

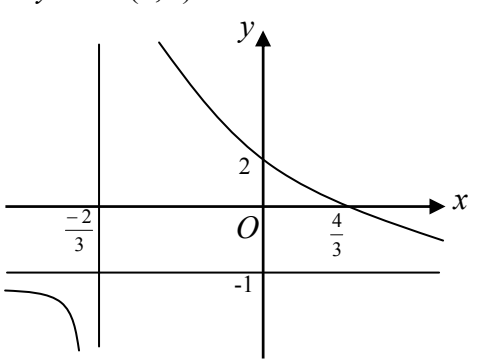
Mark Scheme (Results) January 2009

GCE O Level

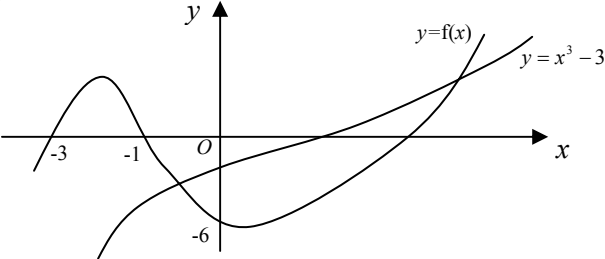
AO Level Pure Mathematics (7362) Paper 1

Pure Mathematics 7362

Paper 1

Q.	Scheme	Marks
1	$\frac{\sin 32}{4.6} = \frac{\sin C}{6.3}, \quad \sin C = \frac{6.3 \sin 32}{4.6}$ $C = 46.5^\circ, \quad 133.5^\circ$	M1, M1 A1A1ft (4)
2	$A = \pi r^2 \quad \frac{da}{dr} = 2\pi r$ $\delta A \approx 2\pi r \delta r$ $\frac{\delta A}{A} \approx \frac{2\pi r \delta r}{\pi r^2}$ $\frac{\delta r}{r} \approx \frac{1}{2} \frac{\delta A}{A}$ $\frac{\delta a}{A} = x\% \quad \therefore \frac{\delta r}{r} = \frac{1}{2} x\%$	B1 M1 M1 M1A1 (5)
3	<p>(a) $\frac{dy}{dx} = 3x^2 - 8x - 2$</p> <p>Grad tgt at A: $3a^2 - 8a - 2 = 1 \quad 3a^2 - 8a - 3 = 0$</p> $(3a+1)(a-3) = 0$ $(a = -\frac{1}{3}) \quad a = 3$ $b = 3^3 - 4 \times 9 - 6 + 10 = -5$ <p>(b) $y + 5 = x - 3$</p>	<div style="border: 1px solid black; width: 20px; height: 20px; margin-left: 100px; margin-top: 10px;"></div> M1 M1 M1 A1 A1 (5) B1 (1)
4	<p>(a) $\frac{dy}{dx} = 12x^2 \cos 3x - 3(4x^3 - 5) \sin 3x$</p> <p>(b) $\frac{dy}{dx} = \frac{2e^{2x}(3x^2 - x) - (6x - 1)e^{2x}}{(3x^2 - x)^2}$</p>	M1A2, 1, 0 (3) M1A2, 1, 0 (3)
5	<p>(a) (i) $y = -1$ (ii) $x = -\frac{2}{3}$</p> <p>(b) (i) $y = 0 \quad 4 - 3x = 0 \quad (\frac{4}{3}, 0)$</p> <p>(ii) $x = 0 \quad y = 2 \quad (0, 2)$</p> <p>(c)</p> 	B1B1 (2) B1 B1 (2) G1 G1ft G1ft (3)

