

Mark Scheme Summer 2008

GCE O Level

GCE AO Level Pure Mathematics (7362)

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Paper 1

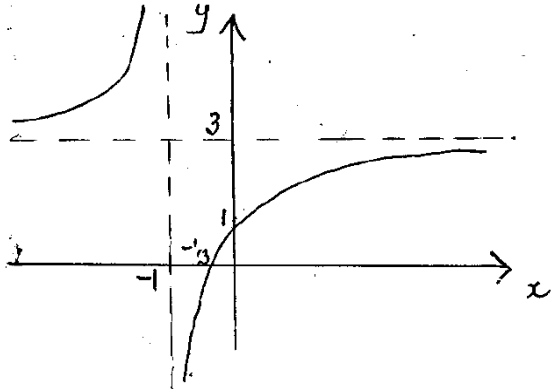
Q.	Scheme	Marks
1	$\cos \theta = \frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5} = \frac{16 + 25 - 36}{40} \left(= \frac{1}{8} \right)$ $\theta = 82.8^\circ$	M1 M1A1 A1 (4)
2	$5 - x = x^2 - 8x + 11$ $x^2 - 7x + 6 = 0$ $(x - 6)(x - 1) = 0$ $x = 6 \quad y = -1$ $x = 1 \quad y = 4$	M1 A1 M1 A1 A1 (5)
3	$\frac{r}{h} = \tan 30 \quad h = \frac{r}{\tan 30}$ $V = \frac{1}{3} \pi r^2 \times \frac{r}{\tan 30}$ $\frac{dV}{dr} = \frac{\pi r^2}{\tan 30}$ $\frac{dV}{dt} = -5 \quad \frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$ $= -5 \times \frac{\tan 30}{\pi r^2} = -5 \times \frac{\tan 30}{\pi (h \tan 30)^2}$ $h = 10 \quad \frac{dr}{dt} = \frac{-5}{100\pi \tan 30} = -0.0275\dots = -0.028 \text{ cm/s}$ Rate of decrease = 0.028 cm/s	B1 M1 M1 B1 M1 A1 A1 (7)
4	(a) $\frac{dy}{dx} = 10xe^{2x} + 2(5x^2 - 2)e^{2x}$ (b) $\frac{dy}{dx} = \frac{3x^2(x - x^2) - (x^3 + 2)(1 - 2x)}{(x - x^2)^2}$ $= \frac{2x^3 - x^4 + 4x - 2}{(x - x^2)^2}$	M1A1A1 (3) M1 A2,1,0 A1 (4)
5	(a) (i) $\overline{AB} = \overline{OB} - \overline{OA} = 5\mathbf{i} + 10\mathbf{j}$ (ii) $\overline{OC} = \overline{OA} + \frac{2}{5}\overline{AB} = 4\mathbf{i} + 5\mathbf{j} + 2\mathbf{i} + 4\mathbf{j} = 6\mathbf{i} + 9\mathbf{j}$ (b) unit vector = $\frac{\overline{AB}}{ \overline{AB} } = \frac{5\mathbf{i} + 10\mathbf{j}}{\sqrt{125}} = \frac{\mathbf{i} + 2\mathbf{j}}{\sqrt{5}}$ (c) $\overline{DA} = -\lambda\mathbf{i} + 4\mathbf{i} + 5\mathbf{j} = (4 - \lambda)\mathbf{i} + 5\mathbf{j}$ Parallel to $5\mathbf{i} + 10\mathbf{j} \quad \therefore 5 = 2(4 - \lambda)$ $\lambda = \frac{3}{2}$	B1 M1A1 (3) M1A1 (2) M1 M1 A1 (3)

