Candidate's Name: _____ Index No. [ ] Class: [ ]

St. Margaret's Secondary School
Year End Examinations 2006

MATHEMATICS 4017/1

Secondary 3 Express

October 2006

Duration: 2 Hours

Total Marks: 80

Instructions to candidates:

1. Write your name, class and register number in the spaces provided at the top of the page.
2. Answer all questions
3. Write your answer in the spaces provided on the question paper.
4. If working is needed for any question, show it in the space below that question.
5. Omission of essential working will result in loss of marks.
6. Do not use paper clips, highlighters, glue or correction fluid.
7. **Neither Electronic Calculators Nor Mathematical Tables may be used in this paper.**

Information for candidates:

The number of marks is given in brackets [ ] at the end of each question or part question.

This paper consists of 12 printed pages.

Answer all questions.
All working must be shown in the space below the question

1.   Express:

   (a) $\dfrac{29}{40}$ as a percentage.

   (b) $\dfrac{156}{195}$ as a fraction in its lowest term

   (a) _____ [1]

   (b) _____ [1]

2.   Evaluate:

   (a) $7\dfrac{2}{3} - 1\dfrac{1}{2} \div 1\dfrac{3}{4}$

   (b) $0.12 \div \dfrac{3}{8} \times 5$

   (a) _____ [2]

   (b) _____ [2]

3. Estimate the value of $\dfrac{12.02^2 \div \sqrt{0.3587}}{\sqrt[3]{986}}$, correct to 1 significant figure.

_____ [3]

4. (a) Find the fraction which is halfway between $\dfrac{9}{13}$ and $\dfrac{10}{13}$, giving your answer as a fraction in its lowest term.

    (b) Express the answer in (a) as a decimal, correct to 3 significant figures.

(a) _____ [2]

(b) _____ [1]

5.　Solve the following equations:

(a) $(x+3)^2 = 3x + 7$

(b) $\dfrac{a+3}{2} - \dfrac{a-1}{3} = \dfrac{a}{4}$

(a) _____ [2]

(b) _____ [2]

6.　Evaluate:

(a) $4^{-1} + 2^3 - 4^0$

(b) $-9^2 \times 3^{-3} \div 27$, leaving your answer as a fraction.

(a) _____ [2]

(b) _____ [2]

7.   (a) Express 48400 as a product of its prime factors.

      (b) Hence find $\sqrt{48400}$.

                                                            (a) _____ [2]

                                                            (b) _____ [1]

8.   Given that $\sqrt{7} = 2.65$ and $\sqrt{0.7} = 0.84$, calculate

      (a) $\sqrt{0.00000007}$, giving your answer to 2 significant figures,

      (b) $\sqrt{1750}$, giving your answer in standard form.

                                                            (a) _____ [2]

                                                            (b) _____ [2]

9. (a) Sarah deposited $5 000 at a bank which offers simple interest. After 2 years, she receives a sum total of $5300 including interest. Find the interest rate per annum.

    (b) Justin sold his apartment for $ 425 000 at a loss of 15%. Find the cost price of the apartment.

    (a) _____ [2]

    (b) _____ [1]

10. Given that $-3 \le x \le 5$ and $-5 \le y \le 4$, where x and y are integers, find

    (a) the largest possible value of $x - y$

    (b) the smallest possible value of $y^2 - x^2$

    (a) _____ [1]

    (b) _____ [2]

11. Express the following as a single fraction in its simplest form.

    $$\frac{4 + 8a}{2a^2 - 32} + \frac{3}{4 - a}$$

    _____ [3]

6

12. Solve the simultaneous equations:

$$x + \frac{3}{2}y = 5$$
$$4x + 5y = 3$$

$x = $ _____ , $y = $ _____ [3]

13. Given that $m + n = 4$ and that $m^2 + n^2 = 12$, calculate the value of $2mn$. Hence find the value of $(m-n)^2$.

$2mn = $ _____ [2]

$(m-n)^2 = $ _____ [1]

14. Given that $x = 3 \times 10^6$ and $y = 7 \times 10^8$, find the value of each of the following, giving your answer in standard form.

(a) $5x + y$

(b) x^2y

(a) _____ [2]

(b) _____ [2]

7

15.

If $F = \sqrt{\dfrac{3 - EW}{W + 1}}$,

(a) Find the value of F when $E = -2$ and $W = 3$

(b) Express W in terms of E and F.

(a) _____ [1]

(b) _____ [2]

16. The first 4 terms of a sequence are -2, 1, 4 and 7.

(a) Write down the next two terms of this sequence.

(b) Write down an expression for the nth term of this sequence in terms of n.

(c) Which term is the number 259?

(a) _____ [1]

(b) _____ [1]

(c) _____ [1]

17.

Given that y is inversely proportional to the square of $(x + 1)$, and that $y = \dfrac{1}{3}$ when $x = 2$,

(a) express y in terms of x,
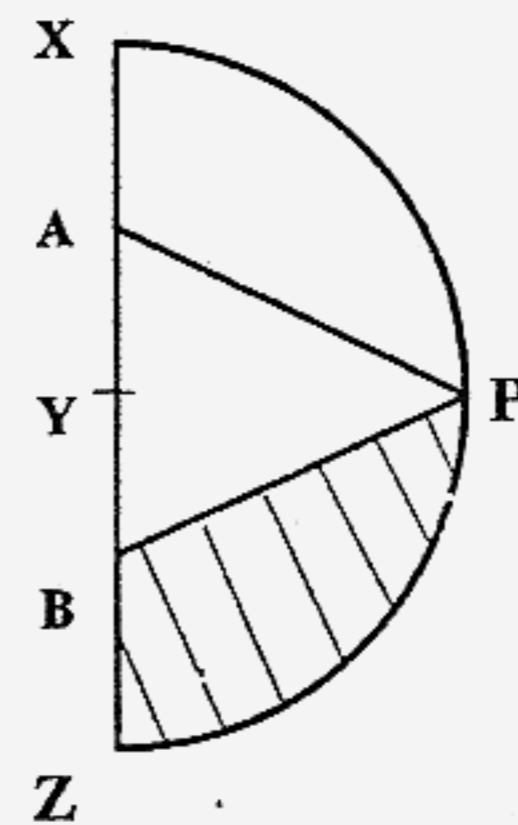
(b) find the value of y when $x = 3$.

