# Mathematics I, II and III (9465, 9470, and 9475)

## General Introduction

There are two syllabuses, one for Mathematics I and Mathematics II, the other for Mathematics III. The syllabus for Mathematics I and Mathematics II is based on a single subject Mathematics Advanced GCE. Questions on Mathematics II are intended to be more challenging than questions on Mathematics I. The syllabus for Mathematics III is wider.

In designing the syllabuses, the specifications for all the UK Advanced GCE examinations, the Scottish Advanced Higher and the International Baccalauriat were consulted.

Each of Mathematics I, II and III will be a 3-hour paper divided into three sections as follows:

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| Section A (Pure Mathematics) | eight questions |
| Section B (Mechanics) | three questions |
| Section C (Probability and Statistics) | two questions |

All questions will carry the same weight. Candidates will be assessed on the six questions best answered; no restriction will be placed on the number of questions that may be attempted from any section. Normally, a candidate who answers at least four questions well will be awarded a grade 1.

The marking scheme for each question will be designed to reward candidates who make good progress towards a complete solution.

## Syllabuses

The syllabuses given below are for the guidance of both examiners and candidates. The following points should be noted.

1. Questions may test candidates' ability to apply mathematical knowledge in novel and unfamiliar ways.
2. Solutions will frequently require insight, ingenuity, persistence and the ability to work through substantial sequences of algebraic manipulation.
3. Some questions may be largely independent of particular syllabus topics, being designed to test general mathematical potential.
4. Questions will often require knowledge of several different syllabus topics.
5. Examiners will aim to set questions on a wide range of topics, but it is not guaranteed that every topic will be examined every year.
6. Questions may be set that require knowledge of elementary topics, such as basic geometry, mensuration (areas of triangles, volume of sphere, etc), sine and cosine formulae for triangles, rational and irrational numbers.

## Formula booklets and calculators

Candidates should use the STEP formula booklet which will be posted out with the question papers and can be downloaded from the website (http://www.stepmathematics.org.uk).

Calculators are not permitted (or required).

## MATHEMATICS I (9465) and MATHEMATICS II (9470)

## Section A: Pure Mathematics

*This section comprises the Advanced GCE common core (typically, modules C1 — C4 of an Advanced GCE specification) broadly interpreted, together with a few additional items ∗enclosed in asterisks∗.*

| Specification | Notes |
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| **General** | |
| Mathematical vocabulary and notation | including: equivalent to; necessary and sufficient; if and only if; $\Rightarrow$ ; $\Leftrightarrow$ ; $\equiv$ . |
| Methods of proof | including proof by contradiction and disproof by counterexample;<br>∗including, in simple cases, proof by induction∗. |
| **Algebra** | |
| Indices and surds | including rationalising denominators. |
| Quadratics | including proving positivity by completing a square. |
| The expansion for $(a+b)^n$ | including knowledge of the general term;<br>notation: $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$ . |
| Algebraic operations on polynomials and rational functions. | including factorisation, the factor theorem, the remainder theorem;<br>including understanding that, for example, if<br>$$x^3 + bx^2 + cx + d \equiv (x-\alpha)(x-\beta)(x-\gamma),$$<br>then $d = -\alpha\beta\gamma$. |
| Partial fractions | including denominators with a repeated or quadratic factor. |
| Sequences and series | including use of, for example, $a_{n+1} = \mathrm{f}(a_n)$ or $a_{n+1} = \mathrm{f}(a_n, a_{n-1})$;<br>including understanding of the terms convergent, divergent and periodic in simple cases;<br>including use of $\sum_{k=1}^{n} k$ to obtain related sums. |
| The binomial series for $(1+x)^k$, where $k$ is a rational number. | including understanding of the condition $\lvert x\rvert < 1$ . |
| Arithmetic and geometric series | including sums to infinity and conditions for convergence, where appropriate. |
| ∗Inequalities∗ | including solution of, eg, $\frac{1}{a-x} > \frac{x}{x-b}$ ;<br>including simple inequalities involving the modulus function;<br>including the solution of simultaneous inequalities by graphical means. |