6	<p>(a) $p^5 = 243, p = 3$</p> <p>(b) $3q + 4 = 4^3$ $3q + 4 = 64 \quad 3q = 60 \quad q = 20$</p> <p>(c) $f(x) = 2x \log_x 3 - 10 \log_x 3 - x + 5$ $= 2 \log_x 3(x - 5) - (x - 5)$ $= (x - 5)(2 \log_x 3 - 1) \quad a = 2, b = 1$</p> <p>(d) $\log_x 3 = \frac{1}{2}$ $x^{\frac{1}{2}} = 3 \quad x = 9$ or $x = 5$</p>	M1,A1 (2) M1A1 (2) M1 M1A1 (3) M1A1 B1 (3)
7	<p>(a) $A = 2(10x^2 + 5xh + 2xh)$</p> <p>(b) $(V =) 10x^2 h = 500$ $A = 20x^2 + 14x \times \frac{50}{x^2}$ $A = 20x^2 + \frac{700}{x}$</p> <p>(c) $\frac{dA}{dx} = 40x - \frac{700}{x^2}$ $\frac{dA}{dx} = 0 \quad x^3 = \frac{700}{40} \quad (x = 2.596\dots)$ $A_{\min} = 20 \times 2.596^2 + \frac{700}{2.596} = 404.4\dots = 404 \text{ cm}^3$</p> <p>(d) $\frac{d^2 A}{dx^2} = 40 + \frac{700 \times 2}{x^3}$ $x > 0 \Rightarrow \frac{d^2 A}{dx^2} > 0 \quad \therefore \text{min } A \text{ at } x = 2.596\dots$</p>	B1 (1) B1 M1 A1 (3) M1 M1A1 M1A1 (5) M1 A1ft (2)
8	<p>(a) $(15x + 6)(x + 4) = (6x - 3)^2$ $15x^2 + 66x + 24 = 36x^2 - 36x + 9$ $21x^2 - 102x - 15 = 0 \quad 7x^2 - 34x - 5 = 0$ $(7x + 1)(x - 5) = 0 \quad x = -\frac{1}{7} \quad x = 5$</p> <p>(b) $x = 5 \quad r = \frac{6x-3}{15x+6} = \frac{27}{81} = \frac{1}{3}$ $x = -\frac{1}{7} \quad r = \frac{-\frac{6}{7}-3}{-\frac{15}{7}+6} = \frac{-27/7}{27/7} = -1$</p> <p>(c) $r = \frac{1}{3} \quad a = 81 \quad S_{\infty} = \frac{a}{1-r} = \frac{81}{2/3} = \frac{243}{2} = 121\frac{1}{2}$</p> <p>(d) $S_n = \frac{81(1 - (\frac{1}{3})^n)}{\frac{2}{3}} = \frac{243}{2}(1 - (\frac{1}{3})^n)$ $\% \text{ error} = \frac{(-) \frac{243}{2} \times (\frac{1}{3})^n}{\frac{243}{2}} \times 100\% = (-)100(\frac{1}{3})^n \%$</p>	M1 M1 A1A1 (4) M1A1ft A1ft (3) B1M1A1 (3) M1 M1A1ft (on r) (3)

9	<p>(a) $\cos 2A \equiv \cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A) = 2\cos^2 A - 1$</p> <p>(b) $\sin 2A = 2\sin A \cos A$</p> <p>(c) $\cos 3A = \cos(2A + A) = \cos 2A \cos A - \sin 2A \sin A$ $= (2\cos^2 A - 1)\cos A - 2\sin^2 A \cos A$ $= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$ $= 4\cos^3 A - 3\cos A$</p> <p>(d) $\cos 3x = 0.6$ $3x = 53.13^\circ, 306.86^\circ, 413.13^\circ$ $x = 17.7^\circ, 102.3^\circ, 137.7^\circ$</p> <p>(e) $\frac{1}{4} \int_0^{\frac{\pi}{3}} (\cos 3\theta + 3\cos \theta) d\theta, = \frac{1}{4} \left[\frac{1}{3} \sin 3\theta + 3 \sin \theta \right]_0^{\frac{\pi}{3}}$ $= \frac{1}{4} \left[\frac{1}{3} \sin \pi + 3 \sin \frac{\pi}{3} - 0 \right] = \frac{3}{4} \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8} \quad a = 3, b = 8, c = 3$</p>	<p>M1A1 (2)</p> <p>B1 (1)</p> <p>M1 M1</p> <p>M1</p> <p>A1 (4) M1</p> <p>M1 A3,2,1,0 (5)</p> <p>M1,M1A1</p> <p>M1A1 (5)</p>
10	<p>(a) $(-2, 0)$ on curve: $-8 + 4p - 2q + 6 = 0$ $4p - 2q = 2 \quad 2p - q = 1$ $(2, -4)$ on curve: $8 + 4p + 2q + 6 = -4$ $2p + q = -9$ Add: $4p = -8$ $p = -2 \quad q = -5$</p> <p>(b) $f(x) = (x+2)(x^2 - 4x + 3) = (x+2)(x-3)(x-1)$ D is $(1, 0)$ E is $(3, 0)$</p> <p>(c) $y = x^3 - 2x^2 - 5x + 6 \quad \frac{dy}{dx} = 3x^2 - 4x - 5$ $x = 2 \quad \frac{dy}{dx} = 12 - 8 - 5 = -1$ grad normal = 1 eqn. normal: $y + 4 = x - 2 \quad (y = x - 6)$</p> <p>(d) $\int_1^2 (x^3 - 2x^2 - 5x + 6) dx = \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x \right]_1^2$ $= \left(4 - \frac{16}{3} - 10 + 12 \right) - \left(\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6 \right) = -2\frac{5}{12}$ Normal cuts x-axis at $x = 6$ Area $\Delta = \frac{1}{2} \times 4 \times 4 = 8$ Total area = $8 + 2\frac{5}{12} = 10\frac{5}{12}$ units²</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1A1 (5) M1</p> <p>A1A1 (3)</p> <p>M1 A1ft</p> <p>B1ft B1 (4)</p> <p>M1</p> <p>M1A1</p> <p>M1A1</p> <p>B1ft (6)</p>

