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Mark Scheme Summer 2009

GCE O Level

GCE AO Level Pure Mathematics (7362)

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PURE MATHEMATICS 7362, MARK SCHEME

Paper 1

Q.	Scheme	Marks
1	$f(3) = 2\times3^3 + p\times3^2 - 5\times3 + 6 = 0$ $9p = -45 \quad p = -5$	M1A1 A1
2	$3x + 19 = 5^4$ $3x = 625 - 19$ $x = 202$	M1 M1A1
3	$\overrightarrow{OE} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}(\mathbf{a}+\mathbf{b})$ $\overrightarrow{OF} = \frac{2}{5}(\mathbf{a}+\mathbf{b}) \quad \left(= \frac{4}{5}\overrightarrow{OE}\right)$ $\therefore$ collinear	M1A1 M1 A1
4	$A = \pi r^2$ $\delta A = 2\pi r\delta r, \quad \delta r = \frac{r}{100}$ $\delta A = \dfrac{2\pi r^2}{100} = \dfrac{2A}{100}$ $\therefore A$ increases by 2%	M1A1,B1 M1 A1(5)
5	(a) x-coord. 5 y-coord. 8 (b) Grad $AB = \frac{14-4}{8-3} = 2$ Grad $CD = \frac{6-8}{9-5} = -\frac{1}{2}$ $2\times-\frac{1}{2} = -1 \quad \therefore CD$ is perp. to AB (c) $y - 4 = -\frac{1}{2}(x-3), \quad 2y + x = 11$ (or equiv.)	B1 B1 M1 A1 B1ft M1,A1 (7)
6	(a) $\dfrac{ds}{dt} = 12t^2 - 44t + 24$ $12t^2 - 44t + 24 = 0 \quad 3t^2 - 11t + 6 = 0$ $(3t-2)(t-3) = 0 \quad t = \frac{2}{3},\ t = 3$ (b) accel $= 24t - 44$ $t = \frac{2}{3}$ accel $= -28\,\text{m/s}^2$	M1A1 M1A1 M1 M1A1 (7)

7	(a) $a+5d+a+6d=5(a+a+d)$ $4a=3d$ $a+3d=15$ $5a=15 \quad a=3 \quad d=4$ (b) $S_{25}-S_9=\frac{25}{2}(6+24\times4)-\frac{9}{2}(6+8\times4)$ $=1104$	M1 A1 B1 M1A1 M1M1A1 A1 (9)
8	(a) $V=3x^2h$ $A=3x^2+8xh$ $25=3x^2+8xh \quad h=\dfrac{25-3x^2}{8x}$ $V=3x^2\left(\dfrac{25-3x^2}{8x}\right)=\frac{3x}{8}(25-3x^2)$ *	B1 B1 M1A1
	(b) $V=\frac{75x}{8}-\frac{9x^3}{8} \quad \frac{dV}{dx}=\frac{75}{8}-\frac{27x^2}{8}$ $\frac{dV}{dx}=0 \quad \frac{75}{8}=\frac{27x^2}{8} \quad x^2=\frac{75}{27} \quad x=\frac{5}{3}$ (awrt 1.67) $V_{\max}=\frac{3}{8}\times\frac{5}{3}(25-\frac{25}{3})=10\frac{5}{12}$ (awrt 10.4)	M1 M1 A1 (12)
9	(a) Use of correct cosine rule to find one angle All nos. correct in formula, correct angle found Cosine or sine rule to find second angle angle sum triangle for 3rd angle $(A=48.6°,\ B=41.8°,\ C=89.6°)$	M1 A1,A1 M1A1 B1ft
	(b) $\angle BAP=A+\frac{1}{2}(180-A)=114.3°$ $\angle APB=23.9°$ $\dfrac{BP}{\sin BAP}=\dfrac{12}{\sin APB}, \quad BP=27\,\text{cm}$	 B1 B1 M1A1,A1 (11)
10	(a) $\sin x(1+\cos x)=0$ $\sin x=0 \quad x=0,\ \pi$ or $\cos x=-1 \quad x=\pi$	M1A2,1,0
	(b) $\frac{dy}{dx}=$ $\cos x+\cos^2 x-\sin^2 x$ $=\cos x+\cos^2 x-(1-\cos^2 x)$ $=2\cos^2 x+\cos x-1=(2\cos x-1)(\cos x+1)$ $\frac{dy}{dx}=0 \quad \cos x=\frac{1}{2} \quad x=\frac{\pi}{3}$ (or subst. $x=\frac{\pi}{3}$ to get $\frac{dy}{dx}=0$) $(\cos x=-1 \qquad x=\pi)$ $\frac{d^2y}{dx^2}=-4\cos x\sin x-\sin x \quad x=\frac{\pi}{3} \quad \frac{d^2y}{dx^2}<0 \quad \therefore \max$ $x=\frac{\pi}{3} \quad y=\frac{\sqrt{3}}{2}(1+\frac{1}{2})=\frac{3\sqrt{3}}{4}$	M1(diff) A1 M1 M1 A1 M1A1 B1 (11)