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| **Functions** | |
| Domain, range, composition, inverse | including use of functional notation such as $y = \mathrm{f}(ax+b)$ , $x = \mathrm{f}^{-1}(y)$ and $z = \mathrm{f}\big(\mathrm{g}(x)\big)$. |
| Increasing and decreasing functions | both the common usages of the term 'increasing' (i.e. $x > y \Rightarrow \mathrm{f}(x) > \mathrm{f}(y)$ or $x > y \Rightarrow \mathrm{f}(x) \geqslant \mathrm{f}(y)$ ) will be acceptable. |
| Exponentials and logarithms | including $x = a^y \Leftrightarrow y = \log_a x$ , $x = \mathrm{e}^y \Leftrightarrow y = \ln x$ ; ∗including the exponential series $$\mathrm{e}^x = 1 + x + \cdots + x^n/n! + \cdots \ . \qquad *$$ |
| The effect of simple transformations | such as $y = a\mathrm{f}(bx+c)+d$. |
| The modulus function. | |
| Location of roots of $\mathrm{f}(x) = 0$ by considering changes of sign of $\mathrm{f}(x)$ . | |
| Approximate solution of equations using simple iterative methods. | |
| **Curve sketching** | |
| General curve sketching | including use of symmetry, tranformations, behaviour as $x \to \pm\infty$, points or regions where the function is undefined, turning points, asymptotes parallel to the axes. |
| **Trigonometry** | |
| Radian measure, arc length of a circle, area of a segment. | |
| Trigonometric functions | including knowledge of standard values, such as $\tan(\pi/4)$ , $\sin 30^\circ$ ; including identities such as $\sec^2\phi - \tan^2\phi = 1$ ; including application to geometric problems in two and three dimensions. |
| Double angle formulae | including their use in calculating, eg, $\tan(\pi/8)$ . |
| Formulae for $\sin(A \pm B)$ and $\cos(A \pm B)$ | including their use in solving equations such as $$a\cos\theta + b\sin\theta = c.$$ |
| Inverse trigonometric functions | definitions including domains and ranges; notation: $\arctan\theta$ , etc |
| **Coordinate geometry** | |
| Straight lines in two-dimensions | including the equation of a line through two given points, or through a given point and parallel to a given line or through a given point and perpendicular to a given line; including finding a point which divides a segment in a given ratio. |
| Circles | using the general form $(x-a)^2 + (y-b)^2 = R^2$ ; including points of intersection of circles and lines. |
| Cartesian and parametric equations of curves and conversion between the two forms. | |

**Calculus**

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| Interpretation of a derivative as a limit and as a rate of change | including knowledge of both notations $\mathrm{f}'(x)$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$ |
| Differentiation of standard functions | including algebraic expressions, trigonometric (but not inverse trigonometric) functions, exponential and log functions. |
| Differentiation of composite functions, products and quotients and functions defined implicitly. | |
| Higher derivatives | including knowledge of both notations $\mathrm{f}''(x)$ and $\frac{d^2y}{\mathrm{d}x^2}$ ; including knowledge of the notation $\frac{d^ny}{\mathrm{d}x^n}$ . |
| Applications of differentiation to gradients, tangents and normals, stationary points, increasing and decreasing functions | including finding maxima and minima which are not stationary points; including classification of stationary points using the second derivative. |
| Integration as reverse of differentiation | |
| Integral as area under a curve | including area between two graphs; including approximation of integral by the rectangle and trapezium rules. |
| Volume within a surface of revolution | |
| Knowledge and use of standard integrals | including the forms $\int \mathrm{f}'(\mathrm{g}(x))\mathrm{g}'(x)\mathrm{d}x$ and $\int \mathrm{f}'(x)/\mathrm{f}(x)\mathrm{d}x$ ; including transformation of an integrand into standard (or some given) form; including use of partial fractions; not including knowledge of integrals involving inverse trigonometric functions. |
| Definite integrals | $*$including calculation, without justification, of simple improper integrals such as $\int_0^\infty \mathrm{e}^{-x}\mathrm{d}x$ and $\int_0^1 x^{-\frac{1}{2}}\mathrm{d}x$ (if required, information such as the behaviour of $x\mathrm{e}^{-x}$ as $x \to \infty$ or of $x\ln x$ as $x \to 0$ will be given).$*$ |
| Integration by parts and by substitution | including understanding their relationship with differentiation of product and of a composite function; including application to $\int \ln x\mathrm{d}x$ . |
| Formulation and solution of differential equations | formulation of first order equations; solution in the case of a separable equation or by some other method given in the question. |

