

Oxford Physics Aptitude Test (PAT) 2006 Solutions

Physics

1) velocity \times time = displacement Ans: C

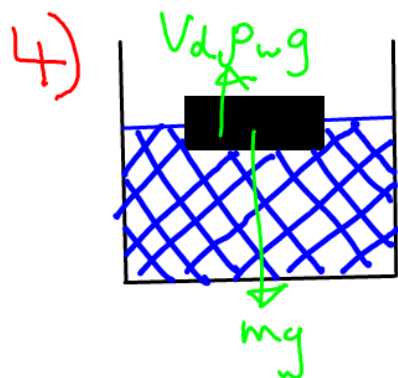
2) $V = 3 \times 4 \times 5 \times 10^{-6} = 6 \times 10^{-5} \text{ m}^3$

$m = \rho V = 8570 \times 6 \times 10^{-5} = 0.5142 \text{ kg}$

$p = \frac{F}{A} = \frac{mg}{\underbrace{3 \times 4 \times 10^{-4}}_{\text{smallest area}}} = \frac{5.142}{1.2 \times 10^{-3}} = 4285 \text{ Pa}$

Ans: A

3) At D, potential energy is higher compared to others so kinetic energy is lower. Ans: A



Volume displaced = V_d Density of water = ρ_w
mass of boat = m

$$mg = V_d \rho_w g$$

$$V_d = \frac{m}{\rho_w}$$

Add an extra mass to the boat, $\delta m = \rho_w \delta V$

$$\Rightarrow V_d' = \frac{m + \delta m}{\rho_w} = \frac{m}{\rho_w} + \frac{\rho_w \delta V}{\rho_w} = V_d + \delta V$$

But δV was taken from the water

$$\therefore V_d'' = V_d + \delta V - \delta V = V_d \quad \text{Ans: C}$$

5) $eV = \frac{p^2}{2m} = \frac{h^2}{2md^2}$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Ans: B

$$6) s = \left(\frac{u+v}{2}\right)t \quad s' = \left(\frac{0+20}{2}\right)0.2d$$

$$t = \frac{2d}{20+0} = 2d$$

$$= 0.1d$$

Ans: D

$$7) D: 1, 4, 9, 16, 25 \dots n^2$$

$$Y: 1, 8, 27, 64, 125 \dots n^3$$

$$\sqrt{1}, 4\sqrt{4}, 9\sqrt{9}, 16\sqrt{16}, 25\sqrt{25}, n^2\sqrt{n^2}$$

Ans: C

$$8) \text{Reading on scale, } R = \frac{W}{8.8} = \frac{mg}{8.8}$$

$$\text{On Mars, } R = 93 = \frac{3.8m}{8.8}$$

$$m = 215 \text{ kg} \quad \text{Ans: C}$$

9) Mass is constant since there is no loss of matter
Given that volume decreases, density must increase

$$m = V\rho \quad \text{Ans: B}$$

$$10) \downarrow +ve \quad s = ? \quad u = 11 \text{ ms}^{-1} \quad v = x \quad a = 10 \text{ ms}^{-1} \quad t = 7 \text{ s}$$

$$s = ut + \frac{1}{2}at^2 = 11(7) + \frac{1}{2}(10)7^2 = 322 \text{ m} \quad \text{Ans: C}$$

$$11) \text{Brightness is determined by power, } P = \frac{V^2}{R}$$

a) Half the voltage \Rightarrow dimmer

b) Same voltage \Rightarrow normal

c) Same voltage (d is off) \Rightarrow normal

d) No current through d \Rightarrow off

e) Half the voltage \Rightarrow dimmer

f) Same voltage \Rightarrow normal

g) Double the voltage \Rightarrow brighter

h) Same voltage \Rightarrow normal

12) a $\Rightarrow r + g = 35$ (1)

b $\Rightarrow 2g + b = 70$ (2)

c $\Rightarrow r + b = 2g$ (3)

d $\Rightarrow r^3 \rho + g^3 \rho + b^3 \rho = 20000$ (4)

(2) - (3): $2g + b - r - b = 70 - 2g$
 $4g - r = 70$ (5)

(4) + (5): $5g = 105$

$g = 21 \text{ cm} \Rightarrow r = 14 \text{ cm} \Rightarrow b = 28 \text{ cm}$

In (4): $\rho = \frac{20000}{14^3 + 21^3 + 28^3} = 0.589 \text{ g/cm}^3$ (cubes)

Density of liquid = $2\rho = 1178 \text{ kg m}^{-3}$

13) a) Rocket started slowing down

b) Just before X. Fuel is used up so mass decreases.

c) Rocket decelerates uniformly

d) Find the area under the graph. The straight line should be extended until it reaches the x-axis.

