

# MATHEMATICS D (CALCULATOR VERSION)

Paper 4024/11

Paper 11

## General comments

To succeed on this paper, candidates need to have completed full subject coverage, remember necessary formulae, show all necessary working clearly and use a suitable level of accuracy.

There were questions on the paper accessible to all candidates but there were many candidates who attempted few questions or had difficulties knowing how to approach a significant number of the topics. There were a few candidates who stopped working well before the end of the paper but this was most likely due to the difficult nature of the questions, rather than a lack of time to complete the paper.

Candidates should ensure they read the questions carefully as errors were made, particularly in **Question 3(a)** and **Question 9**, as a significant number of candidates answered a different question from the one asked.

The standard of presentation was generally good. To ensure the highest possible marks are scored, candidates should place only one answer on the answer line. It is important that sufficient working is shown, as part marks are often available so there is no need to erase working; the working should be placed in the main body of the script and not in the margins. It is often difficult to read answers where the original pencil answer has been overwritten. Any errors made in the working and on the answer line should be deleted and replaced, rather than overwritten as it can be very difficult to read which value is the intended answer.

## Comments on specific questions

### Question 1

(a) This part of this question on fractions was generally answered well. Errors included incorrect multiplications in the fractions resulting in an answer of  $-\frac{17}{30}$  and answers left as  $1 - \frac{13}{30}$ .

(b) This part was generally well answered. Attempts were made, at times, to cancel  $\frac{8}{45}$ .

Answers: (a)  $\frac{17}{30}$  (b)  $\frac{8}{45}$

### Question 2

- (a) The common error in this question on the order of operations was the incorrect use of the rules of arithmetic, resulting in an answer of 0.27.
- (b) There were a good number of correct answers with the common error being the incorrect placing of the decimal point, with  $\frac{3}{2}$  or 1.5 and 0.015 seen. There were occasionally problems in the division with  $9 \div 6 = 1$  remainder 3 changed to 1.3. A few left the answer as  $\frac{0.3}{0.02}$ .

Answers: (a) 0.76 (b) 15

### Question 3

- (a) Many candidates read this question on ratios carefully and realised that the difference between the amounts was required. Others assumed the question simply asked for Brenda's share.
- (b) The correct formula for simple interest was generally known but some did forget to divide by 100. There were a few attempts to use compound interest.

Answers: (a) 120 (b) 16

### Question 4

Many candidates scored at least 1 mark, with 3 lengths in the correct order. The common errors were in reversing 2300 and  $2\frac{1}{4}$ , and in placing 0.021 as the smallest value.

Answer: 220  $2\frac{1}{4}$  2300 0.021

### Question 5

- (a) Candidates were not always sure which way round to subtract the values in this question involving time, with 02 30 a common wrong answer. Other common wrong answers were 10.30 pm (or 22 30) or 09 30.
- (b) The correct multiplication was nearly always attempted in this question on currency conversions, but errors were made in positioning the decimal point and also in giving an answer of 338.000 which is not an acceptable way to express money in pounds.

Answers: (a) 21 30 or 9.30 pm (b) 338

### Question 6

- (a) Many were able to write the answer in Standard Form. Errors included  $34 \times 10^{-6}$  and  $3.4 \times 10^5$ .
- (b) A number of candidates knew the rules about addition of indices, whilst others converted the given values to ordinary numbers, but many left the answer as  $20 \times 10^{15}$ .

Answers: (a)  $3.4 \times 10^{-5}$  (b)  $2 \times 10^{16}$

### Question 7

- (a) The topic of approximating the given values in order to estimate answers did not seem to be fully understood, with a number of candidates giving no response. Using the correct estimates of 72 and 3 resulted in a significant number of answers of  $\sqrt{24}$ , with candidates unable to proceed any further. Answers were not always rounded to the nearest whole number, with both decimal answers, such as 4.9, and surds, such as  $2\sqrt{6}$ , being given. Many candidates attempted long division or, if using  $\frac{22}{7}$  for  $\pi$ , some multiplication also.