11	(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\sin x}{\cos x}\right)=\dfrac{\cos^2 x+\sin^2 x}{\cos^2 x}=\dfrac{1}{\cos^2 x}$	M1M1A1
	(b) A is $(0,1)$	B1
	(c) $y=0$ $\tan x=-1$ $x=-\frac{\pi}{4}$, $\frac{3\pi}{4}$	B1B1
	(d) $x=\frac{\pi}{6}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\cos^2 x}=\frac{4}{3}$	B1
	grad normal $=\frac{-3}{4}$	B1ft
	Eqn normal $y-\left(1+\frac{1}{\sqrt{3}}\right)=-\frac{3}{4}\left(x-\frac{\pi}{6}\right)$	M1A1
	$y=0$ $1+\frac{1}{\sqrt{3}}=\frac{3}{4}\left(x-\frac{\pi}{6}\right)$	M1
	$\frac{3}{4}x=1+\frac{1}{\sqrt{3}}+\frac{\pi}{8}$	
	$x_G=\frac{4}{3}\left(1+\frac{1}{\sqrt{3}}+\frac{\pi}{8}\right)$	A1
	(e) $x_G=2.62...$ at C, $x=\frac{3\pi}{4}=2.35...<x_G$	
	$\Rightarrow$ normal meets x-axis to right of C and $f>0$	M1A1 (14)
12	(a) $1+\left(\frac{1}{5}\right)\left(-\frac{x}{2}\right)+\dfrac{\left(\frac{1}{5}\right)\left(-\frac{4}{5}\right)\left(-\frac{x}{2}\right)^2}{2!}+\dfrac{\left(\frac{1}{5}\right)\left(-\frac{4}{5}\right)\left(-\frac{9}{5}\right)\left(-\frac{x}{2}\right)^3}{3!}+...$	M1
	$=1-\frac{x}{10}-\frac{x^2}{50}-\frac{3x^3}{500}-...$	A2,1,0 (3)
	(b) $1+\left(-\frac{1}{5}\right)\left(\frac{x}{2}\right)+\dfrac{\left(-\frac{1}{5}\right)\left(-\frac{6}{5}\right)\left(\frac{x}{2}\right)^2}{2!}+\dfrac{\left(-\frac{1}{5}\right)\left(-\frac{6}{5}\right)\left(-\frac{11}{5}\right)\left(\frac{x}{2}\right)^3}{3!}+...$	M1
	$=1-\frac{x}{10}+\frac{3x^2}{100}-\frac{11x^3}{1000}-...$	A2,1,0 (3)
	(c) $\left\|\frac{x}{2}\right\|<1$ $\|x\|<2$	B1 (1)
	(d) $\left(\dfrac{2-3y}{2+3y}\right)^{\frac{1}{5}}=\left(1-\frac{3}{2}y\right)^{\frac{1}{5}}\left(1+\frac{3}{2}y\right)^{-\frac{1}{5}}$	M1
	$\left(1-\frac{x}{10}-\frac{x^2}{50}\right)\left(1-\frac{x}{10}+\frac{3x^2}{100}\right)=1-\frac{x}{10}+\frac{3x^2}{100}-\frac{x}{10}+\frac{x^2}{100}-\frac{x^2}{50}$	M1
	$=1-\frac{x}{5}+\frac{x^2}{50}$	A1
	substitute $x=3y$ $1-\frac{3y}{5}+\frac{9y^2}{50}$	M1A1
	$\|y\|<\frac{2}{3}$	B1 (6)
	(e) $\int_0^{0.5}\left(1-\frac{3y}{5}+\frac{9y^2}{50}\right)\mathrm{d}y=\left[y-\frac{3y^2}{10}+\frac{3y^3}{50}\right]_0^{0.5}$	M1A1ft
	$=0.5-\frac{3\times0.5^2}{10}+\frac{3\times0.5^3}{50}-0$	M1
	$=0.4325=0.433$	A1 (4)
		(17)

PURE MATHEMATICS 7362, MARK SCHEME

Paper 2

1

$b^2-4ac=16(2-p)^2-16(3p-8)<0$ M1

$p^2-7p+12<0$

$(p-3)(p-4)<0$ M1A1

$3<p<4$ A1 (4)

