London Examinations GCE

Pure Mathematics

Alternative Ordinary Level

Paper 2

Wednesday 23 January 2008 – Morning

Time: 2 hours

Materials required for examination
Nil

Items included with question papers
Nil

Candidates are expected to have an electronic calculator when answering this paper.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 100.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

Write your answers neatly and legibly.

Turn over

1. The region enclosed by the curve with equation $y = e^{2x} + 4$, the x-axis, the y-axis and the line $x = 2$ is rotated through $360°$ about the x-axis. Find, in terms of e and π, the volume of the solid generated.

(5)

(Total 5 marks)

2. Referred to a fixed origin O, the position vectors of the points P and Q are $(6\mathbf{i} - 5\mathbf{j})$ and $(10\mathbf{i} + 3\mathbf{j})$ respectively. The midpoint of PQ is R.

(a) Find the position vector of R.

(2)

The midpoint of OP is S.

(b) Prove that SR is parallel to OQ.

(3)

Q2

(Total 5 marks)

H 2 6 5 8 0 A 0 3 2 4

3. The volume of a right circular cone is increasing at the rate of 45 cm^3/s. The height of the cone is always three times the radius of the base of the cone. Find the rate of increase of the radius of the base, in cm/s to 3 significant figures, when the radius of the cone is 4 cm.

(6)

Question 3 continued

4. Use $\tan \theta \equiv \dfrac{\sin \theta}{\cos \theta}$

   (a) to show that $\dfrac{\mathrm{d}}{\mathrm{d}\theta} (\tan \theta) = \dfrac{1}{\cos^2 \theta}$,

                                                   (3)

   (b) to solve, in degrees to one decimal place, the equation $5 \sin \theta = 7 \cos \theta$, $0 \leqslant \theta < 360°$.

                                                   (3)

Question 4 continued

5. The fourth term of an arithmetic series is four times the eighth term. The sum of the first four terms is 164. Find

   (a) the common difference of the series,

   (4)

   (b) the first term of the series.

   (2)

   The sum of the first n terms of the series is S_n.

   (c) Find the greatest value of n for which S_n is positive.

   (4)

Question 5 continued

6. (a) Complete the table for $y = 2x - \dfrac{1}{x^2}$, giving the values of y to 3 significant figures where appropriate.

x	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	−3		2.56		4.84			7.94

(2)

(b) Using a scale of 4 cm to 1 unit on the x-axis and 2 cm to 1 unit on the y-axis, draw the graph of $y = 2x - \dfrac{1}{x^2}$ for $0.5 \leqslant x \leqslant 4.0$.
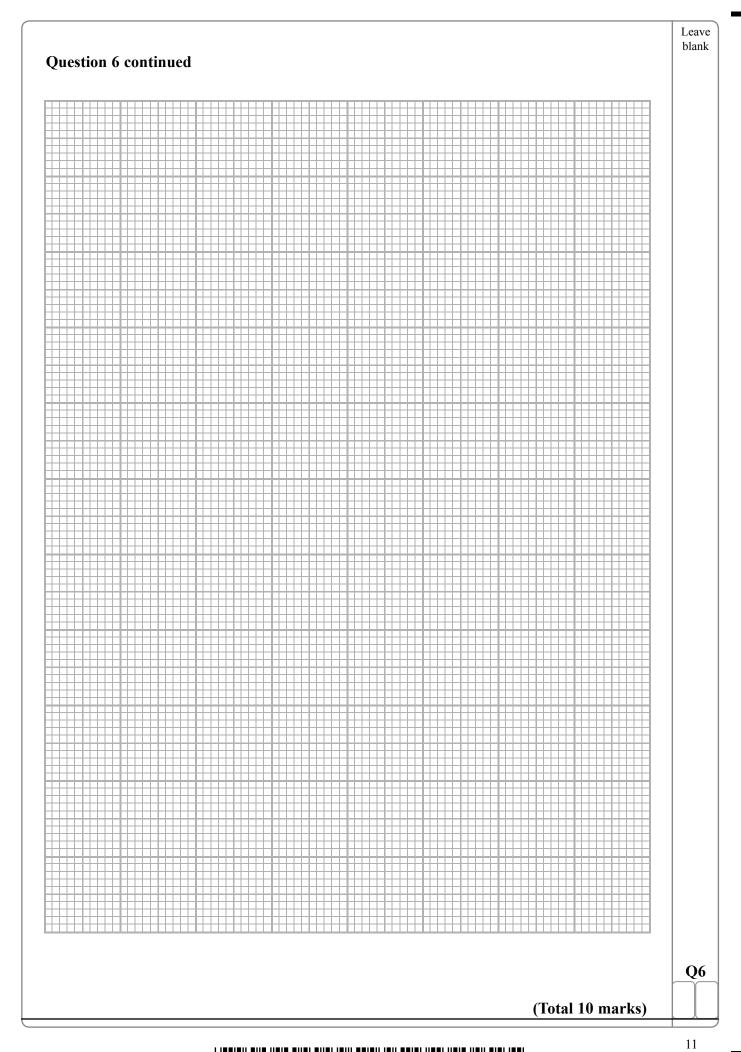
(2)