$$14) a) KE = Pt = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2Pt}{m}}$$

$$b) a = \frac{dv}{dt} = \frac{1}{2} \sqrt{\frac{2P}{m}} \cdot t^{-1/2} = \sqrt{\frac{P}{2mt}}$$

$$d = \int_0^t v dt = \sqrt{\frac{2P}{m}} \int_0^t t^{1/2} dt = \sqrt{\frac{2P}{m}} \left[\frac{2}{3} t^{3/2} \right]_0^t$$

$$= \frac{2}{3} \sqrt{\frac{2Pt^3}{m}}$$

$$c) \text{As } t \rightarrow \infty, v = \sqrt{\frac{2Pt}{m}} \rightarrow \infty$$

This is not reasonable, as infinite velocity is not possible

$$d) \text{As } t \rightarrow \infty, a = \sqrt{\frac{P}{2mt}} \rightarrow 0$$

Reasonable since acceleration cannot go on forever

$$\text{As } t \rightarrow 0, a \rightarrow \infty$$

Reasonable, showing rapid initial acceleration

$$e) Pt = mgh$$

$$h = \frac{Pt}{mg} \Rightarrow v = \frac{h}{t} = \frac{P}{mg}$$

$$f) KE = \frac{1}{2} mv^2 = \frac{1}{2} m \frac{P^2}{m^2 g^2} = \frac{P^2}{2mg^2}$$

$$\frac{KE}{PE} = \frac{P^2}{2mg^2} \cdot \frac{1}{Pt} = \frac{P}{2mg^2 t}$$

This is small for low P and long t .

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Maths

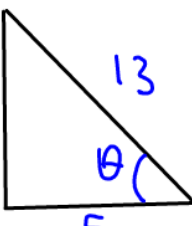
1) i) $2007^2 - 2006^2 = (2007 - 2006)(2007 + 2006)$
 $= 4013$

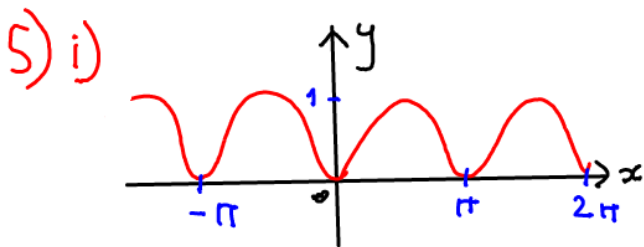
ii) $1.001^6 - 1.001^5 = (1 + 0.001)^6 - (1 + 0.001)^5$
 $= 1 + 0.006 - 1 - 0.005 = 1 \times 10^{-3}$

2) $m = \frac{-10}{1} = -10$

3) i) $\log_e(e^{3x}) = 6$
 $3x = 6$
 $x = 2$

ii) $\log_3 x^2 = 2$
 $x^2 = 3^2$
 $x = \pm 3$

4)  $13 \sin\left[\tan^{-1}\left(\frac{12}{5}\right)\right] = 13 \sin \theta = 13 \times \frac{12}{13} = 12$



ii) $y = \frac{1}{x^2 - 1}$

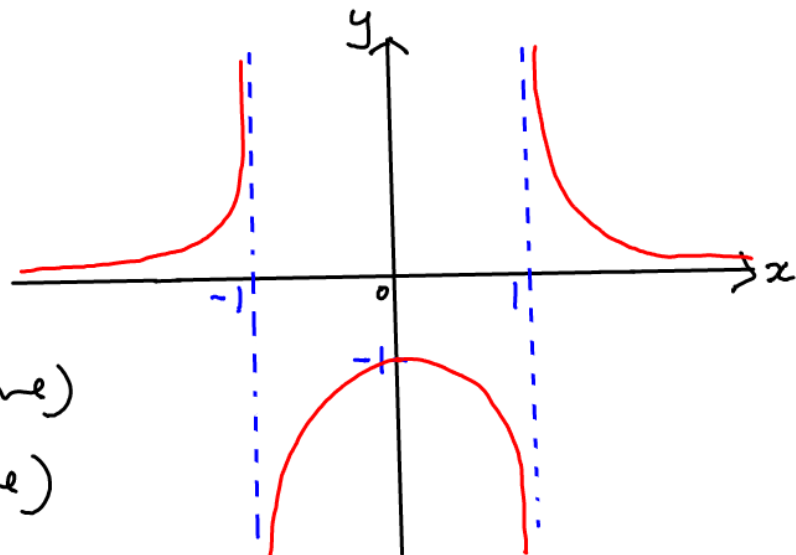
when $x = 0$, $y = -1$

when $y = 0$, no solution

$x = \pm 1$ are asymptotes

As $x \rightarrow \infty$, $y \rightarrow 0$ (+ve)