2

$x(2x-5)=12,\quad 2x^2-5x-12=0$ M1,A1

$(2x+3)(x-4)=0$ M1

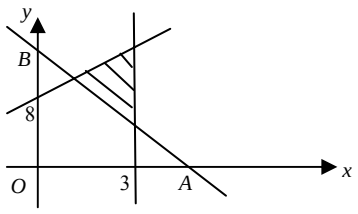
$x=-\frac{3}{2}\quad y=-8$ A1

$x=4\quad y=3$ A1 (5)

3

(a) (i) (4, 0) (b), (c)

(ii) (0, 16)



(a)B1B1

(b)G1

G1

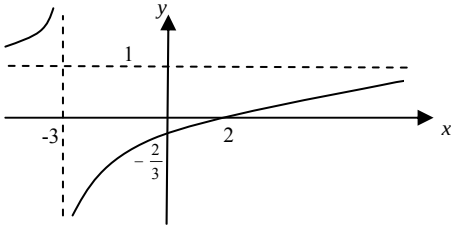
(c)B1 (5)

4

(a) (i) $y=1$ (ii) $x=-3$ B1B1

(b) (i) (2,0) (ii) $(0,-\frac{2}{3})$ B1B1

(c)



B1(2 branches)
B1(asyms.)
B1(X-ing points)
(7)

5

(a)

x	0.3	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	8.71	2	0	0.44	1.25	2.16	3.11	4.08	5.06

B2

(b) Graph drawn G2

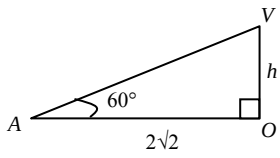
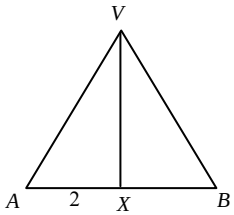
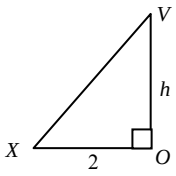
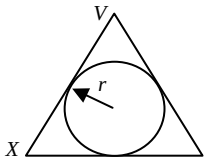
(c) $2+\frac{1}{x}=2x-3+\frac{1}{x^2}$

$2x^2+x=2x^3-3x^2+1$ M1

$2x^3-5x^2-x+1=0$ A1

(d) $x=0.4, 2.6$ B1,B1 (8)

6	(a) $ar^2 - ar = 12 \quad ar(r-1) = 12, \quad ar^4 - ar^3 = 27 \quad ar^3(r-1) = 27$ $r^2 = \frac{27}{12}, \quad r > 1 \quad \therefore r = \frac{3}{2}$ (b) $ar^2 - ar = 12 \quad \left(\frac{9}{4} - \frac{3}{2}\right) = 12 \quad \frac{3}{4}a = 12 \quad a = 16$ (c) $a' = 16 \times \frac{3}{2} = 24, \quad r' = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$ $S_{10} = \dfrac{a\left(r^{10}-1\right)}{r-1} = \dfrac{24\left(2.25^{10}-1\right)}{2.25-1} = 63825.7... = 63826$	M1,A1 M1,A1 M1A1 B1 M1A1A1 (10)
7	(a) $a^{\frac{3}{2}}y = x^{\frac{5}{2}} \quad a^{\frac{3}{2}}\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}}$ At $(a,a) \quad a^{\frac{3}{2}}\frac{dy}{dx} = \frac{5}{2}a^{\frac{3}{2}} \quad \frac{dy}{dx} = \frac{5}{2}$, grad normal $= \frac{-2}{5}$ Eqn normal $y - a = -\frac{2}{5}(x-a)$ $5y + 2x = 7a$ (b) $y = 0 \quad x = \frac{7a}{2}$ (c) Vol under curve $= \int_0^a \pi y^2 dx = \pi \int_0^a \frac{x^5}{a^3} dx, \quad = \pi\left[\frac{x^6}{6a^3}\right]_0^a, \quad = \frac{\pi a^3}{6}$ vol under line $= \frac{1}{3}\pi a^2 \times \left(\frac{7a}{2} - a\right)$ Total vol $= \frac{\pi a^3}{6} + \frac{5\pi a^3}{6} = \pi a^3$	M1A1 A1,B1ft M1 A1 B1 M1,M1,A1 B1 M1A1 (13)
8	(a) $f(x) = -7\left(x^2 + \frac{5}{7}x\right) + 3 = -7\left(x + \frac{5}{14}\right)^2 + 7 \times \frac{25}{196} + 3$ $= 3\frac{25}{28} - 7\left(x + \frac{5}{14}\right)^2$ $A = 3\frac{25}{28}, \quad B = 7, \quad C = \frac{5}{14}$ (b) Max. value $= 3\frac{25}{28}$, when $x = -\frac{5}{14}$ (c) $\alpha\beta = \frac{-3}{7} \quad \alpha + \beta = \frac{-5}{7}$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{25}{49} + 2 \times \frac{3}{7} = \frac{67}{49}$ (d) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{a^2+\beta^2}{\alpha\beta}, = -\frac{67}{49} \times \frac{7}{3} = -\frac{67}{21}$ (e) $x^2 + \frac{67}{21}x + 1, = 0 \quad 21x^2 + 67x + 21 = 0$	M1 A1A1A1 B1ft B1ft B1 M1A1 M1A1,A1 M1A1 (14)

