# EXAMINATIONS OF THE ROYAL STATISTICAL SOCIETY 



## HIGHER CERTIFICATE IN STATISTICS, 2007

(Modular format)

MODULE 7 : Time series and index numbers

## Time allowed: One and a half hours

Candidates should answer THREE questions.
Each question carries 20 marks.
The number of marks allotted for each part-question is shown in brackets.

Graph paper and Official tables are provided.

Candidates may use calculators in accordance with the regulations published in the Society's "Guide to Examinations" (document Ex1).

The notation log denotes logarithm to base $\boldsymbol{e}$.
Logarithms to any other base are explicitly identified, e.g. $\log _{10}$.

$$
\text { Note also that }\binom{n}{r} \text { is the same as }{ }^{n} C_{r} \text {. }
$$

1. (i) Explain how seasonally adjusted and trend estimates are derived from a time series, and what each estimate represents.
(ii) State the key issues involved in seasonal adjustment and discuss any advantages and disadvantages of the seasonal adjustment process.
(iii) Discuss the key issues involved in calculating trend estimates at the end of a series, assuming a filter-based approach to trend estimation.
(iv) Discuss the advantages and disadvantages of using modelling techniques within the seasonal adjustment process, paying particular attention to the impact of modelling on time series estimates at the current end of series. Does modelling necessarily improve the quality of the seasonal adjustment?
2. Referring to the edited output of a filter based approach to seasonal adjustment shown below, answer the following questions.

| Date | Original | S $\times \mathrm{I}$ | Seasonal <br> factor (S) | Seas <br> adj | Trend | Irreg (I) |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| DEC04 | 184.88 | 1.4507 | 1.4486 | 127.62 | 127.36 | 1.0020 |
| JAN05 | 107.29 | 0.8439 | 0.8568 | 125.21 | 128.98 | 0.9708 |
| FEB05 | 111.81 | 0.8789 | 0.8873 | 126.01 | 126.99 | 0.9923 |
| MAR05 | 120.79 | 0.9458 | 0.9283 | 130.13 | 127.49 | 1.0207 |
| APR05 | 122.63 | 0.9541 | 0.9589 | 127.89 | 128.38 | 0.9962 |
| MAY05 | 124.90 | 0.9645 | 0.9621 | 129.82 | 129.48 | 1.0026 |
| JUN05 | 128.93 | 0.9879 | 0.9828 | 131.18 | 130.63 | 1.0042 |
| JUL05 | 130.33 | 0.9917 | 0.9896 | 131.70 | 131.66 | 1.0003 |
| AUG05 | 128.60 | 0.9731 | 0.9499 | 135.39 | 132.47 | 1.0220 |
| SEP05 | 124.89 | 0.9411 | 0.9429 | 132.45 | 133.01 | 0.9958 |
| OCT05 | 131.15 | 0.9857 | 0.9756 | 134.43 | 133.27 | 1.0087 |
| NOV05 | 149.24 | 1.1199 | 1.1219 | 133.02 | 133.32 | 0.9977 |
| DEC05 | 195.77 | 1.4754 | 1.4521 | 134.82 | 132.59 | 1.0168 |
| JAN06 | 113.56 | 0.8512 | 0.8548 | 132.86 | 133.15 | 0.9978 |
| FEB06 | 116.90 | 0.8761 | 0.8826 | 132.44 | 133.06 | 0.9953 |

(i) What decomposition model (additive or multiplicative) was used for this seasonal adjustment? Give a reason for your answer.
(ii) Illustrate arithmetically that the decomposition chosen in part (i) holds, to one decimal place, by using data from November 2005 (NOV05). Discuss and give examples why the decomposition may not hold precisely in all periods.
(iii) Calculate the percentage movement between DEC04 and JAN05 for the original values, the seasonally adjusted estimates and the trend estimates. Compare the three percentages you have found, explaining what each represents and any differences between them.
(iv) DEC05 shows a record value for the original values in column 2. However, the seasonally adjusted estimate for DEC05 is only the second highest as AUG05 is higher. Explain the apparent contradiction between these two figures.
(v) What equation should the seasonal factors satisfy over what calendar period? Demonstrate this using the output above.
3. The Laspeyres price index at time $t$, with base period 0 , can be expressed as

$$
P_{L}(0, t)=\frac{\sum_{i=1}^{N} p_{t i} q_{0 i}}{\sum_{i=1}^{N} p_{0 i} q_{0 i}}
$$

where $p_{u i}$ is the price of commodity $i$ at time $u, q_{u i}$ is the quantity of commodity $i$ at time $u$, and $N$ is the number of commodities.

The corresponding expression for the Paasche price index is

$$
P_{P}(0, t)=\frac{\sum_{i=1}^{N} p_{t i} q_{t i}}{\sum_{i=1}^{N} p_{0 i} q_{t i}}
$$

(i) Starting from these formulae and recalling that value is the product of price and quantity, derive expressions for the Laspeyres and Paasche price indices in terms of values and price relatives.
(ii) State why a Laspeyres price index is typically more convenient to calculate than its corresponding Paasche price index.
(iii) Calculate Laspeyres and Paasche price indices using the data in the following table.

| Commodity | Value in <br> base period 0 | Value in <br> current period $t$ | Price relative |
| :---: | :---: | :---: | :---: |
| Knives | $€ 20$ | $€ 25$ | 1.10 |
| Forks | $€ 20$ | $€ 20$ | 1.15 |
| Tablespoons | $€ 20$ | $€ 15$ | 1.24 |
| Teaspoons | $€ 15$ | $€ 10$ | 1.07 |

(iv) Explain why Laspeyres price index numbers are usually greater than their Paasche equivalents.
4. (a) Prove that the ratio of the Paasche price index to the Laspeyres price index is equal to the ratio of the Paasche volume index to the Laspeyres volume index when these indices are calculated from the same data.
(b) Suppose that a country is divided into a number of administrative provinces. Using data from the table below,
(i) calculate the Laspeyres, Paasche and Fisher price indices for South province, using Central province as a base province,
(ii) calculate the Laspeyres, Paasche and Fisher volume indices for South province, using Central province as a base province.

| Commodity | Central province |  | South province |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Price | Quantity | Price | Quantity |
| Plain chocolate | 70 c | 100 million | 55 c | 20 million |
| Milk chocolate | 40 c | 100 million | 30 c | 40 million |
| White chocolate | 45 c | 50 million | 38 c | 10 million |