(c) Express $2x - \dfrac{1}{x^2}$ as a single fraction, and hence use your graph to estimate, to 2 significant figures, the value of $\sqrt[3]{0.5}$.

(3)

(d) By drawing a suitable straight line on your graph, find an estimate, to 2 significant figures, of the root of the equation $3x - 6 - \dfrac{1}{x^2} = 0$, in the interval $0.5 \leqslant x \leqslant 4.0$.

(3)

Question 6 continued

Q6

7. The points A, B and C have coordinates $(2, 6)$, $(6, 8)$ and $(4, 2)$ respectively.

   (a) Find the exact lengths of

   (i) AB,   (ii) BC,   (iii) AC.

   (4)

   (b) Find the size of each angle of $\triangle ABC$.

   (3)

   A circle is drawn to pass through the points A, B and C. Find

   (c) the coordinates of the centre of the circle,

   (2)

   (d) the exact length of the radius of the circle.

   (2)

Question 7 continued

8. (a) Expand $(1+\frac{1}{4}x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2, simplifying each term as far as possible.

(3)

(b) Expand $(1-\frac{1}{4}x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2, simplifying each term as far as possible.

(3)

(c) State the range of values of x for which both of your expansions are valid.

(1)

Using your answers to parts (a) and (b),

(d) expand $\left(\dfrac{4+x}{4-x}\right)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2, simplifying each term as far as possible.

(3)

(e) Hence obtain an estimate, to 3 significant figures, of $\displaystyle\int_0^{0.3}\left(\dfrac{4+x}{4-x}\right)^{\frac{1}{3}}\,dx$.

(4)

Question 8 continued

H 2 6 5 8 0 A 0 1 5 2 4

Question 8 continued

H 2 6 5 8 0 A 0 1 6 2 4

Question 8 continued

9. Solve

(a) $\log_q 343 = 3,$

(2)

(b) $\log_4 (5n + 9) = 3,$

(3)

(c) $\log_m 4 + 8 \log_4 m = 6,$

(6)

(d) $2 \log_3 x - 3x \log_3 x + 6x = 4.$

(5)

Question 9 continued

H 2 6 5 8 0 A 0 1 9 2 4

Question 9 continued

H 2 6 5 8 0 A 0 2 0 2 4

Question 9 continued

(Total 16 marks)

H 2 6 5 8 0 A 0 2 1 2 4

10. A curve has equation $y = \dfrac{2x-1}{4x+2}$, $x \neq -\frac{1}{2}$.

(a) Write down an equation for the asymptote to the curve which is parallel to

(i) the x-axis,

(ii) the y-axis.

(2)

(b) Find the coordinates of the points where the curve crosses the coordinate axes.

(2)

(c) Sketch the curve, showing clearly the asymptotes and the coordinates of the points where the curve crosses the coordinate axes.

(3)

The curve intersects the y-axis at the point P.

(d) Find an equation for the normal to the curve at P.

(5)

The normal at P meets the curve again at Q.

(e) Find the coordinates of Q.

(5)

Question 10 continued

H 2 6 5 8 0 A 0 2 3 2 4

Question 10 continued

(Total 17 marks)

TOTAL FOR PAPER: 100 MARKS

END

Q10

H 2 6 5 8 0 A 0 2 4 2 4