Class :_______ Name : __________________________________ (   )

**BUKIT PANJANG GOVERNMENT HIGH SCHOOL**

**SECONDARY THREE EXPRESS**

**MID-YEAR EXAMINATION 2007**

**ADDITIONAL MATHEMATICS 4038**

**Date : 9 May 2007** **Duration : 2 hours**

**Time : 1040 to 1240 hours**

## INSTRUCTIONS TO CANDIDATES:

Write your name, class and index number in the spaces provided on the answer papers.

Answer **ALL** the questions.

If you use more than one sheet of paper, fasten the sheets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION FOR CANDIDATES:

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

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*This page consists of 2 pages*

**[Turn over]**

(**1**) Express $\frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}}$ in the form $a+b\sqrt{c}$. [4]

(**2**) Sketch the graph of $y = x^2 - 2x - 3$ for $-5 \le x \le 5$. [4]

(**3**) By writing $e^x = u$, or otherwise, solve the equation $e^x - 2e^{-x} = 1$. [4]

(**4**) Solve the simultaneous equations: $\frac{3^x}{9^y} = 27$ and $4^{2x}(2^{6y}) = \frac{1}{4}$ [5]

(**5**)Express $x^2 + 6kx + 144$ in the form $(x+p)^2 + q$. Find the range of values of k such that $x^2 + 6kx + 144$ is positive for all values of $x$. [5]

(**6**) Solve the equation: $2x^2 - 3 = x$ [5]

(**7**) For what range of values of $x$ is $(2x+1)(x+3) \ge 7$? [5]

(**8**) Solve the simultaneous equations $5x + 3y + 7 = 0$ and $3y^2 = x^2 - 4y + 3$. [6]

(**9**) Find the numerical values of the constants $A$, $B$, $C$, $D$ such that

$$x^4 + Ax^3 + 5x^2 + x + 3 = (x^2+4)(x^2 - x + B) + Cx + D$$ [6]

(**10**) Determine the range of values of $p$ so that the roots of the equation

$3x^2 - 3px + (p^2 - p - 3) = 0$ are real. [6]

(**11**) Show that $\log_3 x = \log_9 x^2$. Hence solve the equation

$$\log_3(x+1) - \log_9(x-1) = 1$$ [8]

(**12**) The polynomial $2x^3 - 3ax^2 + ax + b$ has a factor $x - 1$ and when divided by $x + 2$, a remainder of -54 is obtained. Find the values of $a$ and $b$. With these values of $a$ and $b$, factorise the polynomial completely. Hence, or otherwise, solve $2x^6 - 9x^4 + 3x^2 + 4 = 0$. [10]

(**13**) If $\alpha$, $\beta$ are the roots of $2x^2 + 3x - 4 = 0$, calculate the numerical values of
(i) $\alpha^2 + \beta^2$ (ii) $\alpha^2 - \beta^2$ (iii) $\alpha^3 + \beta^3$
Form the equation whose roots are (iv) $2\alpha$, $2\beta$ (v) $\alpha^2$, $\beta^2$ [12]