(a) _____ [2]

(b) _____ [1]

18. Solve the inequality $\dfrac{3}{4}(3x+2) \leq 4x-9$.

_____ [2]

19. XPZ is a semicircle with centre at Y. Given that PA = PB = 13 cm and AY = 5 cm, calculate, leaving your answers in terms of π if applicable,

(a) the radius of the semicircle,

(b) the perimeter of the shaded region PBZ

(c) the area of the shaded region
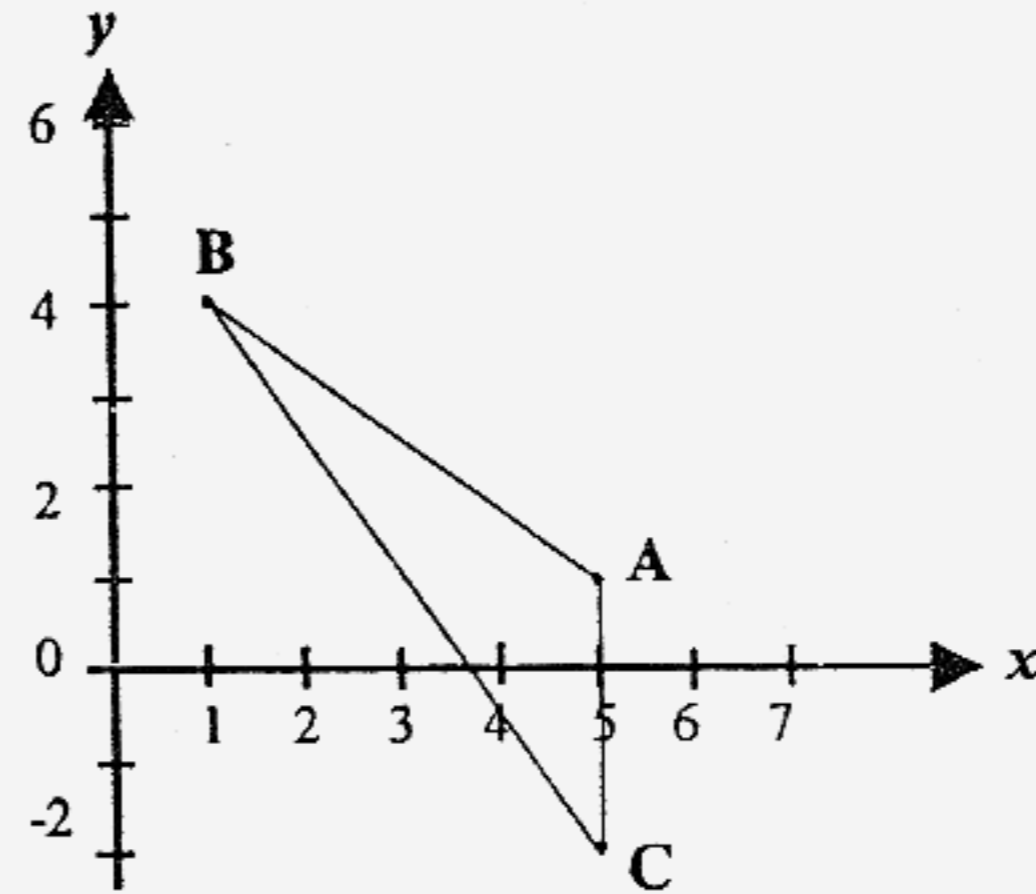
(d) $\sin P\hat{A}Y$

(a) _____ [1]

(b) _____ [2]

(c) _____ [2]

(d) _____ [1]

9

20. The diagram shows 3 points A (5, 1), B(1, 4) and C (5, -2).

Find:

(a) the gradient of the line BC

(b) the equation of the line BC

(c) the length AB

(d) the value of cos $B\hat{A}C$

(e) the area of $\triangle ABC$

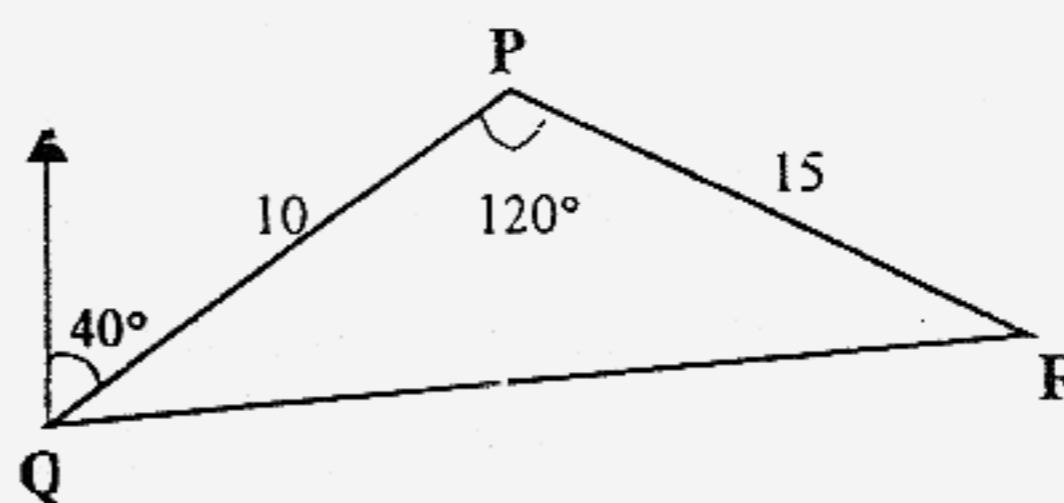(a) _____ [1]

(b) _____ [2]

(c) _____ [2]

(d) _____ [2]

(e) _____ [2]

10

21.

In the diagram, P, Q and R are three towns. The bearing of P from Q is $040°$. $Q\hat{P}R = 120°$, $PQ = 10$ km and $PR = 15$ km.

(a) Calculate

    (i) the bearing of Q from P,

    (ii) the bearing of P from R.

(b) Using as much of the information given below as is necessary, calculate the value of QR^2.

	sin	cos	tan
60°	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{2}$	$\sqrt{3}$

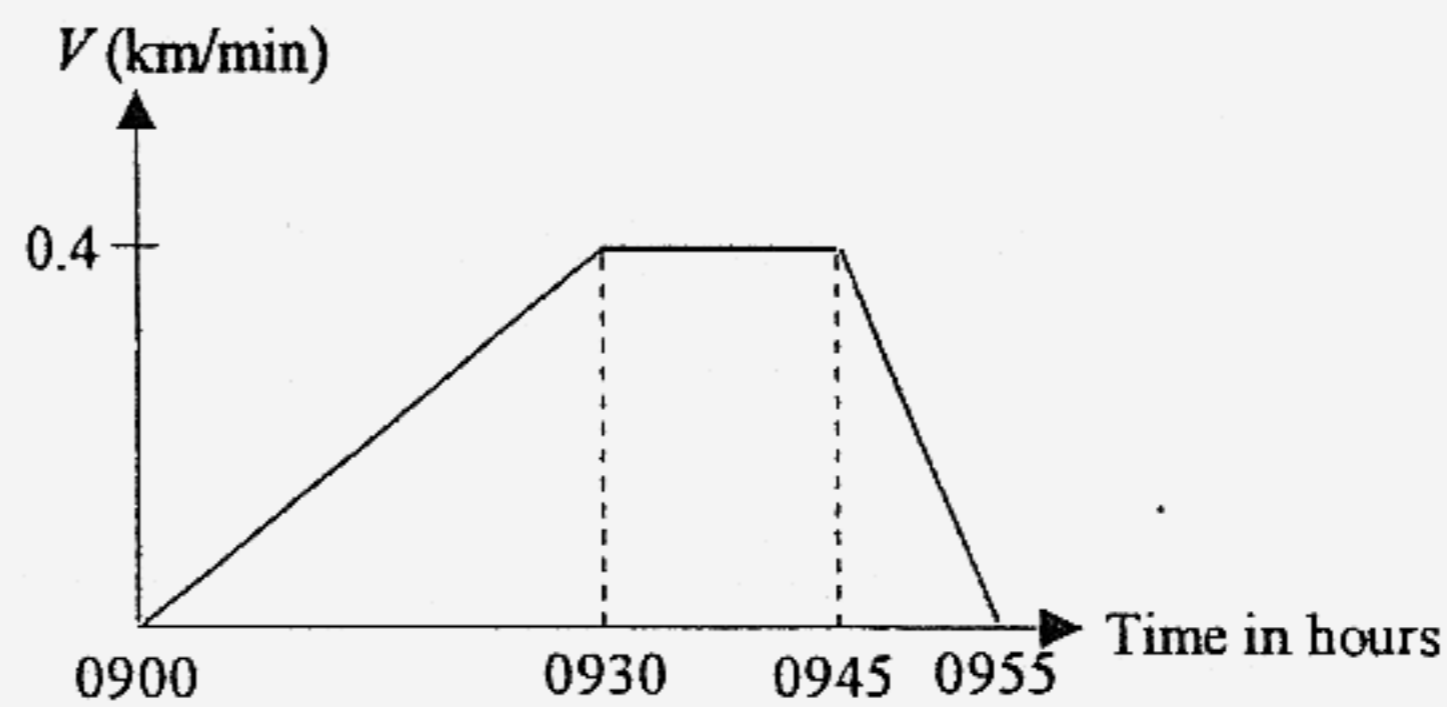(a) (i) _____ ° [1]