**Vectors**

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| Vectors in two and three dimensions | including use of column vector and $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ notation. |
| Magnitude of a vector | including the idea of a unit vector. |
| Vector addition and multiplication by scalars | including geometrical interpretations. |
| Position vectors | including application to geometrical problems. |
| The distance between two points. | |
| Vector equations of lines | including the finding the intersection of two lines; understanding the notion of skew lines (knowledge of shortest distance between skew lines is not required). |
| The scalar product | including its use for calculating the angle between two vectors. |

## Section B: Mechanics

*This section is roughly equivalent to two GCE modules M1 and M2; candidates who have only studied one module of mechanics are likely to lack both the knowledge and the experience of mechanics required to attempt the questions. Questions may involve any of the material in the Pure Mathematics syllabus.*

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| Force as a vector | including resultant of several forces acting at a point and the triangle or polygon of forces;<br>including equilibrium of a particle;<br>forces include weight, reaction, tension and friction. |
| Centre of mass | including obtaining the centre of mass of a system of particles, of a simple uniform rigid body (possible composite) and, in simple cases, of non-uniform body by integration. |
| Equilibrium of a rigid body or several rigid bodies in contact | including use of moment of a force;<br>for example, a ladder leaning against a wall or on a movable cylinder;<br>including investigation of whether equilibrium is broken by sliding, toppling or rolling;<br>including use of Newton's third law;<br>excluding questions involving frameworks. |
| Kinematics of a particle in a plane | including the case when velocity or acceleration depends on time (but excluding use of acceleration $= v\frac{\mathrm{d}v}{\mathrm{d}x}$) ;<br>questions may involve the distance between two moving particles, but detailed knowledge of relative velocity is not required. |
| Energy (kinetic and potential), work and power | including application of the principle of conservation of energy. |
| Collisions of particles | including conservation of momentum, conservation of energy (when appropriate);<br>coefficient of restitution, Newton's experimental law;<br>including simple cases of oblique impact (on a plane, for example);<br>including knowledge of the terms *perfectly elastic* $(e = 1)$ and *inelastic* $(e = 0)$;<br>questions involving successive impacts may be set. |
| Newton's first and second laws of motion | including motion of a particle in two and three dimensions and motion of connected particles, such as trains, or particles connected by means of pulleys. |
| Motion of a projectile under gravity | including manipulation of the equation<br>$$y = x\tan\alpha - \frac{gx^2}{2V^2\cos^2\alpha},$$<br>viewed, possibly, as a quadratic in $\tan\alpha$ ;<br>not including projectiles on inclined planes. |

## Section C: Probability and Statistics

*The emphasis, in comparison with Advanced GCE and other comparable examinations, is towards probability and formal proofs, and away from data analysis and use of standard statistical tests. Questions may involve use of any of the material in the Pure Mathematics syllabus.*

**Probability**

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| Permutations, combinations and arrangements | including sampling with and without replacement. |
| Exclusive and complementary events | including understanding of $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ , though not necessarily in this form. |
| Conditional probability | informal applications, such as tree diagrams. |

**Distributions**

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| Discrete and continuous probability distribution functions and cumulative distribution functions. | including calculation of mean, variance, median, mode and expectations by explicit summation or integration for a given (possibly unfamiliar) distribution (eg exponential or geometric or something similarly straightforward); notation: $\mathrm{f}(x) = \mathrm{F}'(x)$. |
| Binomial distribution | including explicit calculation of mean. |
| Poisson distribution | including explicit calculation of mean; including use as approximation to binomial distribution where appropriate. |
| Normal distribution | including conversion to the standard normal distribution by translation and scaling; including use as approximation to the binomial or Poisson distributions where appropriate; notation: $X \sim \mathrm{N}(\mu, \sigma^2)$ . |

**Hypothesis testing**

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| Basic concepts in the case of a simple null hypothesis and simple or compound alternative | including knowledge of the terminology *null hypothesis* and *alternative hypothesis*, *one* and *two tailed tests.* |

## MATHEMATICS III (9475)

*In each section knowledge of the corresponding parts of the syllabus for Mathematics I and II is assumed, and may form the basis of some questions.*

## Section A: Pure Mathematics

### Algebra

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| Summation of series | including use of method of differences, recognition of Maclaurin's series (including binomial series) and partial fractions;<br>including investigation of convergence, by consideration of the sum to $n$ terms. |
| Polynomial equations | including complex solutions (and the fact that they occur in conjugate pairs if the cooefficients are real);<br>including the relationship between symmetric functions of the roots and the coefficients;<br>including formation of a new equation whose roots are related to the roots of the old equation. |
| Induction | including application complicated situations such as divisibility problems. |