\- END OF PAPER –

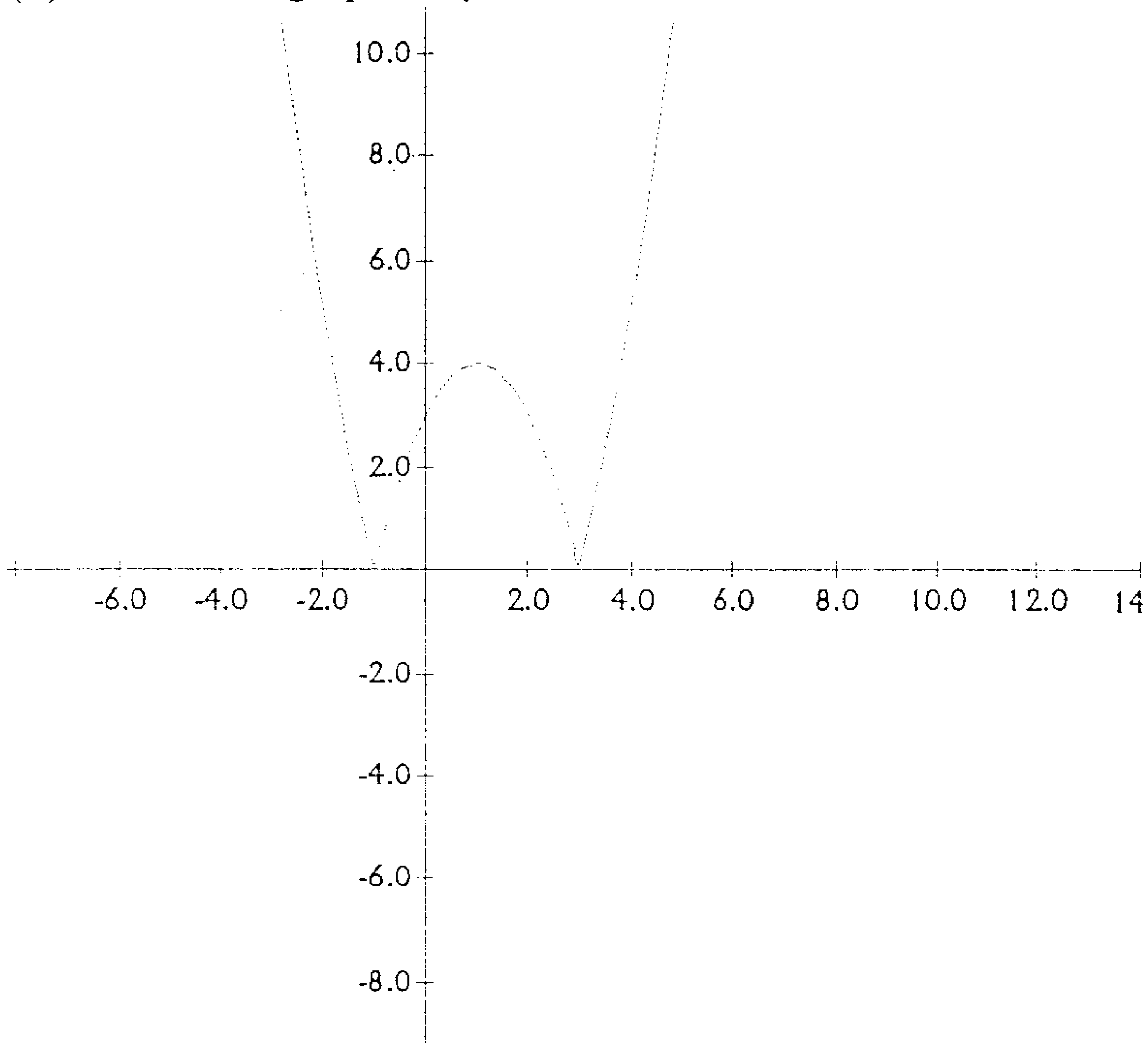
**Solution to AM 3E Mid-year 2007**

**(1)** Express $\frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}}$ in the form $a+b\sqrt{c}$. [4]

$$\frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}} \times \frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}+3\sqrt{2}} = \frac{12+12\sqrt{6}+18}{12-18} = \frac{30+12\sqrt{6}}{-6} = -5-2\sqrt{6}$$

| |
|---|
| M1 on multiplying conjugate<br>M1 on expanding the product<br>A1 for denominator value and<br>A1 for final answer |

**(2)** Sketch the graph of $y = |x^2 - 2x - 3|$ for $-5 \le x \le 5$ [4]



| |
|---|
| M1 for shape of graph<br>A2 for correct x-intercepts<br>A1 for correct y-intercept |

**(3)** By writing $e^x = u$, or otherwise, solve the equation $e^x - 2e^{-x} = 1$. [4]

$e^x = u$, $u - 2\frac{1}{u} = 1$, $u^2 - u - 2 = 0$, $(u-2)(u+1) = 0$

*So* $e^x = 2$ *or* $e^x = -1$ *(NA since* $e^x > 0$*)*

*So* $x = \ln 2 = 0.69$

| |
|---|
| M1 for getting quadratic equation in u<br>M1 for factorise equation in u<br>A1 for finding x<br>M1 for rejecting u = -1 |

**(4)** Solve the simultaneous equations: $\frac{3^x}{9^y} = 27$ and $4^{2x}(2^{6y}) = \frac{1}{4}$ [5]

$3^{x-2y} = 3^3$, so $x - 2y = 3$ -----(1)