(ii) _____ ° [1]

(b) $QR^2 =$ _____ km [2]

11

22. The diagram below shows the speed-time graph of a cyclist. The cyclist starts from rest at O and accelerates uniformly until he reaches 0.4 km/min. He then continues to travel at this constant speed for 15 minutes before decreasing his speed uniformly until he comes to rest at Q.

V (km/min)

0.4

0900   0930   0945 0955   Time in hours

(a) Find the distance he travelled in the first 30 minutes.

(b) Find the retardation during the last 10 minutes.

(c) In the space provided below, sketch the distance-time graph for the whole journey.

(c) [2 marks]

Distance (km)

0900   0930   0945 0955

Time in hours

(a) _____ [1]

(b) _____ [1]

END OF PAPER

12
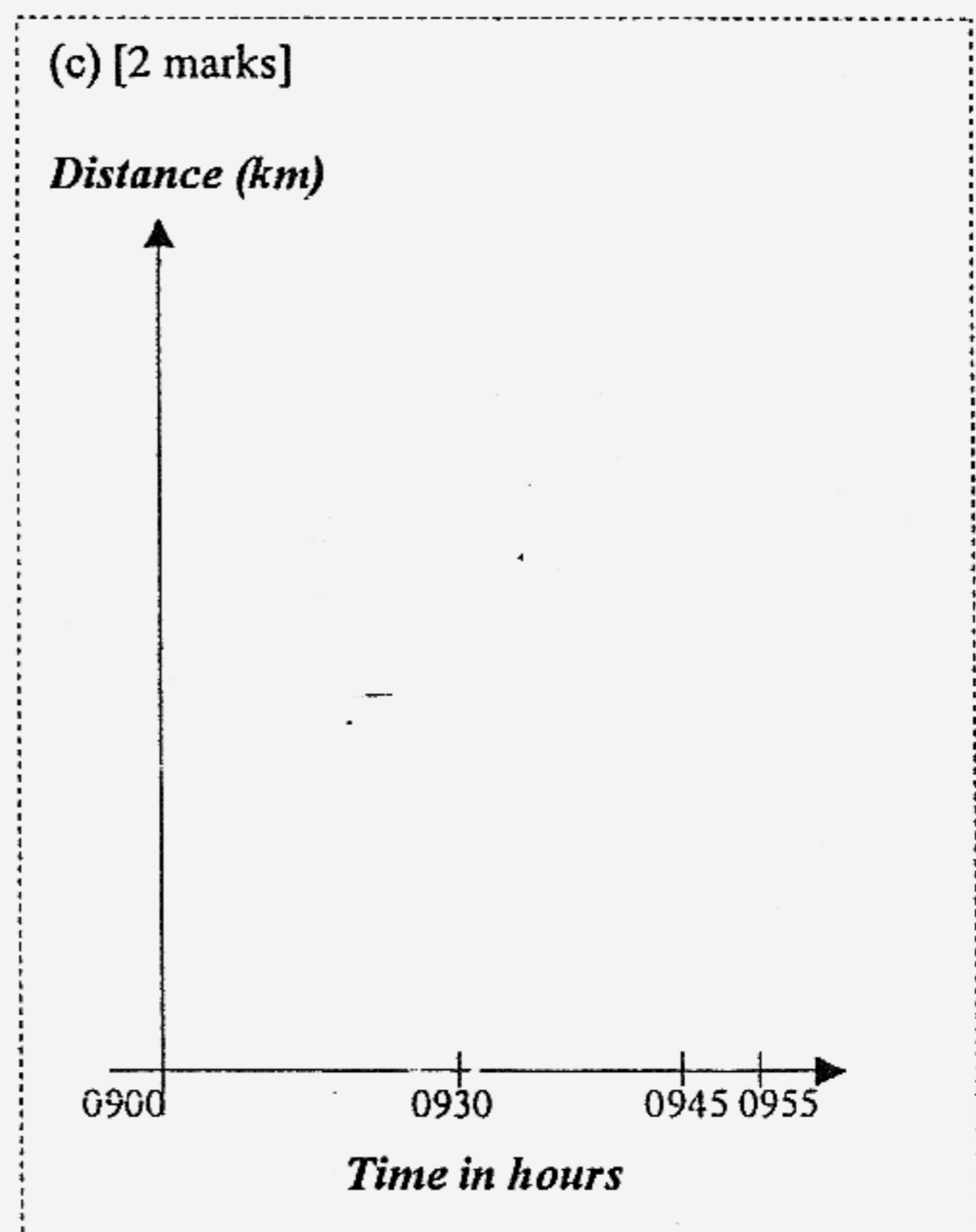
St. Margaret's Secondary School

End of Year Examinations 2006

Secondary Three Express

3 October 2006

Duration: 2 hours 30 minutes

Total Marks: 100

Instructions to candidates:

1. Write your answers and workings on the separate answer paper provided.
2. Omission of essential workings will result in loss of marks.
3. Answer **all** questions in Section A and any **one** in Section B.
4. The number of marks is given in brackets [ ] at the end of each question or part question.
5. You are expected to use an electronic calculator to evaluate explicit numerical expressions.
6. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
7. For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π

This paper consists of <u>9</u> printed pages.

Answer all the questions in this section.