As $x \rightarrow -\infty$, $y \rightarrow 0$ (+ve)



$$6) r_A = r_B + 1 \quad (1) \quad A_A = A_B + 2\pi$$

$$\pi r_A^2 = \pi r_B^2 + 2\pi$$

$$r_A^2 = r_B^2 + 2 \quad (2)$$

$$\textcircled{1} \text{ in } \textcircled{2} : (r_B + 1)^2 = r_B^2 + 2$$

$$r_B^2 + 2r_B + 1 = r_B^2 + 2$$

$$r_B = \frac{1}{2} \Rightarrow r_A = \frac{3}{2}$$

$$7) i) P(666) = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$$

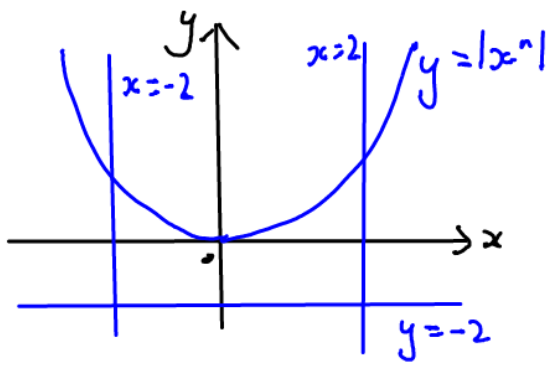
$$ii) P(111) + P(222) + \dots + P(666) = 6 \times \frac{1}{216} = \frac{1}{36}$$

$$iii) P(6'6'6) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

$$8) \frac{dV}{dt} = 1 \text{ cm}^3 \text{ s}^{-1} \quad V = \frac{4}{3} \pi r^3 \quad \text{When } S = 100,$$

$$\frac{dV}{dr} = 4\pi r^2 = 100$$

$$\frac{dr}{dt} = \frac{dr}{dV} \cdot \frac{dV}{dt} = \frac{1}{100} \times 1 = 0.01 \text{ cm s}^{-1}$$

$$9) \text{Area under } y = |x^n| \text{ from } x = -2 \text{ to } x = 2 \text{ and } y = -2$$


$$A = (4 \times 2) + 2 \int_0^2 x^n dx = 8 + 2 \left[\frac{x^{n+1}}{n+1} \right]_0^2$$

$$= 8 + \frac{2 \times 2^{n+1}}{n+1} = 8 + \frac{2^{n+2}}{n+1}$$

$$10) i) \text{G.P. with } a = 1, r = e^y$$

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1-e^y}$$

$$ii) \log_2 1 + \log_2 2 + \log_2 4 + \dots + \log_2 2^n$$

$$= 0 + 1 + 2 + \dots + n$$

A.P. with n $a=1, d=1, l=n$

$$S_n = \frac{n}{2} (a+l) = \frac{n(n+1)}{2}$$

$$ii) y = 5 + 24x - 9x^2 - 2x^3$$

$$\frac{dy}{dx} = 24 - 18x - 6x^2 = 0$$

$$x^2 + 3 - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$x = -4 \text{ or } 1$$

$$\frac{d^2y}{dx^2} = -12x - 18$$

$$\text{When } x = -4, y = 5 + 24(-4) - 9(16) + 2(64) \\ = -107$$

$$\frac{d^2y}{dx^2} = -12(-4) - 18 = 30 > 0$$

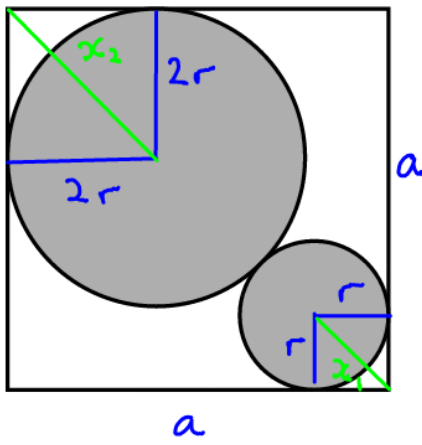
$\therefore (-4, -107)$ is a min. pt.

$$\text{When } x = 1, y = 5 + 24 - 9 - 2 = 18$$

$$\frac{d^2y}{dx^2} = -12(1) - 18 = -30 < 0$$

$\therefore (1, 18)$ is a max. pt.

12)



$$x_1^2 = r^2 + r^2 \quad x_2^2 = (2r)^2 + (2r)^2$$

$$x_1 = \sqrt{2} r \quad x_2 = 2\sqrt{2} r$$

Diagonal of square:

$$D = x_1 + x_2 + r + 2r = (3\sqrt{2} + 3)r$$

$$D^2 = a^2 + a^2 = 2a^2$$

$$\text{Area of square, } A_s = a^2 = \frac{D^2}{2} = \frac{(3\sqrt{2} + 3)^2 r^2}{2}$$

$$\text{Area of circles, } A_c = \pi r^2 + \pi (2r)^2 = 5\pi r^2$$

$$\Rightarrow \text{Fraction covered} = \frac{5\pi r^2}{\frac{(3\sqrt{2} + 3)^2 r^2}{2}} = \frac{10\pi}{27 + 18\sqrt{2}}$$