### Trigonometry

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| Formula for $\sin A \sin B$ (etc) and $\sin A + \sin B$ (etc) | |

### Coordinate geometry

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| Ellipse, parabola, hyperbola and rectangular hyperbola | including standard Cartesian forms and parametric forms;<br>including geometrical understanding, but not detailed knowledge of, the terms *focus*, *directrix*, *eccentricity*. |
| Polar coordinates | including relation with Cartesian coordinates;<br>including ability to sketch simple graphs;<br>including integral formula for area. |

### Complex numbers

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| Algebra of complex numbers | including Cartesian and polar forms. |
| Argand diagram | including geometrical representation of sums and products;<br>including identification of loci and regions (eg $\|z-a\| = k\|z-b\|$). |
| Euler's identity: $\mathrm{e}^{\mathrm{i}\theta} \equiv \cos\theta + \mathrm{i}\sin\theta$ | including the relationship between trigonometrical and exponential functions;<br>including application to summation of trigonometric series. |
| de Moivre's theorem | including application to finding $n$th roots. |

**Hyperbolic functions**

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| Definitions and properties of the six hyperbolic functions | including integrals and derivatives of $\cosh$, $\sinh$, $\tanh$ and $\coth$. |
| Formulae corresponding to trigonometric formulae | such as $\cosh^2 x - \sinh^2 x = 1$ and $\sinh(x+y) = \cdots$. |
| Inverse functions | including ability to derive logarithmic forms and graphs; including derivatives. |

**Calculus**

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| Integrals resulting in inverse trig. or hyperbolic functions | including integrands such as $(x^2 - ax)^{-\frac{1}{2}}$. |
| Maclaurin's series | including knowledge of expansions for $\sin x$, $\cos x$, $\cosh x$, $\sinh x$ and $\ln(1+x)$. |
| Arc length of a curve expressed in Cartesian coordinates | including curves described parametrically. |
| First order linear differential equations | including the general form $$\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{P}(x)y = \mathrm{Q}(x).$$ |
| Second order linear differential equations with constant coefficients | including the inhomogeneous case $$a\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + b\frac{\mathrm{d}y}{\mathrm{d}x} + cy = \mathrm{f}(x),$$ where $\mathrm{f}(x)$ is an exponential function, a sin/cos function or a polynomial (but excluding resonant cases);<br>including equations that can be transformed into the above form;<br>including initial or boundary conditions. |

## Section B: Mechanics

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| Motion of a particle subject to a variable force | including use of $v\frac{\mathrm{d}v}{\mathrm{d}x}$ for acceleration. |
| Elastic strings and springs | including Hooke's law and the potential energy;<br>notation: $T = kx = \lambda x/l$, where $k$ is stiffness, $\lambda$ is modulus of elasticity. |
| Motion in a circle | including motion with non-uniform speed. |
| Simple harmonic motion | including obtaining from a given model an equation of the form $\ddot{x} + \omega^2 x = a$, where $a$ is a constant, and quoting the solution. |
| Relative motion | |
| Moment of inertia | including calculations involving integration (such as a non-uniform rod, or a lamina);<br>including parallel and perpendicular axis theorems. |
| Motion of a rigid body about a fixed axis | including understanding of angular momentum and rotational kinetic energy and their conservation;<br>including the understanding of couple and the work done by a couple;<br>including calculation of forces on the axis. |

## Section C: Probability and Statistics

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| Algebra of expectations | for example, $\mathrm{E}(aX + bY + c) = a\mathrm{E}(X) + b\mathrm{E}(Y) + c$,<br>$\mathrm{Var}\,(X) = \mathrm{E}(X^2) - \mathrm{E}(X)^2$,<br>$\mathrm{Var}\,(aX + b) = a^2\,\mathrm{Var}\,(X)$. |
| Further theory of distribution functions | including, for example, use of the cumulative distribution function to calculate the distribution of a related random variable (eg $X^2$). |
| Generating functions | including the use of generating functions to obtain the distribution of the sum of independent random variables. |
| Discrete bivariate distributions | including understanding of the idea of independence;<br>including marginal and conditional distributions;<br>including use of $\mathrm{Var}\,(aX + bY) = a^2\,\mathrm{Var}\,(X) + b^2\,\mathrm{Var}\,(Y) + 2ab\mathrm{Cov}(X, Y)$. |
| Sampling | including sample mean and variance;<br>including use of the central limit theorem. |