1. Given that $u = \dfrac{2v+6}{v}$.

   a) Find the value of u when $v = 6$. [2]

   b) Find the value of v when $u = 8$. [2]

   c) Express v in terms of u. [1]

2. a) Simplify $(64a^4b)^{\frac{1}{2}} \times (b^3)^{-\frac{3}{2}}$, giving your answer in positive indices only. [2]

   b) Express the following as a single fraction in its lowest term:
   $$\frac{4x}{2x-4} + \frac{3}{2-x}$$
   [2]

   c) Solve: $11^{x^2-4} = 49 \div 7^2$ [2]

3. Sue decided to open a joint bank account with her mother.

   a) Sue deposited a minimum sum of $500 into this new account. The account pays a compound interest at the rate of 2% per annum. Calculate the total amount in their account at the end of 2 years. [3]

   b) If the amount in Sue's bank account falls below $500, she would be fined an amount of $2 per month. Unfortunately, in February 2006, the amount in Sue's joint account fell to $450. Calculate the total amount that was left in her bank account in February 2007 (not including interest). [2]

4. A salesgirl earns a basic salary of $x every month by selling handphones. For every handphone she sells, she earns a commission of $y\%$ of the selling price. [4]

   In the month of January, her total salary was $3580 when her total sales was $8600. In the month of February, her total salary was $4150 when her total sales was $10500.

   Form 2 equations and find her basic salary and commission.

5. A fish can swim at a speed of 5 km/h in still water. The total time taken to swim 2 km upstream and back is 1hour. If the speed of the current is x km/h,

    a) form an expression for the time taken for the fish to swim upstream in terms of x.   [1]

    b) form an expression for the time taken for the fish to swim downstream in terms of x.   [1]

    c) Hence, find the speed of the current.   [2]

6. Jen threw a dice 30 times and recorded her results as follows:

    6   2   2   1   3   1   4   2   1   4

    5   2   5   1   2   4   4   3   5   3

    5   2   5   6   2   1   2   5   1   2

    a) <u>Copy</u> and complete the table below.   [1]

Number shown on dice	1	2	3	4	5	6
Frequency						

    b) For this random distribution, find

        (i)   the mode   [1]

        (ii)   the median   [1]

        (iii)   the percentage of throws in which a 6 appeared   [1]

    c) In a particular quiz, the marks obtained by a class of 39 students are shown in the table below.

| Marks obtained	0	1	2	3	4	5
No. of students	2	5	x	9	y	6

        (i)   If 2 and 3 are the modal scores, state the value of y.   [1]

        (ii)   Find the mean score if $x = 7$ and $y = 10$.   [2]

        (iii)   If the median score is 4, find the possible values of x.   [2]

7. In the diagram, B is the point (0, 6) and C is the point (6, 0). The sloping line through B and the vertical line through C meet at the point A. Given that the gradient of the line AB is $\frac{1}{2}$, calculate

    a) the coordinates of point A   [3]
    b) the area of $\triangle ABC$ [2]
    c) the length of line AB   [2]
    d) the perpendicular distance from C to AB   [2]
    e) the coordinates of another point D if AC is the line of symmetry of the quadrilateral ABCD.   [1]
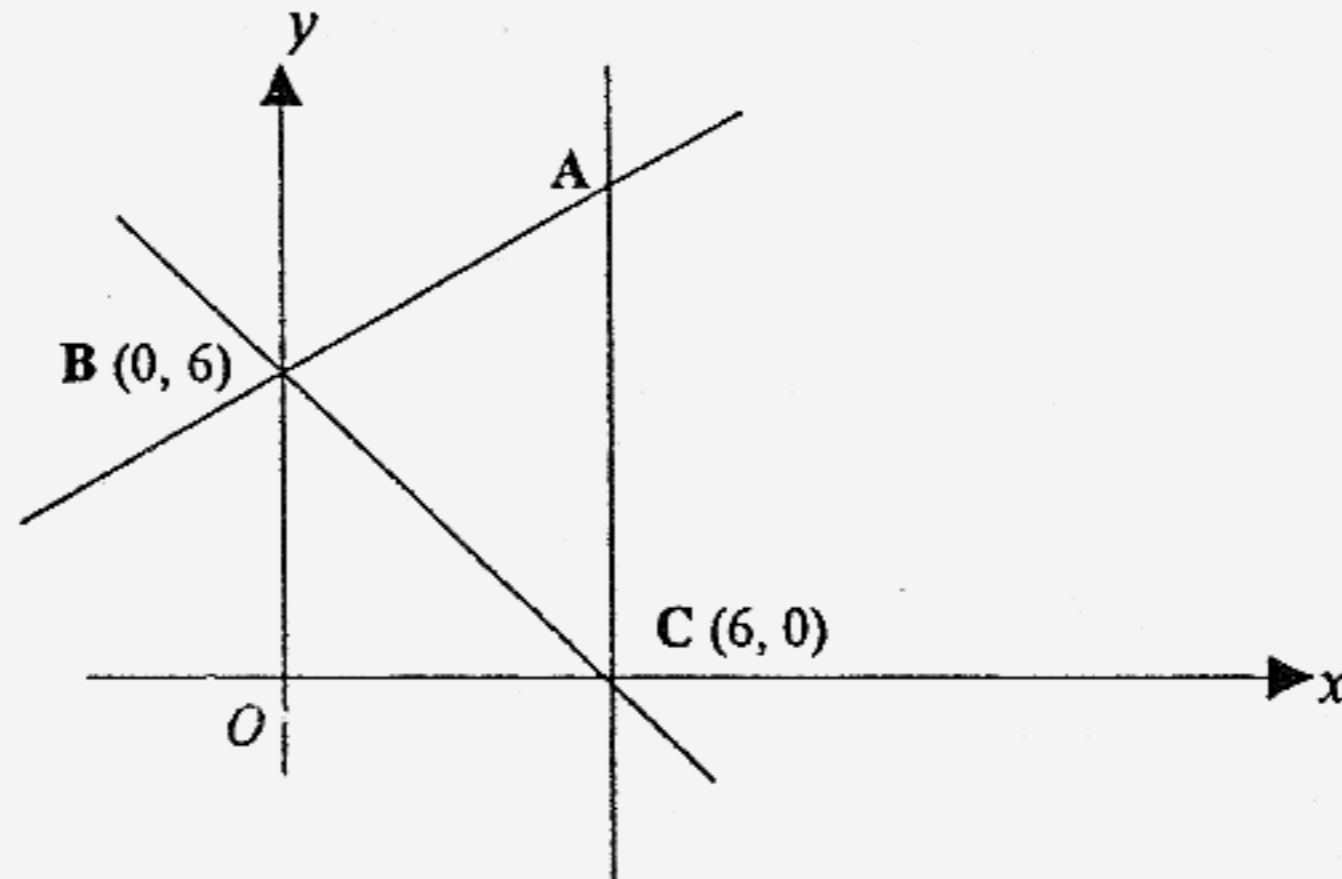
8. The diagram shows the speed-time graph of a toy car and a toy van.
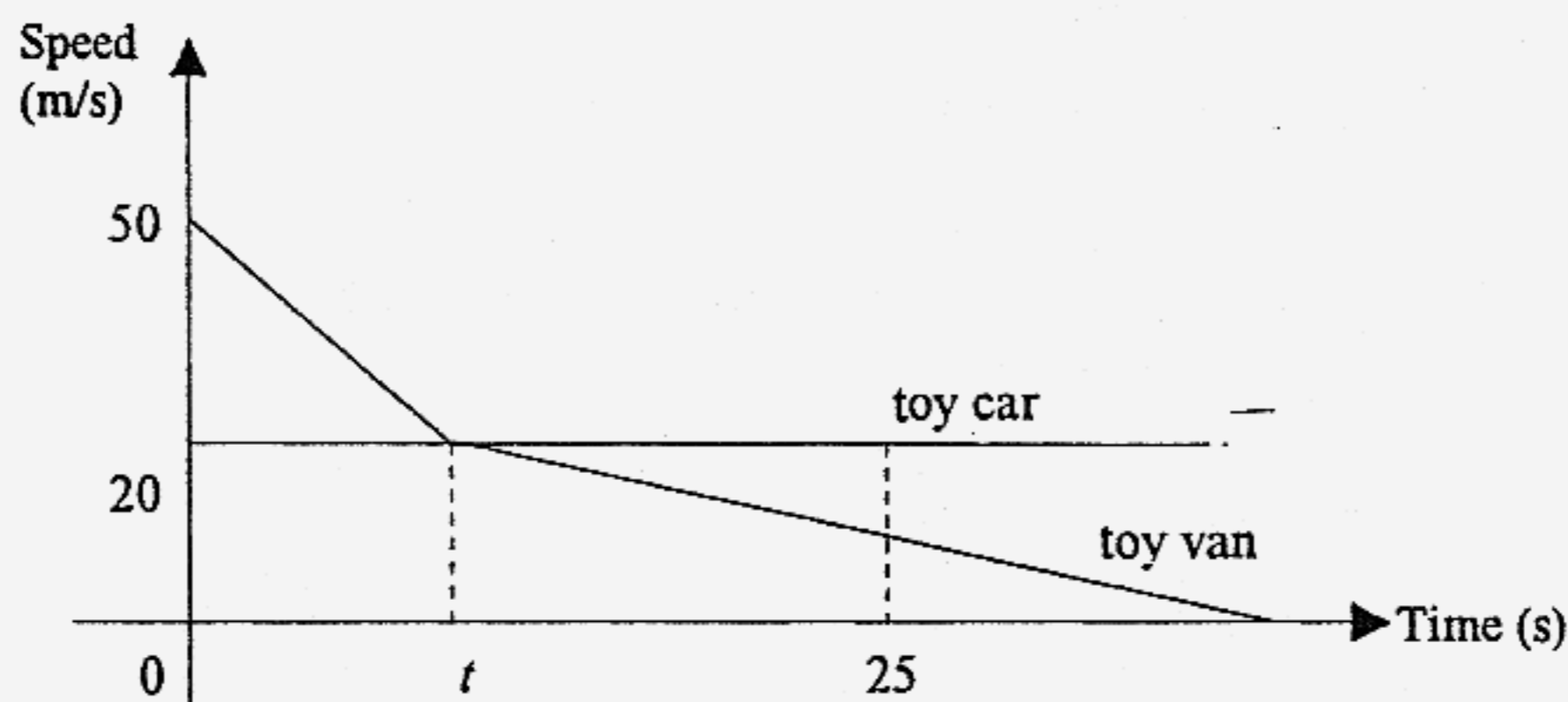
The toy van retards uniformly from 50m/s to 20m/s in t seconds, and continues retarding at a different uniform rate until it finally comes to rest after a further 30 s.

