1

When $h(x) = 0$, $(x-3)(x+2)(x+1) = 0$

$x = 3,\ or\ -2\ or\ -1$

Comment [S21]: A1 X 3

3. Find the range of values of k for which $y = -kx^2 - 6kx + 18$ is always positive. [4]

For $y = -kx^2 - 6kx + 18$ to be always positive, $b^2 - 4ac < 0$

$(-6k)^2 - 4(-k)(18) < 0$

$36k^2 + 72k < 0$

$k(k+2) < 0$

Using either number line or graphical method

$-2 < k < 0$

Ans: $-2 < k < 0$

Comment [S22]: A1 (not awarded if wrong sign)

Comment [S23]: M1

Comment [S24]: M1

Comment [S25]: A1 If numerical answers are correct, deduct for EACH wrong sign

4. The roots of the quadratic equation $x^2 - 4x - 5 = 0$ are α and β.

(i) State the values of $\alpha + \beta$, $\alpha\beta$ and $\alpha^2 + \beta^2$ [3]

(ii) Hence, find the quadratic equation in x whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [5]

$x^2 - 4x - 5 = 0$

(i) $\alpha + \beta = -(\frac{-4}{1}) = 4$

$\alpha\beta = (\frac{-5}{1}) = -5$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\alpha^2 + \beta^2 = (4)^2 - 2(-5) = 26$

(ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$

$= \frac{26}{(5)^2}$

$= \frac{26}{25}$

$\left(\frac{1}{\alpha^2}\right)\left(\frac{1}{\beta^2}\right) = \frac{1}{25}$

Comment [S26]: A1

Comment [S27]: A1

Comment [S28]: A1

Comment [S29]: M1

Comment [S30]: A1

Comment [S31]: M1

Equation of quadratic curve:

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$x^2 - \frac{26}{25}x + \frac{1}{25} = 0$

$25x^2 - 26x + 1 = 0$

Comment [S32]: M1

Comment [S33]: A1 For simplification

5. (i) By expressing in the form of $a(x+b)^2 + c$, show that $y = 2x^2 - 4x + 7$ is always positive for all real values of x. [3]

(ii) Sketch the graph of $y = 2x^2 - 4x + 7$ for $-3 \le x \le 2$. [2]

(iii) By adding a line $y = |10x|$ or otherwise, find the number of solutions for $2x^2 - 4x + 7 = |10x|$ for $-3 \le x \le 2$. [2]

(i) $y = 2x^2 - 4x + 7$

$y = 2(x^2 - 2x) + 7$

$y = 2[(x-1)^2 - 1] + 7$

$y = 2(x-1)^2 + 5$

Since $(x-1)^2 \ge 0$, $y = 2x^2 - 4x + 7$ is always positive for all real values of x.

OR

Since the minimum value is 5, $y = 2x^2 - 4x + 7$ is always positive for all real values of x.