9	(a)  (b) (c)  (d)  (e)  	$AC^2 = 2\times 4^2 \quad AC = 4\sqrt{2}, \quad AO = 2\sqrt{2}$ $\tan 60 = \frac{VO}{2\sqrt{2}}$ $VO = 4.898... = 4.90$ $\cos 60 = \frac{2\sqrt{2}}{VA}$ $VA = \frac{2\sqrt{2}}{\cos 60} = 5.656... = 5.66$ cm (or by Pythagoras) $VX^2 = VO^2 + 2^2$ $VX = 5.291... = 5.29$ cm $\tan VXO = \frac{VO}{2}$ $VXO = 67.78... = 67.8°$ $\theta = \frac{1}{2}VXO$ $r = 2\tan\theta = 2\tan 33.89...$ $= 1.343... = 1.34$ cm	M1,A1 M1 A1 (4) M1A1ft A1 (3) M1A1ft A1 (3) M1A1ft A1 (3) B1 M1 A1 (3)

10	(a) $\cos 2A = \cos^2 A - \sin^2 A = \cos^2 A - \left(1 - \cos^2 A\right) = 2\cos^2 A - 1$	M1A1
	(b) (i) $\cos 5\theta + \cos 3\theta$	
	$= \cos 4\theta \cos\theta - \sin 4\theta \sin\theta + \cos 4\theta \cos\theta + \sin 4\theta \sin\theta$	M1M1
	$= 2\cos 4\theta \cos\theta$	
	(ii) $2\cos 4\theta \cos\theta + 2\cos\theta$	M1
	$= 2\cos\theta\left(\cos 4\theta + 1\right)$	M1
	$= 4\cos\theta \cos^2 2\theta$	
	$= 4\cos\theta\left(2\cos^2\theta - 1\right)^2$	M1
	$= 16\cos^5\theta - 16\cos^3\theta + 4\cos\theta$	A1
	(c) $\cos 5\theta + \cos 3\theta - 2\cos\theta = 0$	
	$16\cos^5\theta - 16\cos^3\theta + 4\cos\theta - 4\cos\theta = 0$	M1
	$\cos^5\theta - \cos^3\theta = 0$	A1
	$\cos^3\theta\left(\cos^2\theta - 1\right) = 0$	M1
	$\cos\theta = 0 \quad \theta = \pm\frac{\pi}{2}$	A1
	$\cos\theta = \pm 1 \quad \theta = 0, \pm\pi$	A1
	(d) $\int_0^{\frac{\pi}{3}} \left(\cos^5\theta - \cos^3\theta\right) \mathrm{d}\theta$	
	$= \int_0^{\frac{\pi}{3}} \left(\cos^5\theta - \cos^3\theta + \frac{1}{4}\cos\theta - \frac{1}{4}\cos\theta\right) \mathrm{d}\theta$	
	$= \int_0^{\frac{\pi}{3}} \left\{\frac{1}{16}\left(\cos 5\theta + \cos 3\theta + 2\cos\theta\right) - \frac{1}{4}\cos\theta\right\} \mathrm{d}\theta$	M1
	$= \left[\frac{1}{16}\left(\frac{1}{5}\sin 5\theta + \frac{1}{3}\sin 3\theta + 2\sin\theta\right) - \frac{1}{4}\sin\theta\right]_0^{\frac{\pi}{3}}$	M1A1
	$= \left[\frac{1}{16}\left(\frac{1}{5}\sin 5\frac{\pi}{3} + \frac{1}{3}\sin 3\frac{\pi}{3} + 2\sin\frac{\pi}{3}\right) - \frac{1}{4}\sin\frac{\pi}{3} - 0\right]$	M1
	$= -0.1190... = -0.119$	A1
		(18)

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