    a) Find the rate of deceleration of the toy van during the last 30 s of its journey.   [1]
    b) Calculate the value of t if the distance covered by the van during the first t seconds is 420m.   [2]
    c) Given that the speed-time graph of the toy car is a horizontal straight line. Find its acceleration from t sec to 25 sec.   [1]

9. The figure below shows the cross-section of a ball bearing stuck in a crack in the ground. O is the centre of the circle and PX and RX are tangents. Given that the radius of the ball bearing is 7cm, PX = 19.2cm and $\angle POX = 70°$.

    a) Write down angle OPX.   [1]

    b) Find the area of $\triangle$OPX.   [1]

   c) Find the area of the shaded region PQRX.   [3]
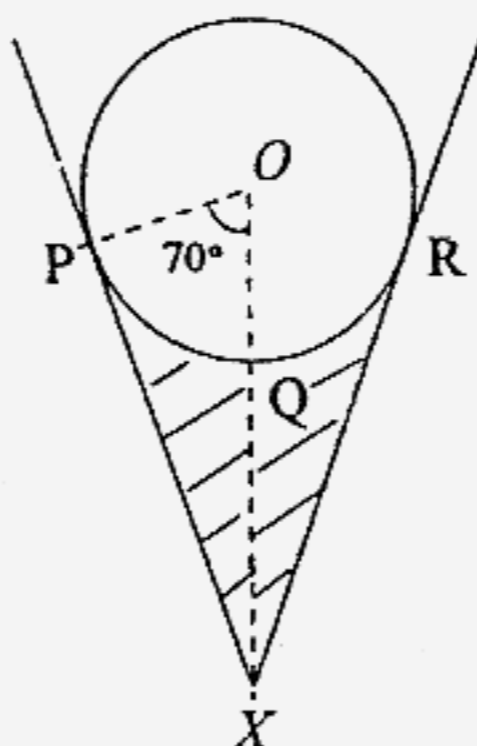
10. This diagram is not drawn to scale.
In this diagram, O is the centre of the circle. Diameter WY bisects chord XZ and ZA = 2BX. WAY, ZAX, and WCP are straight lines and line YP is the tangent drawn at point Y. It is given that $\angle WZX = 70°$ and WA is parallel to CB.
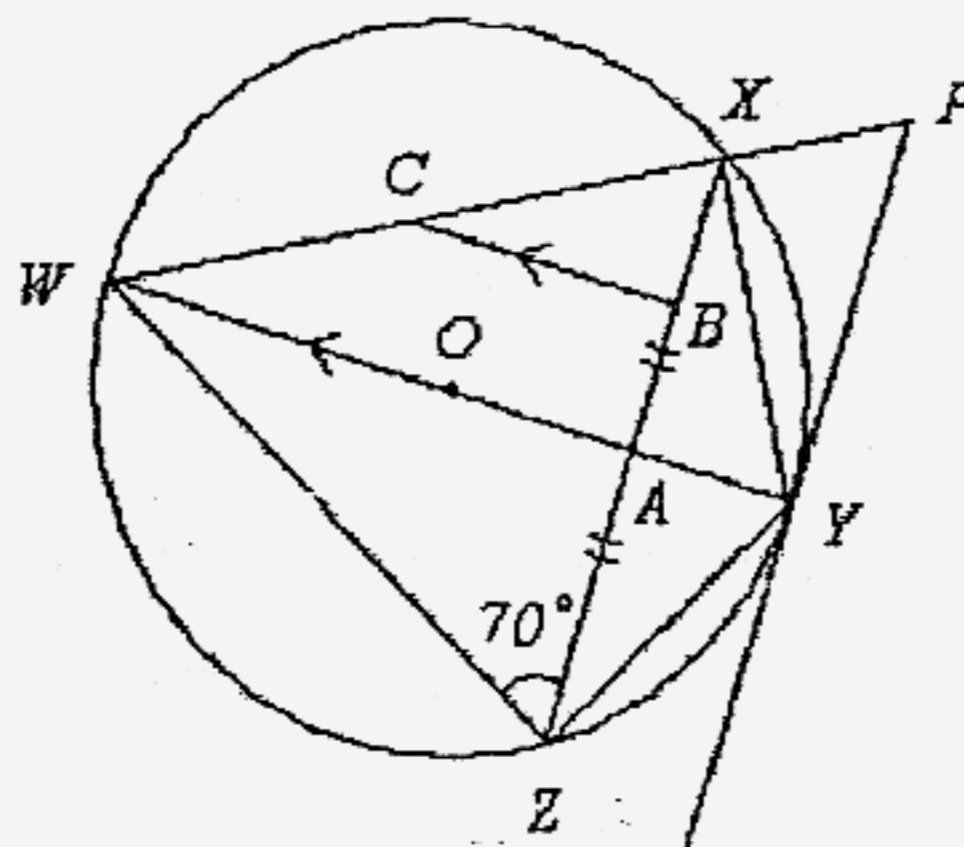
    a) Name a triangle that is similar to $\triangle$WAX.   [1]