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Paper 2

1	$b^2 - 4ac > 0$ $4p^2 - 4(10 - 3p) > 0$ $4p^2 + 12p - 40 > 0$ $p^2 + 3p - 10 > 0$ $(p + 5)(p - 2) > 0$ crit. values $p = -5$ $p = 2$ $p < -5$ $p > 2$	M1 M1A1 A1ft (4)
2	$\sum_{r=5}^{195} r - \sum_{r=1}^{39} 5r$ $= \frac{191}{2}(5 + 195) - 5 \times \frac{39}{2}(1 + 39), = 19100 - 3900 = 15200$	M1A1 M1A1, A1 (5)
3	Grad. line $= \frac{12}{-6} = -2$ Grad. perp. $= \frac{1}{2}$ Mid pt. is $(8, 3)$ Eqn. perp. $y - 3 = \frac{1}{2}(x - 8)$ $2y = x - 2$	M1 A1 B1 M1A1 (5)
4	$y = 0$ $x^2 = 9$ $x = \pm 3$ $V = \int_{-3}^3 \pi y^2 dx = \int_{-3}^3 \pi(9 - x^2)^2 dx = \pi \int_{-3}^3 (81 - 18x^2 + x^4) dx$ $= \pi \left[81x - 6x^3 + \frac{x^5}{5} \right]_{-3}^3$ $= \pi \left[\left(81 \times 3 - 6 \times 27 + \frac{3^5}{5} \right) - \left(-81 \times 3 + 6 \times 3^3 - \frac{3^5}{5} \right) \right] = 814$	B1 M1 M1A1 M1A1 (6)
5	(a) $v = t^2 - 2t + 9$ $\frac{dv}{dt} = 2t - 2$ $t = 3$ accel. $= 4\text{m/s}^2$ (b) $s = \int (t^2 - 2t + 9) dt = \left[\frac{t^3}{3} - t^2 + 9t \right]_0^6$ $= 72 - 36 + 54 = 90\text{m}$	M1 A1 (2) M1A1 M1A1 (4)
6	(a) (i) $y = 3$ (ii) $x = -1$ (b) (i) $y = 0$ $3 = \frac{2}{x+1}$ $x = -\frac{1}{3}$ $(-\frac{1}{3}, 0)$ (ii) $x = 0$ $y = 1$ $(0, 1)$ (c) 	B1B1 (2) B1 B1 (2) G1 G1ft G1ft (3)

7	<p>(a) $\alpha + \beta = -k$ $\alpha\beta = -5$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = k^2 + 10$ $\alpha^2\beta^2 = 25$</p> <p>(b) $5(k^2 + 10) - 7 \times 25 = 0$ $5k^2 = 175 - 50$ $k^2 = 25$ $k = \pm 5$</p> <p>(c) $k = 5$ $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{25+10}{25} = \frac{7}{5}$ $\frac{1}{\alpha^2\beta^2} = \frac{1}{25}$ Eqn. $x^2 - \frac{7}{5}x + \frac{1}{25} = 0$ $25x^2 - 35x + 1 = 0$</p>	<p>B1 M1A1</p> <p>B1 (4)</p> <p>M1</p> <p>A1 (2)</p> <p>M1A1</p> <p>B1</p> <p>M1 A1 (5)</p>
8	<p>(a) $\sin \theta = \frac{1}{4}$ ($\sin \theta = -\frac{5}{2}$ not poss.) $\theta = 0.253, 2.89$</p> <p>(b) $(2\theta - \frac{\pi}{3}) = 1.176, 4.317$ $2\theta = 2.223, 5.364$ $\theta = 1.11, 2.68$</p> <p>(c) $9(1 - \cos^2 \theta) - 9\cos \theta = 11$ $9\cos^2 \theta + 9\cos \theta + 2 = 0$ $(3\cos \theta + 1)(3\cos \theta + 2) = 0$ $\cos \theta = -\frac{1}{3}, \cos \theta = -\frac{2}{3}$ $\theta = 1.91, \theta = 2.30$</p>	<p>M1 A1A1 (3)</p> <p>M1 M1 A1A1 (4)</p> <p>M1</p> <p>M1A1 A1,A1 (5)</p>
9	<p>(a) $a + 2d = \log pq^4$ $a + 4d = \log pq^8$ $2d = \log pq^8 - \log pq^4$ $= \log \frac{pq^8}{pq^4} = \log q^4 = 4\log q$ $d = 2\log q$ $b = 2$</p> <p>(b) $a = \log pq^4 - 4\log q = \log \frac{pq^4}{q^4} = \log p$</p> <p>(c) $S_n = \frac{n}{2} \{2\log p + 2(n-1)\log q\}$ $= n \{ \log pq^{n-1} \}$ $s = n$ $r = n-1$</p> <p>(d) $S_{16} = 16\log pq^{15}$ $S_4 = 4\log pq^3$ $16\log pq^{15} = 40\log pq^3$ $2\log p + 30\log q = 5\log p + 15\log q$ $3\log p = 15\log q, \log p = 5\log q$</p>	<p>M1</p> <p>M1</p> <p>A1 (3) M1A1 (2)</p> <p>M1A1</p> <p>M1A1 (4)</p> <p>M1A1ft</p> <p>M1A1 (4)</p>