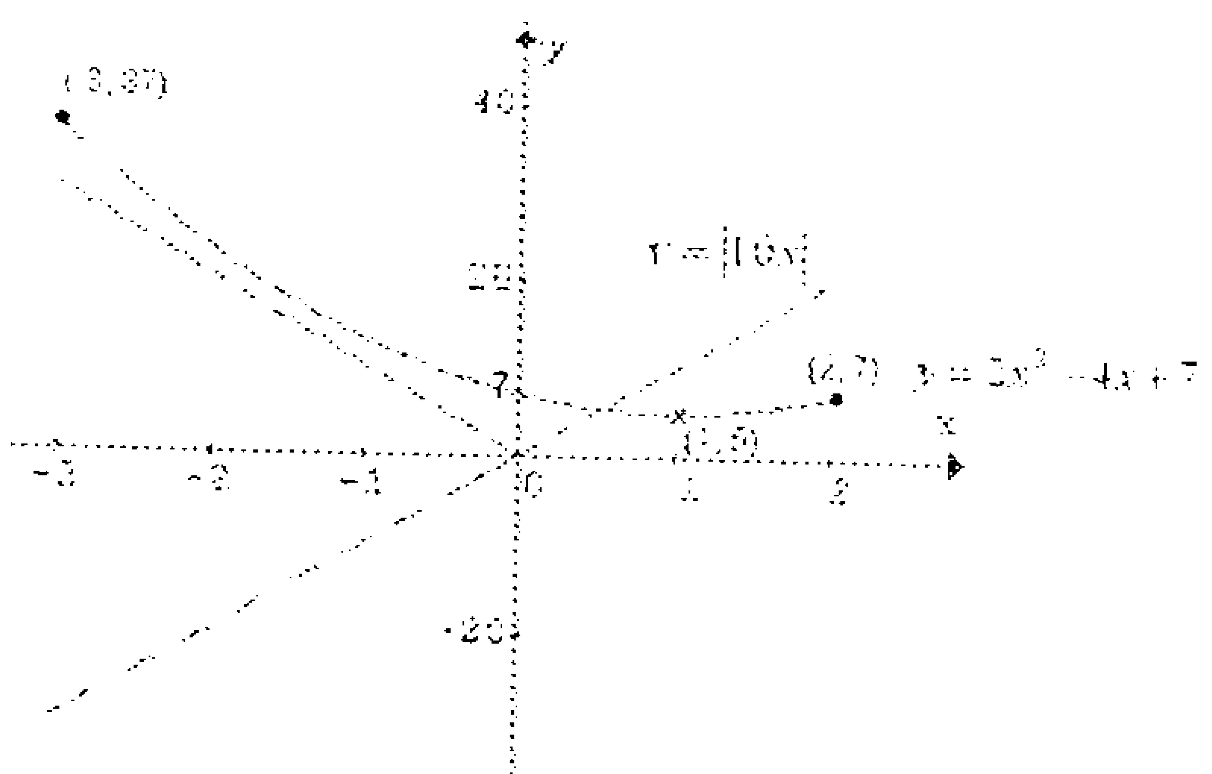
Comment [S34]: M1

Comment [S35]: A1

Comment [S36]: A1

Comment [S37]: OR A1

(ii), (iii):



(ii)
Correct shape = A½
Turning pt (1, 5) = A½
(-3, 37) & (2, 7) = A½
y intercept at 7 = A½

If labelling (equation, each points and/or axis, outline not drawn to (unshaded)), deduct ½ for each mistake, but max deduct 1m

(iii)
y = 10x A½
y = |10x| A½
Ans: 1 solution (A1)

6. Express $\frac{4x^3 - 3x + 2}{2x^2 + 3x - 2}$ into its partial fractions. [7]

$\frac{4x^3 - 3x + 2}{2x^2 + 3x - 2}$

$= 2x - 3 + \frac{10x - 4}{2x^2 + 3x - 2}$

$= 2x - 3 + \frac{10x - 4}{(2x-1)(x+2)}$

Let $\frac{10x - 4}{(2x-1)(x+2)} = \frac{A}{(2x-1)} + \frac{B}{(x+2)}$

$10x - 4 = A(x+2) + B(2x-1)$

When $x = -2$,

$10(-2) - 4 = B(2(-2) - 1)$

$B = \frac{24}{5}$

When $x = \frac{1}{2}$,

$10(\frac{1}{2}) - 4 = A(\frac{1}{2} + 2)$

$A = \frac{2}{5}$

Thus,

$= 2x - 3 + \frac{2}{5(2x-1)} + \frac{24}{5(x+2)}$

Comment [S38]: A1

Comment [S39]: M1

Comment [S40]: M1

Comment [S41]: M $\frac{1}{2}$

Comment [S42]: A1

Comment [S43]: M $\frac{1}{2}$

Comment [S44]: A1

Comment [S45]: A1

2007 S3 Additional Mathematics MYE Solution

7. Solve the following equations:

(a) $4^{x+1} + 7(2^x) = 2$ [4]

(b) $\log_5 x - \log_{25}(x+10) = \frac{1}{2}$ [4]

Solution:

(a)
$$4^{x+1} + 7(2^x) = 2$$
$$2^{2(x+1)} + 7(2^x) = 2$$
$$4(2^{2x}) + 7(2^x) = 2$$

Let $y = 2^x$
$$4y^2 + 7y - 2 = 0$$
$$(4y-1)(y+2) = 0$$
$$y = \frac{1}{4} \qquad y = -2 \ (rejected)$$
$$2^x = \frac{1}{4}$$
$$2^x = 2^{-2}$$
$$x = -2$$

(b)
$$\log_5 x - \log_{25}(x+10) = \frac{1}{2}$$
$$\log_5 x - \frac{\log_5(x+10)}{\log_5 25} = \frac{1}{2}$$
$$\log_5 x - \frac{\log_5(x+10)}{2} = \frac{1}{2}$$
$$2\log_5 x - \log_5(x+10) = 1$$
$$\log_5 x^2 - \log_5(x+10) = 1$$
$$\log_5 \frac{x^2}{(x+10)} = 1$$
$$\frac{x^2}{(x+10)} = 5$$
$$x^2 = 5x + 50$$
$$x^2 - 5x - 50 = 0$$
$$(x-10)(x+5) = 0$$
$$x = 10 \quad \text{or} \quad x = -5 \ (rejected)$$