$2^{4x+6y} = 2^{-2}$, so $4x + 6y = -2$, $2x+3y = -1$ -----(2)

(1)x2: $2x - 4y = 6$ -----(3)

(2)-(3): $7y = -7$, $y = -1$; $x = 3+2y = 1$.

| |
|---|
| M1 for getting equation (1)<br>M1 for getting equation (2)<br>M1 for solving two equations<br>A2 for correct x and y values |

(5) Express $x^2 + 6kx + 144$ in the form $(x + p)^2 + q$. Hence find the range of values of k such that $x^2 + 6kx + 144$ is positive for all values of $x$. [5]

$x^2 + 6kx + 144 = (x + 3k)^2 + 144 - 9k^2$.

So $p = 3k$ and $q = 144 - 9k^2$.

For $x^2 + 6kx + 144$ to be positive, it means the minimum curve is above the x-axis. That is the minimum value is above the x-axis.

Min value $= 144 - 9k^2 > 0$

So $16 - k^2 > 0$ or $k^2 - 16 < 0$, $(k+4)(k-4) < 0$, so $-4 < k < 4$.

> M1A1 for the correct completing square method and values for p and q
> M1 for deduce $144 - 9k^2 > 0$
> M1 and A1 for solving the quadratic inequality

(6) Solve the equation: $2x^2 - 3 = x$ [5]

$2x^2 - 3 = x$ means

$2x^2 - 3 = x$ or $2x^2 - 3 = -x$

$2x^2 - x - 3 = 0$ or $2x^2 + x - 3 = 0$

$(2x - 3)(x + 1) = 0$ or $(2x + 3)(x - 1) = 0$

$x = -1, 1, -\frac{3}{2}, \frac{3}{2}$

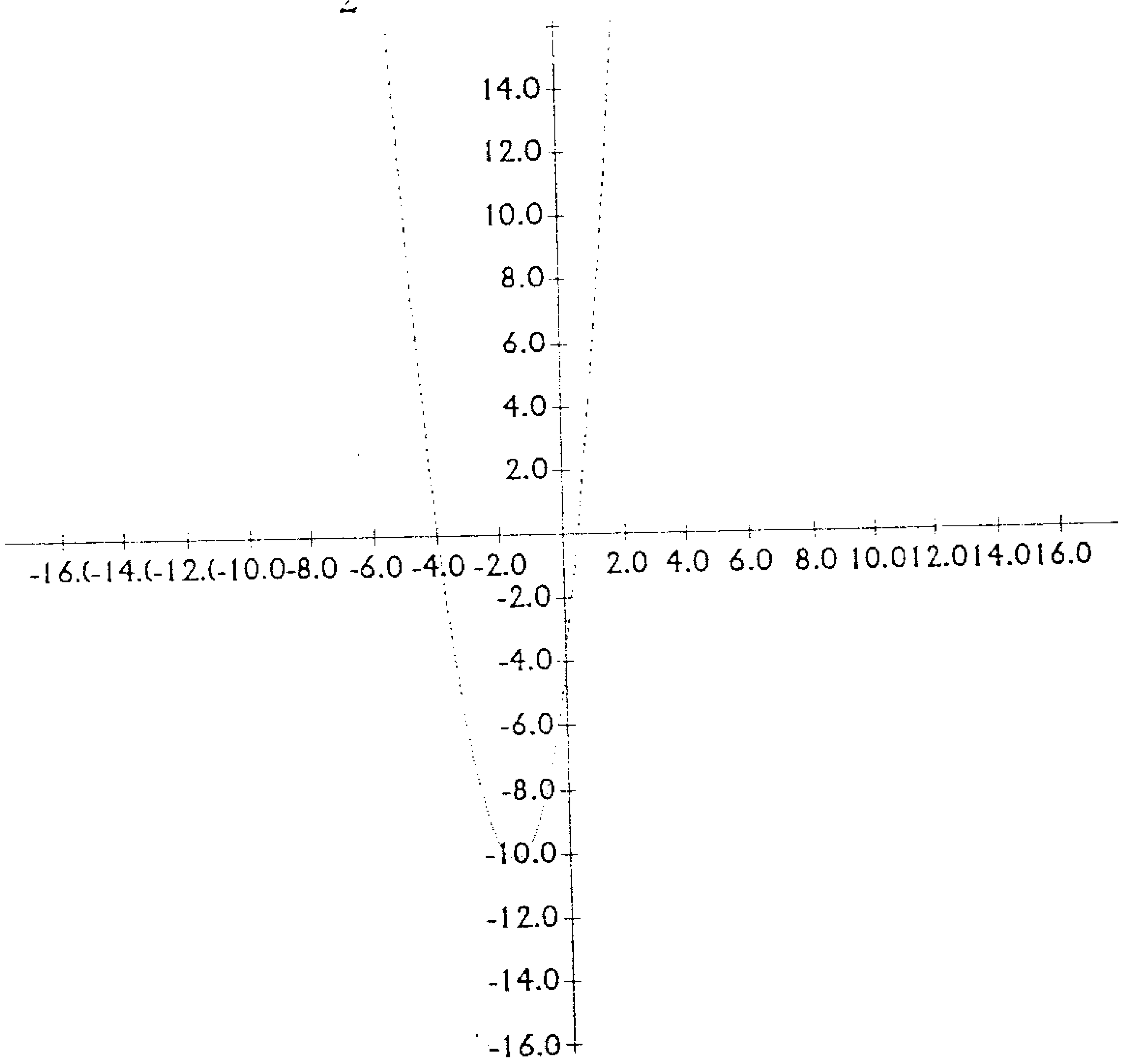
Since $x = 2x^2 - 3 > 0$, so $x = 1$ or $\frac{3}{2}$.