    b) (i) Find the ratio of lengths WA and CB.   [1]

         (ii) If the area of $\triangle$WAX is 16cm^2, find the area of $\triangle$CBX.   [2]
             Hence, find the area cf $\triangle$WZX.

    c) Calculate   (i) $\angle$XWY   [2]

                      (ii) $\angle$ZXY   [2]

                      (iii) $\angle$WPY   [2]

    d) Show that $\angle$ZOY = 40°.   [1]

11. This diagram is not drawn to scale.
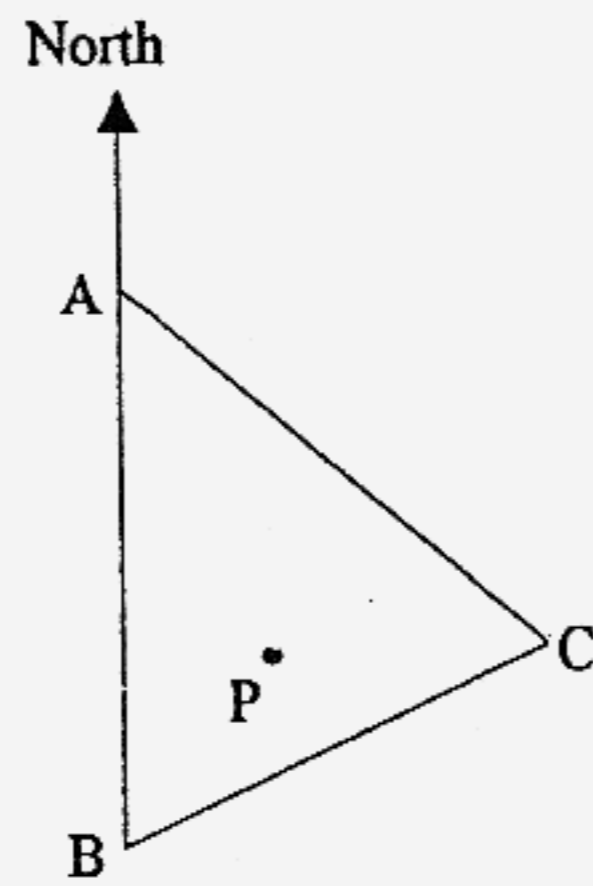
A, B and C are 3 points in the sea. AB = 23.7 m, AC = 22.1 m and the bearing of C from A is 124°.

Calculate

a) how far B is away from C   [2]

b) the area of ΔABC   [2]

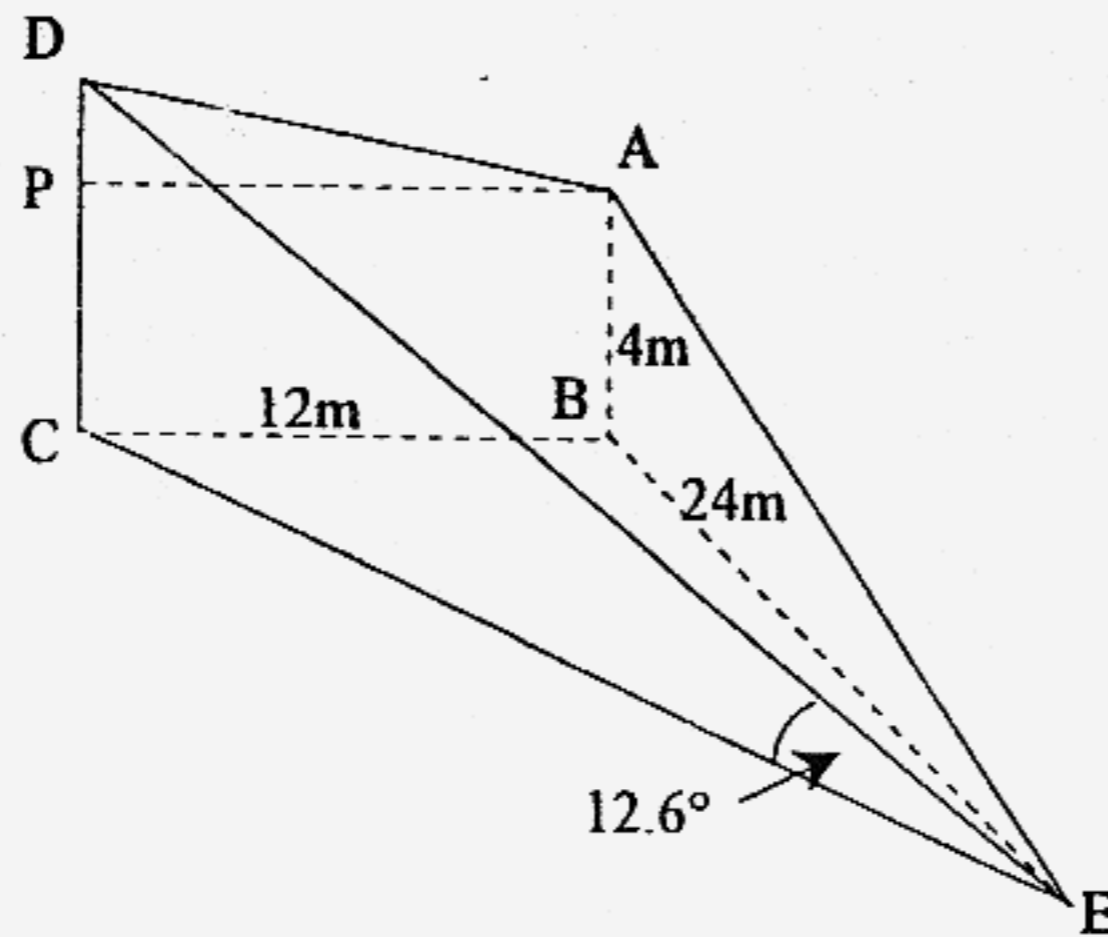c) the bearing of C from B   [2]

giving your answers correct to 1 decimal place.

A whale at point B swims towards point P at a speed of 6 m/s. The bearing of P from B is 036° and line AP bisects ∠BAC.