Comment [S46]: M1

Comment [S47]: M1

Comment [S48]: Deduct if not written rejected

Comment [S49]: (M $\frac{1}{2}$) X 2

Comment [S50]: (A $\frac{1}{2}$) X 2

Comment [S51]: M1 for changing base

Comment [S52]: A1

Comment [S53]: A1

Comment [S54]: A1 deduct if not written rejected

8. Express $\left(\dfrac{13}{4+\sqrt{3}}\right)^2$ in the form $p + q\sqrt{3}$, where p and q are integers. [4]

$$\left(\frac{13}{4+\sqrt{3}}\right)^2$$

$$= \frac{169}{16+8\sqrt{3}+3}$$

Comment [S55]: A1

$$= \frac{169}{19+8\sqrt{3}} \times \frac{19-8\sqrt{3}}{19-8\sqrt{3}}$$

Comment [S56]: M1

$$= \frac{169(19-8\sqrt{3})}{169}$$

$$= 19 - 8\sqrt{3}$$

Comment [S57]: A1 X 2

Thus, $p = 19$, $q = -8$

Comment [S58]: Good to state, but no deduction of marks if not stated

9. Given that $\dfrac{a^x}{b^{x-3}} \times \dfrac{b^y}{(a^{y-1})^2} = ab^5$, find the value of x and of y. [6]

$$\frac{a^x}{b^{x-3}} \times \frac{b^y}{(a^{y-1})^2} = ab^5$$

$$\frac{a^x}{b^{x-3}} \times \frac{b^y}{a^{2y-2}} = ab^5$$

Comment [S59]: A1

$$a^{x-2y+2}b^{y-x+3} = ab^5$$

Comment [S60]: A1

By comparison,

$x - 2y + 2 = 1$

Comment [S61]: M1

$x - 2y = -1$ ------------------- (1)

$y - x + 3 = 5$

Comment [S62]: M1

$-x + y = 2$ ------------------- (2)

(1) + (2): $-y = 1$

$y = -1$

Comment [S63]: A1

When $y = -1$, $x = -3$

Comment [S64]: A1, deduct if not written

10. Given that $\log_2 x = a$ and $\log_8 y = b$, express $\log_2 xy$ in terms of a and b [4]

Solution:

$\log_2 xy$

$= \log_2 x + \log_2 y$

Comment [S65]: M1 for recognizing product law

$= \log_2 x + \dfrac{\log_8 y}{\log_8 2}$

Comment [S66]: M1 for correct change of base (any base)

$= \log_2 x + \dfrac{\log_8 y}{\log_8 8^{\frac{1}{3}}}$

Comment [S67]: A1

$= \log_2 x + \dfrac{\log_8 y}{\frac{1}{3}\log_8 8}$

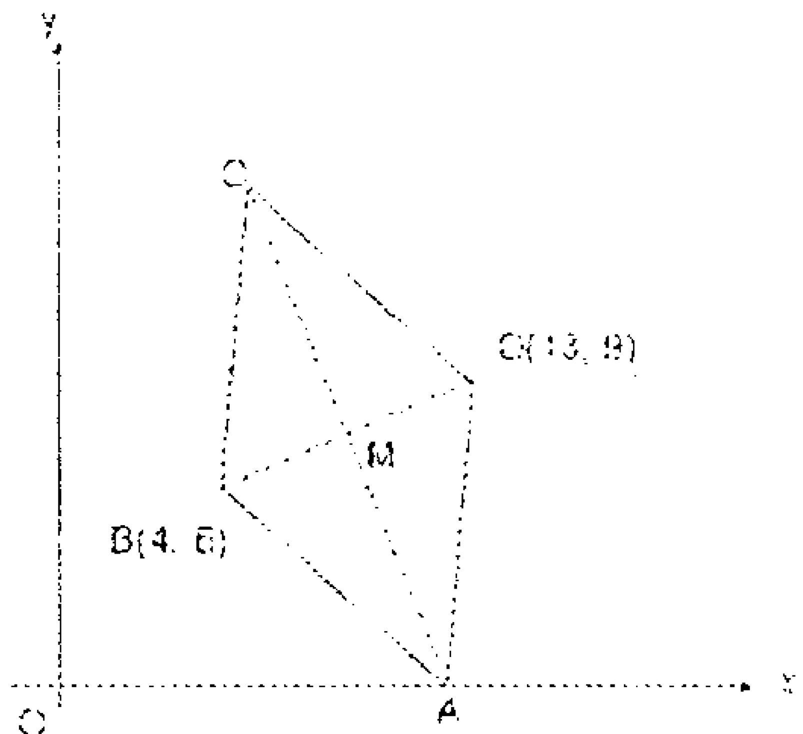
$= \log_2 x + 3\log_8 y$

$= a + 3b$

Comment [S68]: A1

11. **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows a rhombus ABCD with AB = BC = CD = DA. The point A lies on the x-axis. B and D are the points (4, 6) and (13, 9) respectively.