> M1 for using result of modulus
> M1 for getting the two quadratic equations
> M1 for getting the four values of x
> A2 for the final correct x

(7) For what range of values of $x$ is $(2x + 1)(x + 3) \geq 7$? [5]

$(2x+1)(x+3) \geq 7$, $2x^2 + 7x + 3 - 7 \geq 0$, $2x^2 + 7x - 4 \geq 0$, $(2x - 1)(x + 4) \geq 0$

So $x \leq -4$ or $x \geq \frac{1}{2}$ by referring to the graph of $y = (2x - 1)(x + 4)$




> M1 for simplify the quadratic expression
> M1 for factorise the quadratic expression
> M1 for quadratic graph
> A2 for correct range of values

(**8**) Solve the simultaneous equations $5x + 3y + 7 = 0$ and $3y^2 = x^2 - 4y + 3$. [6]

Let $x = -\frac{3y+7}{5}$, and substitute into $3y^2 = x^2 - 4y + 3$

$3y^2 = (-\frac{3y+7}{5})^2 - 4y + 3$, so $75y^2 = 9y^2 + 42y + 49 - 100y + 75$

$66y^2 + 58y - 124 = 0$

$33y^2 + 29y - 62 = 0$

$(33y + 62)(y - 1) = 0$

$y = -\frac{62}{33} = -1\frac{29}{33},\ y = 1$

$x = -\frac{3}{11}$ or $x = -2$

M1 for substitution method for x
M1 for simplify equation in y
M2 for solving x and y
A2 for correct x and y

(**9**) Find the numerical values of the constants $A, B, C, D$ such that

$$x^4 + Ax^3 + 5x^2 + x + 3 = (x^2 + 4)(x^2 - x + B) + Cx + D.$$ [6]

RHS $= x^4 - x^3 + Bx^2 + 4x^2 - 4x + 4B + Cx + D = x^4 - x^3 + (B+4)x^2 + (C-4)x + 4B+D$

