CHIJ

1 Simplify each of the following, giving your answer in positive index form.

(a) $x^2 \div x^{-3} \times x^{0.5}$ [1]

(b) $\left(2\sqrt[3]{y}\right)^3 \div y^2$ [2]

2 Given that $a = \log_b c$,

(a) express c in terms of a and b. [1]

(b) express b in terms of a and c, give your answer in surd form. [2]

3 Rationalize the denominator of the following surd, giving your answer in the simplest form. [3]

$$\frac{5 + 3\sqrt{2}}{4 - 2\sqrt{2}}$$

4 Solve the simultaneous equations

$$\frac{x}{2} + \frac{y}{3} = 4 \qquad \frac{xy}{18} = 1$$ [4]

5 Kenny who is 27 years old this year bought 1 set of the pig's year coin at Suntec City sales counter for \$14.65. He estimates that the value of the coin set will increase at 2% per annum, so that its price will be $\$14.65(1.02)^n$ after n years.

(a) What will the value of the pig's year coin set when Kenny is 39 years old? Give your answer correct to 2 decimal places. [1]

(b) How old will Kenny be when the price of the pig's year coin series first exceed \$28.00. Give your answer in **years and months** correct to the nearest month. [4]

6 Find the equation of the line passing through the mid-point of (−1, 4) and (7, −2), and parallel to the line which joins (3, −2) and (6, 4). [5]

7 Find the inverse of the matrix $\begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}$ and use it to solve the simultaneous equations $3x + 2y = 7$ and $5x = -y + 3$. Explain why this method fails when applied to the equations $5x = -y + 3$ and $15x = -3y + 9$. [6]

8 (a) Express $\frac{3x - 2x^2}{(x+1)(x+2)^2}$ in partial fractions. [4]

(b) Given that $\log_c x^2 y = 8$ and $\log_c \frac{y^2}{x} = 6$, find the values of $\log_c \sqrt{xy}$. [5]

9 Find the range of values of m for which the line $y = mx - 4$ does not intersect the curve $y = x^2 - 2x + 5$. State also the values of m for which this line is a tangent to the curve. [6]

10 Solve the equation $2x^3 + 9x^2 = 20x + 12$.

Hence, solve the equation $2e^{3x} + 9e^{2x} = 20e^x + 12$ [8]

11 The roots of the quadratic equation $3x^2 - 4x + 6 = 0$ are α and β,

(i) state the value of

(a) $\alpha + \beta$,
(b) $\alpha\beta$,
(c) $\alpha^2 + \beta^2$,
(d) $\alpha^2\beta^2$. [5]

(ii) hence, find the quadratic equation in x whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [3]

12 Without using a calculator

(i) find the value of a and b for which

$$\frac{6}{\sqrt{2}}\left(\frac{5\sqrt{32}}{2} + \frac{15}{\sqrt{50}} - \frac{14}{7\sqrt{6}}\right) = a - b\sqrt{3}$$ [5]

(ii) evaluate $\dfrac{\log_4 18 + 2\log_4 3 - \log_4 6}{\log_4 81}$ [4]

13 Solve each of the following equations

(a) $16^{4x} = 8^{3+2x}$ [3]

(b) $\log_3 x - 4\log_x 3 = \log_6 1$ [4]

(c) $3^{2x+1} - 7(3^x) + 2 = 0$ [4]

The End

Marking Scheme for Sec 3 AM 2007 Mid-Year Examination

1(a) $x^{5.5}$ [B1]

1(b) $8y \div y^2$ [M1]

$= \frac{8}{y}$ [A1]

2(a) $c = b^a$ [B1]

2(b) $b = c^{\frac{1}{a}}$ [M1]

$b = \sqrt[a]{c}$ [A1]

3 $\frac{5 + 3\sqrt{2}}{4 - 2\sqrt{2}} \times \frac{4 + 2\sqrt{2}}{4 + 2\sqrt{2}}$ [M1]

$= \frac{20 + 10\sqrt{2} + 12\sqrt{2} + 6(2)}{16 - 4(2)}$ [M1]

$= 4 + 2.75\sqrt{2}$ [A1]

4 $3x + 2y = 24$ --- (1)

$xy = 18$ --- (2)

From (2), $x = \frac{18}{y}$ --- (3) [M1]

Substitute (3) into (1),

$2y^2 - 24y + 54 = 0$

$2(y - 3)(y - 9) = 0$ [M1]

$y = 3$ *or* $y = 9$ [A1]

when $y = 3$, $x = 6$
when $y = 9$, $x = 2$ [A1]

5(a) \$18.58 [B1]

5(b) $\$14.65(1.02)^n > \$28.00$ [M1]

$\lg 1.02^n > \lg\left(\frac{28}{14.65}\right)$ [M1]

$n > 32.7$ [B1]

59 years and 9 months [A1]

6 mid- point $= (3, 1)$ [B1]

gradient $= \frac{-2 - 4}{3 - 6}$ [M1]

$= 2$

Equation of the line is

$y - 1 = 2(x - 3)$ [M1] for equation of line
[M1] same gradient = 2

$y = 2x - 5$ [A1]