10	<p>(a) $\left(1 + \frac{x}{2}\right)^{\frac{1}{5}} = 1 + \left(\frac{1}{5}\right)\left(\frac{x}{2}\right) + \frac{\left(\frac{1}{5}\right)\left(-\frac{4}{5}\right)}{2!}\left(\frac{x}{2}\right)^2$ $= 1 + \frac{x}{10} - \frac{x^2}{50}$</p> <p>(b) $\left(1 - \frac{x}{2}\right)^{-\frac{1}{5}} = 1 + \left(-\frac{1}{5}\right)\left(-\frac{x}{2}\right) + \frac{\left(-\frac{1}{5}\right)\left(-\frac{6}{5}\right)}{2!}\left(-\frac{x}{2}\right)^2$ $= 1 + \frac{x}{10} + \frac{3x^2}{100}$</p> <p>(c) $x < 2$</p> <p>(d) $\left(\frac{2+x}{2-x}\right)^{\frac{1}{5}} = \left(\frac{1+\frac{x}{2}}{1-\frac{x}{2}}\right)^{\frac{1}{5}} = \left(1 + \frac{x}{10} - \frac{x^2}{50}\right)\left(1 + \frac{x}{10} + \frac{3x^2}{100}\right)$ $= 1 + \frac{x}{5} + \frac{x^2}{50}$</p> <p>(e) $\int_0^{0.5} \left(\frac{2+x}{2-x}\right)^{\frac{1}{5}} dx = \int_0^{0.5} \left(1 + \frac{x}{5} + \frac{x^2}{50}\right) dx$ $= \left[x + \frac{x^2}{10} + \frac{x^3}{150}\right]_0^{0.5}$ $= 0.5 + \frac{0.5^2}{10} + \frac{0.5^3}{150} = 0.5258$</p>	<p>M1 A2,1,0 (3)</p> <p>M1 A2,1,0 (3)</p> <p>B1 (1)</p> <p>M1 M1A1 (3)</p> <p>M1A1ft M1A1 (4)</p>
11	<p>(a) $AC = \ddot{O} (12^2 + 12^2) = 12 \ddot{O} 2$ $VP = 6 \ddot{O} 2 \tan 45 = 6 \ddot{O} 2$</p> <p>(b) $VA^2 = (6 \ddot{O} 2)^2 + (6 \ddot{O} 2)^2 = 144$ $VA = 12 \text{ cm}$</p> <p>(c) $DX^2 = 12^2 - 6^2$ $DX = \ddot{O} 108 = 6 \ddot{O} 3$</p> <p>(d) $\tan \theta = \frac{6 \ddot{O} 2}{6} \quad \theta = 54.7^\circ$</p> <p>(e) Identify the angle $\cos \phi = \frac{(6 \ddot{O} 3)^2 + (6 \ddot{O} 3)^2 - (12 \ddot{O} 2)^2}{2 \times 6 \ddot{O} 3 \times 6 \ddot{O} 3}$ $= -\frac{1}{3}$ $\phi = 109.5^\circ$</p>	<p>M1A1 M1A1 (4)</p> <p>M1A1ft A1 (3)</p> <p>M1A1 A1 (3)</p> <p>M1A1A1 (3)</p> <p>B1 M1A1ft A1 (4)</p>

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