Find

(a) the coordinates of M, the midpoint of BD, [2]

(b) the equation of AC [3]

(c) the coordinates of A and of C, [4]

(a) $M = \left(\dfrac{4+13}{2}, \dfrac{6+9}{2}\right)$

Comment [S69]: M1

$M = \left(8\frac{1}{2},\ 7\frac{1}{2}\right)$ — Comment [S70]: A1

(b) Gradient of line BD $= \frac{9-6}{13-4}$ — Comment [S71]: M1

$= \frac{1}{3}$

$M = \left(8\frac{1}{2},\ 7\frac{1}{2}\right)$

Gradient of line AC = -3 — Comment [S72]: M

Equation of line AC:

$y - \frac{15}{2} = (-3)(x - \frac{17}{2})$ — Comment [S73]: M

$y - \frac{15}{2} = -3x + \frac{51}{2}$

$y = -3x + 33$ — Comment [S74]: A1

(c) When $y = 0$, $0 = -3x + 33$ — Comment [S75]: M1

$x = 11$ — Comment [S76]: A1

Coordinates of A are $(11,\ 0)$ — Comment [S77]: Deduct if not written

Since $M\left(8\frac{1}{2},\ 7\frac{1}{2}\right)$ is also the mid point of AC,

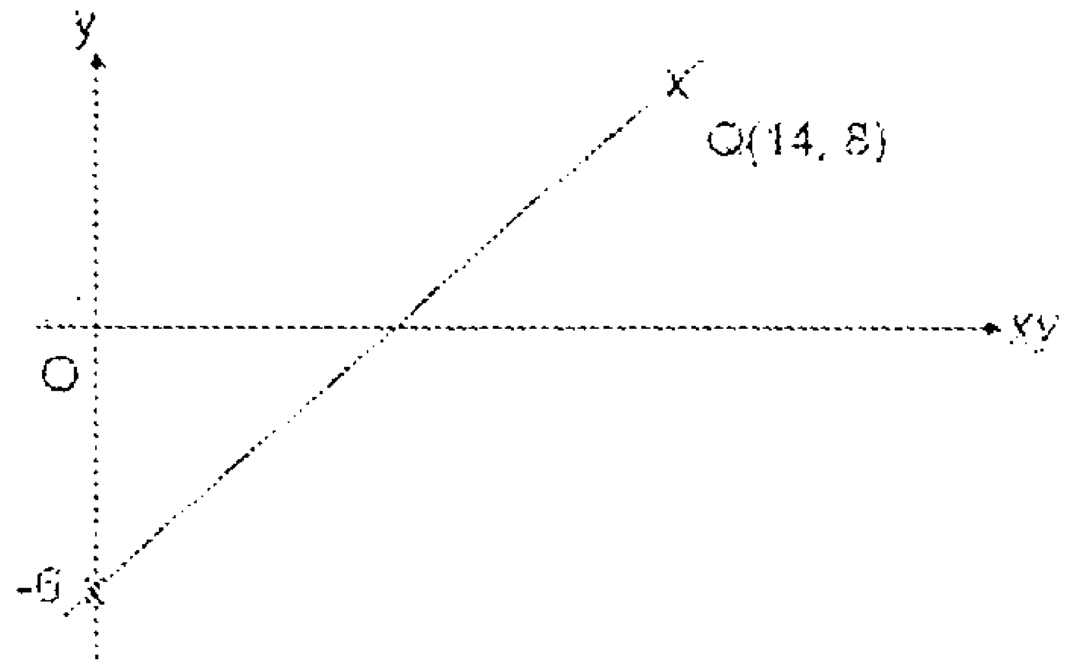
Let $C = (p,\ q)$

$\left(\frac{11+p}{2},\ \frac{0+q}{2}\right) = \left(\frac{17}{2},\ \frac{15}{2}\right)$ — Comment [S78]: M1

$p = 6$ and $q = 15$

Coordinates of C are $(6,\ 15)$ — Comment [S79]: Deduct if not written

12. The equation $y = \dfrac{a}{b-x}$, where a and b are constants, can be represented by a straight line when y is plotted against xy as shown in the diagram. Calculate the value of a and of b. [4]



$y = \dfrac{a}{b-x}$

$yb - xy = a$

$y = (\dfrac{1}{b})xy + \dfrac{a}{b}$

Comment [S80]: A1

Gradient $= \dfrac{1}{b}$, $(0,\ -6)$ and $(14,\ 8)$,

Thus,

$\dfrac{1}{b} = \dfrac{8-(-6)}{14-0}$

Comment [S81]: M

$b = 1$

Comment [S82]: A

y-intersection is $P(0,\ -6)$

Comment [S83]: M

$\dfrac{a}{b} = -6$

Comment [S84]: M

$a = -6$

Comment [S85]: A1

Ans: $a = -6$ and $b = 1$

Comment [S86]: Deduct if not written

End of Paper