7 $\begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}^{-1} = \frac{1}{3 - 10}\begin{pmatrix} 1 & -2 \\ -5 & 3 \end{pmatrix}$ [M1] for working out the determinant

$= \frac{1}{-7}\begin{pmatrix} 1 & -2 \\ -5 & 3 \end{pmatrix}$ [A1]

$\begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$ [M1]

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-7}\begin{pmatrix} 1 & -2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 7 \\ 3 \end{pmatrix}$ [M1]

$= \frac{1}{-7}\begin{pmatrix} 1 \\ -26 \end{pmatrix}$ [A1]

$\left|\begin{pmatrix} 5 & 1 \\ 15 & 3 \end{pmatrix}\right| = 0$

Since determinant = 0 $\Rightarrow$ the matrix is a singular matrix
$\Rightarrow$ infinite solutions [B1] or students could identify that the 2 lines are parallel

8(a) $C = 14$ [B1]

$A = -5$ [B1]

$B = 3$ [B1]

$$\frac{3x - 2x^2}{(x+1)(x+2)^2} = \frac{-5}{x+1} + \frac{3}{x+2} + \frac{14}{(x+2)^2}$$ [B1]

8(b) $2\log_c x + \log_c y = 8$ --- (1)

$-\log_c x + 2\log_c y = 6$ --- (2) [B1]

$\log_c y = 4$ [B1]

$\log_c x = 2$ [M1]

$\log_c \sqrt{xy} = \frac{1}{2}(\log_c x + \log_c y)$ [B1]

$= 3$ [A1]

9 $x^2 - (2 + m)x + 9 = 0$ [B1]

$(2 + m)^2 - 4(1)(9) < 0$ [M1] substitute correctly

$(m + 8)(m - 4) < 0$ [M1]

$-8 < m < 4$ [A1]

$(m + 8)(m - 4) = 0$ [M1]

$m = -8$ or $m = 4$ [A1]

10 $(x - 2)$ is a factor. [B1]

$(x - 2)(ax^2 + bx + c) = 0$

$a = 2, \quad c = 6, \quad b = 13$ [B1]

$(x - 2)(x + 6)(2x + 1) = 0$ [M1]

$x = 2$ or $x = -6$ or $x = -0.5$ [A1]

$e^x = 2$ or $e^x = -6$ (NA) or $e^x = -0.5$ (NA) [M1] [M1]

$\ln e^x = \ln 2$ [M1]

$x \approx 0.693$ [A1]

11(i)(a) $\alpha + \beta = 1\frac{1}{3}$ [B1]

11(i)(b) $\alpha\beta = 2$ [B1]

11(i)(c) $(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\left(1\frac{1}{3}\right)^2 = \alpha^2 + \beta^2 + 2(2)$ [M1]

$-2\frac{2}{9} = \alpha^2 + \beta^2$ [A1]

11(i)(d) $\alpha^2\beta^2 = (\alpha\beta)^2 = 4$ [B1]

11(ii) sum of roots $= -\frac{5}{9}$ [B1]

product of roots $= \frac{1}{4}$ [B1]

quadratic equation in x:

$x^2 + \frac{5}{9}x + \frac{1}{4} = 0$ [A1]

12(i) $\frac{6}{\sqrt{2}}\left(\frac{5 \times 4\sqrt{2}}{2} + \frac{15}{5\sqrt{2}} - \frac{14}{7\sqrt{6}}\right)$ [B1][B1]

$= \frac{6}{\sqrt{2}}\left(\frac{60\sqrt{2} + 9\sqrt{2} - 2\sqrt{6}}{6}\right)$ [M1]

$= \frac{\sqrt{2}\left(69 - 2\sqrt{3}\right)}{\sqrt{2}}$ [M1]

$= 69 - 2\sqrt{3}$

$a = 69, \quad b = 2$ [A1]

12(ii) $\frac{\log_4 18 + \log_4 3^2 - \log_4 6}{\log_4 81}$ [M1]

$= \frac{\log_4 (18 \times 9 \div 6)}{\log_4 81}$ [M1]

$= \frac{\log_4 3^3}{\log_4 3^4}$ [M1]

$= \frac{3}{4}$ [A1]

13(a) $\left(2^4\right)^{4x} = \left(2^3\right)^{3 + 2x}$ [M1]

$16x = 9 + 6x$ [M1]

$x = 0.9$ [A1]

13(b) $\log_3 x - \log_x 3^4 = 0$ [M1]

$\log_3 x - \frac{\log_3 3^4}{\log_3 x} = 0$ [M1]

$\log_3 x = \pm 2$ [M1]

$x = 9$ or $x = \frac{1}{9}$ [A1]

13(c) $3(3^x)^2 - 7(3^x) + 2 = 0$ [M1]

Let $y = 3^x$

$y = \frac{1}{3}$ or $y = 2$

$3^x = \frac{1}{3}$

$x = -1$ [A1]

or $3^x = 2$

$x = \frac{\lg 2}{\lg 3}$ [M1]

$x \approx 0.631$ (3SF) [A1]