6	<p>(a) $V = (80 - 2x)(50 - 2x)x$ $= 4000x - 260x^2 + 4x^3$ *</p> <p>(b) $\frac{dV}{dx} = 4000 - 520x + 12x^2$ $0 = 3x^2 - 130x + 1000$ $0 = (3x - 100)(x - 10)$ $x = \frac{100}{3}$ (not poss) or $x = 10$</p> <p>$\frac{d^2V}{dx^2} = -520 + 24x$ $x = 10$ $\frac{d^2V}{dx^2} = -520 + 240 < 0$ \therefore Max.</p> <p>(c) $V_{\max} = (80 - 20)(50 - 20) = 18000$</p>	<p>M1 A1 (2) M1 M1A1 M1M1A1 (6) M1A1 (2)</p>
7	<p>(a) (i) $\overline{AD} = \frac{1}{4}\mathbf{b} - \mathbf{a}$ (ii) $\overline{OE} = \frac{1}{4}\mathbf{b} - \frac{1}{3}\left(\frac{1}{4}\mathbf{b} - \mathbf{a}\right) = \frac{1}{3}\mathbf{a} + \frac{1}{6}\mathbf{b}$ (iii) $\overline{BE} = \frac{1}{3}\mathbf{a} - \frac{5}{6}\mathbf{b}$</p> <p>(b) $\overline{BF} = -\mathbf{b} + \mu\mathbf{a}$ $\overline{BE} = \frac{1}{3}\mathbf{a} - \frac{5}{6}\mathbf{b}$, $\therefore F, E, B$ collinear when $\overline{BF} = \frac{6}{5}\overline{BE}$ $\therefore \mu = \frac{6}{5} \times \frac{1}{3} = \frac{2}{5}$</p> <p>(c) $\overline{EG} = -\frac{2}{3}(-\mathbf{a} + \frac{1}{4}\mathbf{b}) + \lambda(-\mathbf{a} + \mathbf{b}) = (\frac{2}{3} - \lambda)\mathbf{a} + (\lambda - \frac{1}{6})\mathbf{b}$ DB parallel to EG $\overline{DB} = \frac{3}{4}\mathbf{b}$ $\therefore \lambda = \frac{2}{3}$</p>	<p>B1 M1A1 B1 (4) M1 M1A1 A1 (4) M1A1 M1A1 (4)</p>
8	<p>(a) $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $= (1 - \sin^2 \theta) - \sin^2 \theta = \cos^2 - (1 - \cos^2 \theta)$ $2\sin^2 \theta = 1 - \cos 2\theta$ $2\cos^2 \theta = \cos 2\theta + 1$ $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$ * $\cos^2 \theta = \frac{1}{2}(\cos 2\theta + 1)$ *</p> <p>(b) $8\sin^4 \theta - 2\sin^2 \theta - 2 = 8 \times \frac{1}{4}(1 - \cos 2\theta)^2 - 2 \times \frac{1}{2}(1 - \cos 2\theta) - 2$ $= 2(1 - 2\cos 2\theta + \cos^2 2\theta) - 1 + \cos 2\theta - 2$ $= 2 - 4\cos 2\theta + 2 \times \frac{1}{2}(\cos 4\theta + 1) - 3 + \cos 2\theta$ $= \cos 4\theta - 3\cos 2\theta$ *</p> <p>(c) $\cos 4\theta - 3\sin 2\theta + 2 + 3\cos 2\theta = 2.5$ $\cos 4\theta = 0.5$ $4\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \theta = \frac{\pi}{12}, \frac{5\pi}{12}$</p> <p>(d) $\int_0^{\frac{\pi}{2}} (4\sin^4 \theta - \sin^2 \theta + \cos \theta) d\theta$ $= \int_0^{\frac{\pi}{2}} \left(\frac{1}{2}(\cos 4\theta - 3\cos 2\theta) + 1 + \cos \theta\right) d\theta$ $= \left[\frac{1}{8}\sin 4\theta - \frac{3}{4}\sin 2\theta + \theta + \sin \theta\right]_0^{\frac{\pi}{2}}$ $= \left(\frac{\pi}{2} + \sin \frac{\pi}{2}\right) - 0 = \frac{\pi}{2} + 1$</p>	<p>M1 M1 A1A1 (4) M1 M1 A1 (4) M1 M1A1A1 (4) M1 M1A1 A1 (4)</p>

<p>9</p>	<p>(a) $a = (x-1) \quad ar^2 = 3x \quad ar^4 = (10x+8)$ $r^2 = \frac{3x}{x-1} = \frac{10x+8}{3x}$ $9x^2 = 10x^2 - 2x - 8 \quad x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0 \quad x = 4, \quad x = -2$</p> <p>(b) Pos terms $\Rightarrow x = 4 \quad a = 3$</p> <p>(c) $ar^2 = 12 \quad r^2 = 4 \quad r = 2$</p> <p>(d) $S_8 = \frac{a(r^8-1)}{(r-1)} = \frac{3(2^8-1)}{(2-1)} = 765$</p> <p>(e) $a = (x-1) = 3 \quad a + 5d = 48 \quad d = 9$</p> <p>(f) $T_n = \frac{n}{2}(6 + 9(n-1)) = \frac{n}{2}(9n-3)$ $\frac{n}{2}(9n-3) > 765$ $9n^2 - 3n - 1530 > 0 \quad 3n^2 - n - 510 > 0$ Crit values $n = \frac{1 \pm \sqrt{1 + 4 \times 3 \times 510}}{6} = 13.2 \quad (n > 0)$ least $n = 14$</p>	<p>M1A1</p> <p>M1A1 (4) B1 (1)</p> <p>M1A1 (2)</p> <p>M1A1 (2)</p> <p>M1A1 (2)</p> <p>M1 M1</p> <p>M1A1</p> <p>A1 ft (5)</p>
<p>10</p>	<p>(a) $8 + 4p - 2q - 6 = 0$ $-27 + 9p + 3q - 6 = 0$ $2p - q = -1$ $3p + q = 11 \quad p = 2, \quad q = 5$</p> <p>(b) $f(x) = (x-2)(x+3)(x+1)$</p> <p>(c) $x^3 - 3 = x^3 + 2x^2 - 5x - 6$ $2x^2 - 5x - 3 = 0$ $(2x+1)(x-3) = 0$ $x = -\frac{1}{2} \quad x = 3$ $y = -3\frac{1}{8} \quad y = 24$</p> <p>(d)</p>  <p>(i) G1 G2, 1, 0 (3)</p> <p>(ii) G1 (1)</p> <p>(f) $\int_{-\frac{1}{2}}^3 (x^3 - 3) - (x^3 + 2x^2 - 5x - 6) dx$ $= \int_{-\frac{1}{2}}^3 (-2x^2 + 5x + 3) dx$ $= \left[-\frac{2}{3}x^3 + \frac{5}{2}x^2 + 3x \right]_{-\frac{1}{2}}^3$ $= (-18 + \frac{45}{2} + 9) - (\frac{2}{24} + \frac{5}{8} - \frac{3}{2}) = 14\frac{7}{24} \quad \text{or} \quad (\frac{343}{24})$</p>	<p>M1 A1</p> <p>M1A1 (4)</p> <p>B1 (1) M1</p> <p>M1</p> <p>A1A1 (4)</p> <p>(i) G1 G2, 1, 0 (3)</p> <p>(ii) G1 (1)</p> <p>M1</p> <p>M1A1</p> <p>M1A1 (5)</p>