So $A = -1,\ B + 4 = 5,\ B = 1;\ \ C-4 = 1,\ C = 5$ and $4B + D = 3,\ D = -1.$

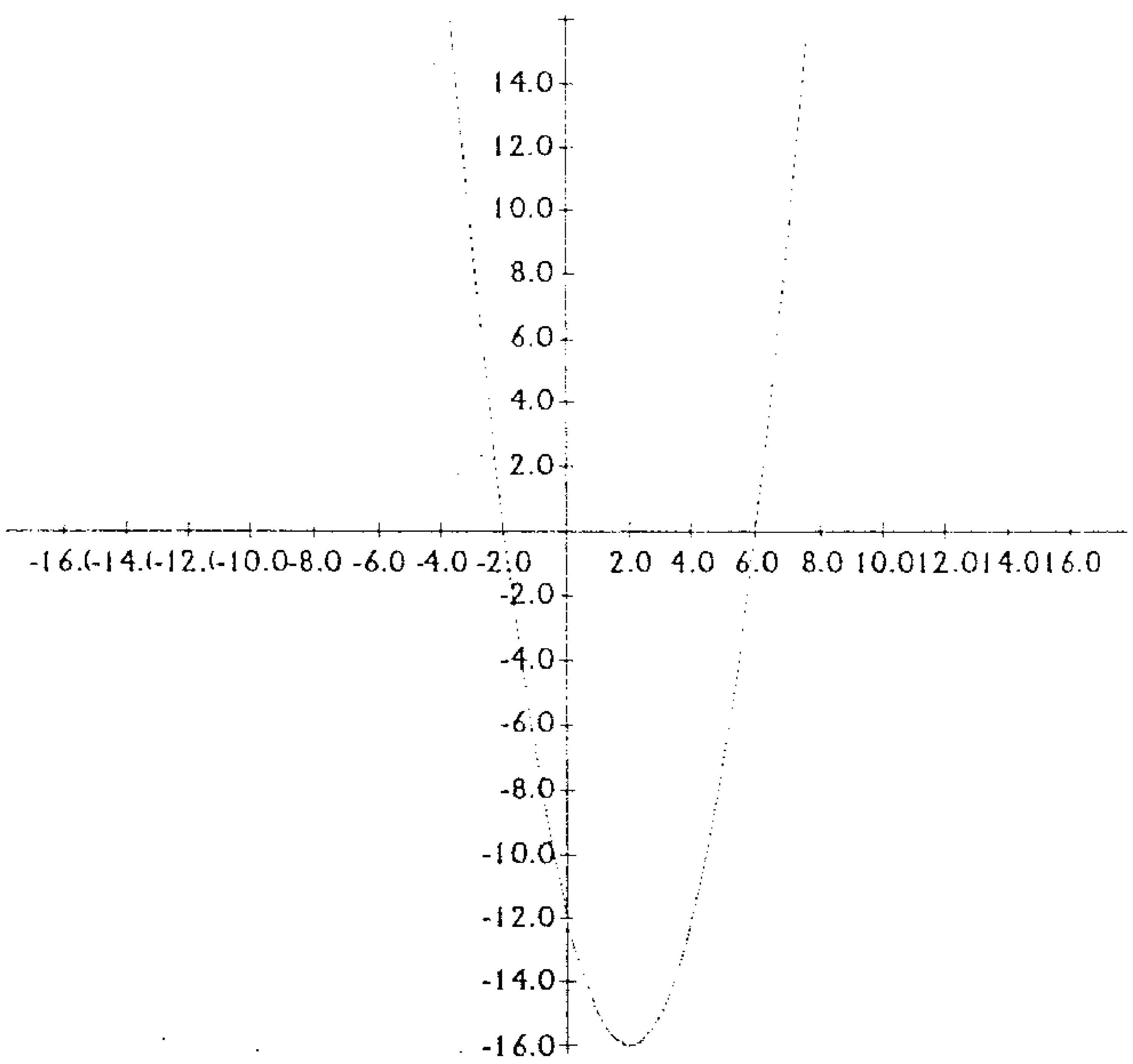
M1 for expanding RHS
M1 for comparing coefficients method
A4 for correct A, B, C and D

(**10**) Determine the range of values of $p$ so that the roots of the equation

$3x^2 - 3px + (p^2 - p - 3) = 0$ are real. [6]

Discriminant $= (-3p)^2 - 4(3)(p^2 - p - 3) = 9p^2 - 12(p^2 - p - 3) = -3p^2 + 12p + 36 = -3(p^2 - 4p - 12) \geq 0$ since the roots are real

So $p^2 - 4p - 12 \leq 0$, $(p - 6)(p + 2) \leq 0$, so $-2 \leq p \leq 6$




M1 for finding the discriminant
A1 for correct discriminant
M2 for solving quadratic inequality for p
M1 for quadratic graph
A2 for correct range of p

**(11)** Show that $\log_3 x = \log_9 x^2$. Hence solve the equation

$$\log_3(x+1) - \log_9(x-1) = 1$$ [8]