Calculate

d) the distance that the whale at position B has to travel to reach P   [2]

e) the time taken for whale at position B to reach P   [1]

12.

ABCD is the wall of the garage which stands vertically to the ground. [4]
AD is the roof which makes an angle with the horizontal AP.
C, B and E are points on the ground where E is due south of B and C is
due west of B. Given that AB = 4 m, BC = 12 m and BE = 24 m. The
angle of elevation of D when viewed from E is 12.6°. Calculate the angle
of elevation of D from A.

13. **Answer the whole of this question on a sheet of graph paper.**

a) Given that $y = \dfrac{4x^2 + 5}{2} - \dfrac{7}{3}x$, find the value of b. [1]

x	-1.0	-0.5	0	0.5	1.0	1.5	2.0
y	6.83	4.17	2.50	1.83	2.17	3.50	b

b) Using a scale of 4cm to 1 unit on the x-axis and 2cm to 1 unit on [3]
the y-axis, draw the graph of the equation $y = \dfrac{4x^2 + 5}{2} - \dfrac{7}{3}x$ for
$-1 \le x \le 2$.

c) Use your graph to estimate [1]

(i)   the value of x when $y = 3$ [1]
(ii)   the least value of y

d) By drawing a suitable straight line, obtain the approximate [2]
solutions of the equation $\dfrac{4x^2 + 5}{2} - 2x = \dfrac{7}{3}x + 1$.

e) Use your graph to solve the inequality $\dfrac{4x^2 + 5}{2} - 2x \le \dfrac{7}{3}x + 1$. [2]

f) By drawing a tangent, find the gradient of the curve at $x = -0.3$. [2]

14.   A and B are 2 points on the ground, 20 m apart. A Ferris wheel is moving in a vertical circle at a uniform speed when in operation about a point C, which is 12 m vertically above A. P is a point on the Ferris wheel where CP // AB. The angle of elevation from B to P is 47.5° as shown in the figure below.
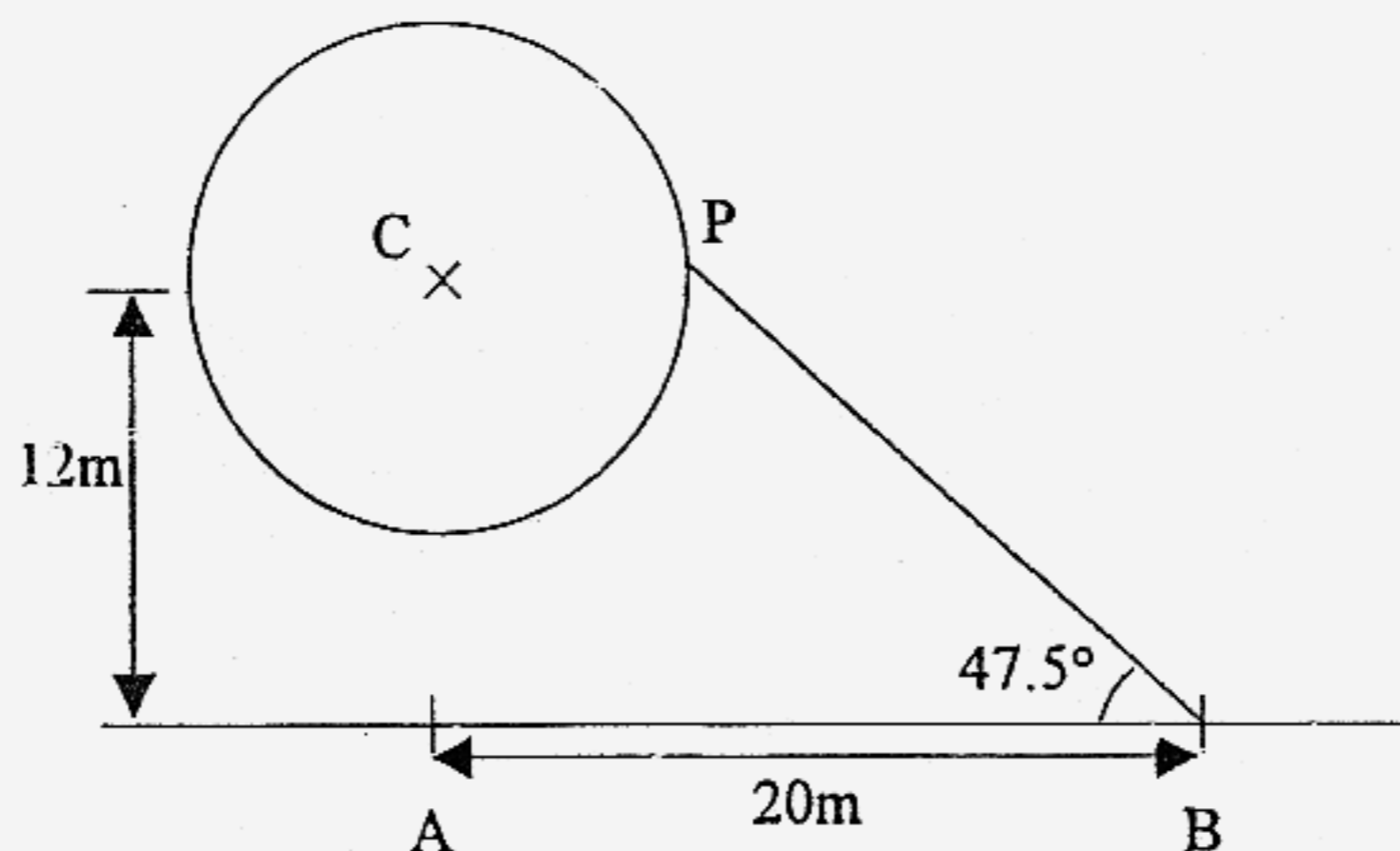
If the radius of the wheel is always shorter than the distance AB. Calculate

    a) the radius of the circle   [3]

    b) the angle of elevation from B to the lowest point of the wheel   [3]
       north from A.

    c) (i) Find the length of BC   [2]

        (ii) Find the value of ∠CBA   [1]

        (iii) Hence, find the smallest angle of elevation from B to the   [3]
            wheel.