RHS $= \log_9 x^2 = \dfrac{\log_3 x^2}{\log_3 9} = \dfrac{2\log_3 x}{2\log_3 3} = \log_3 x =$ LHS

$$\log_3(x+1) - \log_9(x-1) = 1$$

$$\log_9(x+1)^2 - \log_9(x-1) = 1$$

$\log_9 \dfrac{(x+1)^2}{x-1} = 1$, $(x+1)^2 = 9(x-1)$, $x^2+2x+1 = 9x-9$

$x^2 - 7x + 10 = 0$

$(x-2)(x-5) = 0$

$x = 2$ or $5$

> M2 for proving the identity using change of base formula
> M1 for using the result
> M2 for solving log equation
> M1 solving finding x values
> A2 for correct x values

**(12)** The polynomial $2x^3 - 3ax^2 + ax + b$ has a factor $x - 1$ and when divided by $x + 2$, a remainder of -54 is obtained. Find the values of $a$ and $b$. With these values of $a$ and $b$, factorise the polynomial completely. Hence, or otherwise, solve $2x^6 - 9x^4 + 3x^2 + 4 = 0$. [10]

Let $f(x) = 2x^3 - 3ax^2 + ax + b$

$f(1) = 0$, $2 - 3a + a + b = 0$, $2a - b = 2$ ------(1)

$f(-2) = -54$, $-16 - 12a - 2a + b = -54$, $14a - b = 38$ -----(2)

(2) – (1): $12a = 36$, $a = 3$, $b = 2a - 2 = 4$.

So $f(x) = 2x^3 - 9x^2 + 3x + 4 = (x-1)(2x^2 - 7x - 4) = (x-1)(2x+1)(x-4)$

Observe $2x^6 - 9x^4 + 3x^2 + 4 = f(x^2) = 0$, so $x^2 - 1 = 0$ or $2x^2 + 1 = 0$ or $x^2 - 4 = 0$, hence $x = 1, -1, 2, -2$ since $2x^2 + 1 > 0$.

> M2 for getting equation (1) and (2)
> M1 for solving (1) and (2)
> A2 for getting correct a and b
> M1 for finding quadratic factor of f(x)
> M1 for finding all linear factor of f(x)
> M1 for identifying $f(x^2)$
> A2 for finding correct values of x

**(13)** If $\alpha, \beta$ are the roots of $2x^2 + 3x - 4 = 0$, calculate the numerical values of

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^2 - \beta^2$ (iii) $\alpha^3 + \beta^3$

Form the equation whose roots are (iv) $2\alpha$, $2\beta$ (v) $\alpha^2$, $\beta^2$ [12]

$\alpha + \beta = -\dfrac{3}{2}$ and $\alpha\beta = -2$

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (-\dfrac{3}{2})^2 - 2(-2) = \dfrac{9}{4} + 4 = \dfrac{25}{4}$

> (i) M1 and A1 for finding the value

(ii) $\alpha^2 - \beta^2 = (\alpha+\beta)^2 - 4\alpha\beta = (-\dfrac{3}{2})^2 - 4(-2) = \dfrac{9}{4} + 8 = \dfrac{41}{4}$

> (ii) M1 and A1 for finding the value

(iii) From $2x^2 + 3x - 4 = 0$, we have $2\alpha^2 + 3\alpha - 4 = 0$ and $2\beta^2 + 3\beta - 4 = 0$

So $2\alpha^3 + 3\alpha^2 - 4\alpha = 0$ and $2\beta^3 + 3\beta^2 - 4\beta = 0$

Sum up to get $2(\alpha^3+\beta^3) + 3(\alpha^2 + \beta^2) - 4(\alpha + \beta) = 0$

$$2(\alpha^3+\beta^3) + 3\left(\frac{25}{4}\right) - 4\left(-\frac{3}{2}\right) = 0$$

$$\alpha^3+\beta^3 = -\frac{1}{2}\left(\frac{75}{4}+6\right) = -\frac{1}{2}\left(\frac{99}{4}\right) = -\frac{99}{8} = -12\frac{3}{8}$$

M1 for using the equation
M1 for getting the cubic terms
M1 and A1 for finding the value

(iv) Sum of roots = $2\alpha + 2\beta = -3$ and product of roots = $4\alpha\beta = -8$

Equation is $x^2 + 3x - 8 = 0$

(v) Sum of roots = $\alpha^2 + \beta^2 = \frac{25}{4}$ and product of roots = $(\alpha\beta)^2 = 4$

Equation is $4x^2 - 25x + 16 = 0$

A2 for finding sum and product of root
A1 for correct equation obtained

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