15. ABCD and PQRC are 2 identical squares each of side 8 cm. Given that
     $\angle BCP = 40°$ and PX=BX,
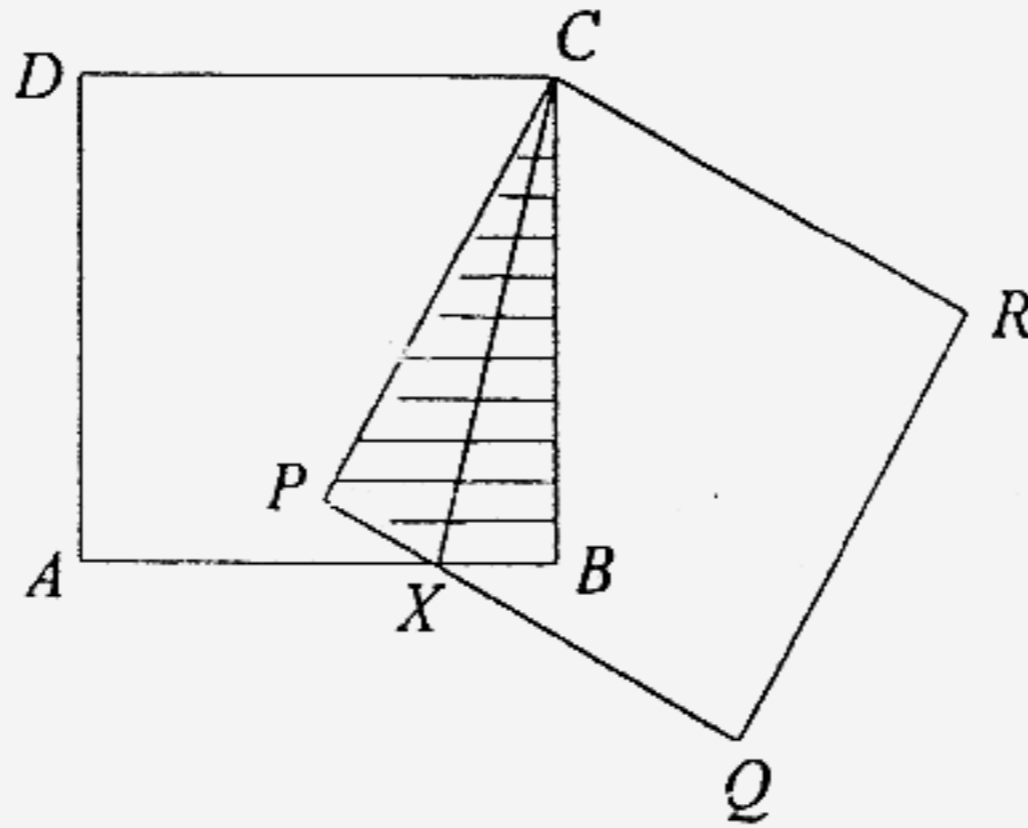
a) Prove that CX is an angle bisector of $\angle BCP$.   [4]

b) Find the length of PX.   [2]

c) Find the area of the shaded region.   [2]

d) What percentage of the whole figure AXQRCD is shaded?   [4]

End of Paper

1(a)   72.5%

(b) $\dfrac{4}{5}$

2 (a) $6\dfrac{17}{21}$

(b) $1\dfrac{3}{5}$

3.   20

4(a) $\dfrac{19}{26}$

(b)   0.731

5(a) $x = -1$ or $x = -2$

(b) $a = 22$

6 (a) $7\dfrac{1}{4}$

(b) $\dfrac{1}{9}$

7(a) $48400 = 2^4 \times 5^2 \times 11^2$

(b) 220

8(a) 0.00027

(b) 4.2×10

9(a)   3%

(b) $500 000

10(a) 10

(b) -25

11) $\dfrac{a-10}{(a+4)(a-4)}$ or $\dfrac{10-a}{(a+4)(4-a)}$

12) $x = -20\dfrac{1}{2}, \quad y = 17$

13) $2mn = 4, (m\text{-}n)^2 = 8$

14(a) 7.15×10^8

(b) 6.3×10^{21}

15(a) $1\dfrac{1}{2}$

(b) $\dfrac{3-F^2}{F^2+E}$

16 (a) 10 and 13

(b) $3n-5$

(c) 88

17 (a) $y = \dfrac{3}{(x+1)^2}$

(b) $\dfrac{3}{16}$

18) $x \geq 6$

19) (a) 12 cm

(b) $6\pi + 20$

(c) $36\pi - 30$

(d) $\dfrac{12}{13}$

20 (a) $-1\dfrac{1}{2}$

(b) $y = -\dfrac{3}{2}x + \dfrac{11}{2}$

(c) 5

(d) $-\dfrac{3}{5}$

(e) 6 units^2

21 (a) 220

(b) 280

(c) 475 km

22 (a) 6 km

(b) 0.04 km/min

Answers

1a) $u = 3$ b) $v = 1$ c) $\dfrac{6}{u-2}$

2a) $\dfrac{8a^2}{t^4}$ b) $\dfrac{x-3}{x-2}$ c) $x = 2$ or -2

3a) \$520.20   b) \$426

4)   Basic salary (x) = \$1000;
       Commission $(y\%)$ = 30%

5a) Time taken to swim upstream = $\dfrac{2}{5-x}$

b) Time taken to swim downstream = $\dfrac{2}{5+x}$

c) $\dfrac{2}{5-x} + \dfrac{2}{5+x} = 1$ (form equation)
   Ans: $x = 2.24$ km/h (3 s.f.)

6a)

No. on dice	1	2	3	4	5	6
Frequency	6	9	3	4	6	2

bi) Mode = 2

bii) (Middle no. = 15, 16)
     Median = 2.5

biii) $6\dfrac{2}{3}\%$ or 6.67%

ci) 8   (cii) 2.97

ciii) Possible values of x = 0, 1, 2, 3

7a) A = (6, 9)   b) 27 units^2
  c) 6.71 units   d) 8.05 units
  e) D = (12, 6)

8a) $\dfrac{2}{3}$ m/s^2 (or 0.667 m/s^2)   b) 12 s
  c) 0 m/s^2

9a) $O\hat{P}X = 90°$ b) 67.2 cm^2 c) 74.6 cm^2

10a) $\triangle$WAX is similar to $\triangle$CBX (or $\triangle$ZAY)
   bi) 2:1 (or $\dfrac{2}{1} = 2$)
   bii) Area of $\triangle$CBX = 4 cm^2;
        Area of $\triangle$WZX = 32 cm^2

ci) 20°   cii) 20°   ciii) 70°

d) To show that $Z\hat{O}Y = 40°$;

Since $Z\hat{X}Y = 20°$

And $Z\hat{O}Y = 2\ Z\hat{X}Y$ (angle at centre is twice

angle at circumference)

Hence,

$$Z\hat{O}Y = 2 \times 20° = 40° \text{ (shown)}$$

11a) 21.5 m   b) 217.1 m²   c) 058.4°

    d) 12.4 m   e) 2.07 s

12) 9.5°

13a) $b = 5.83 \approx 5.8$

b) correct scale used; smooth curve; accurate points

ci) $x = -0.2$ and $1.35\ [\pm 0.1]$

cii) Least value of $y = 1.8$

d) Sketch $y = 2x + 1$

$x = 0.43, y = 1.85$ and

$x = 1.73, y = 4.45$

e) $0.43 \leq x \leq 1.73$

f) Draw tangent at $x = -0.3$

Gradient $= -3.53\ [\pm 0.2]$

14a) 9.00cm   b) 8.5°

ci) 23.3 m   cii) 31.0°   ciii) 8.3°

15a)   CP = CB (sides of square)

           PX = BX (given)

           $\angle CPX = \angle CBX = 90°$

           $\therefore \triangle CPX \equiv \triangle CBX$ (RHS)

Since the $\triangle$s are $\equiv$, then $\angle PCX = \angle BCX$,

Hence, CX is an $\angle$ bisector of $\angle BCP$.

b) 2.91cm   c) 23.3cm²   d) 22.2%