- (b) Writing 0.005 as  $\frac{5}{1000}$  or equivalent, in order to use the given cube roots, was not well understood. Common errors included 1.7 and 0.0017 but many answers bore no resemblance to any of the 3 given cube roots.

Answers: (a) 5 (b) 0.17

### Question 8

Those candidates who realised the need to find the total for A, B and C first were generally successful, although some answers were spoilt by poor arithmetic. Many candidates simply found  $(40 + 48) / (2 \text{ or } 4)$  giving answers of 44 or 22.

Answer: 42

### Question 9

The common error was to use 'y is directly proportional to x', rather than inversely proportional. Those who set their work out logically, starting with  $y = k/x$ , generally scored at least 1 mark for  $k = 4$ . Dealing then with division by  $\frac{1}{7}$  did cause some problems with a number of candidates giving their answer as  $\frac{4}{7}$ . Finding  $4 \times 7$  was sometimes incorrect with 21, 32 and 35 being common errors.

Answer: 28

### Question 10

The topic of bearings did not appear to be well understood.

- (a) There was often no working shown. Wrong answers of 135 (from  $75 + 60$ ) and 285 (from  $360 - 75$ ) were common.
- (b) Answers were often given as less than  $90^\circ$ , even though the diagram indicated a larger size.

Answers: (a) 135 (b) 195

### Question 11

- (a) There were a significant number of correct answers given for the mode. The common error was to give the highest frequency of 5 as the answer.
- (b) The idea of listing numbers to find the median was known but an incorrect answer of 2 was very common either from listing 1, 1, 2, 3, 5, or 0, 1, 2, 3, 4. The mean of 0, 1, 2, 3 and 4 was sometimes calculated (again 2). There were a number of answers given without evidence of working.

Answers: (a) 3 (b) 2.5

### Question 12

- (a) The left hand branches in this question on probability tree diagrams were often correct but those on the right often contained answers such as  $\frac{0}{4}$  or  $\frac{1}{2}$ . The pairs of branches did not always total 1. Some answers were whole numbers or colours.
- (b) The topic of combining probabilities was not well understood. Answers were often given without working.

Answers: (a)  $\left(\frac{1}{4} \text{ and } \frac{3}{4}\right)$ , (0 and 1) and  $\left(\frac{1}{3} \text{ and } \frac{2}{3}\right)$  (b)  $\frac{1}{4}$

### Question 13

The topic of bounds continues to be not well understood.

- (a) There were a significant number of correct answers, with common wrong answers of 1.95 and 1.9.
- (b) There was little recognition that bounds were needed to find the total weight of the cans so most answers used  $350 \text{ g} \times 20$  with 5 g then subtracted, rather than  $345 \text{ g} \times 20$ . Those who recognised there was also a change of units needed before adding on the weight of the box were often successful. There were a number of candidates who gave no response to this part of the question.

Answers: (a) 1.5 (b) 8.4

### Question 14

- (a) (i) This part of this question on indices was generally well answered. The common error was to write  $5^0$  as 0.
- (ii) This part was sometimes correct. The common incorrect answer was  $-\frac{16}{9}$ .
- (b) There were some correct answers. Common wrong answers were  $2x^6$ ,  $8x^2$  and  $6x^6$ .

Answers: (a)(i) 1 (ii)  $\frac{9}{16}$  (b)  $8x^6$

### Question 15

- (a) Candidates found it difficult to build up an answer from the relevant parts of the diagram given in this question on compound shapes. Some included the line across the lawn as they split the shape into two rectangles. 28 was a common wrong answer, often given without working. Occasionally the area was found.
- (b) Candidates were generally more successful in finding the area of the path.
- (c) Recognition that 4 squares were needed for each square metre was rare. There were a number of candidates who gave no response.

Answers: (a) 36 (b) 28 (c) 112

### Question 16

- (a) The common errors in this question on matrices were missing out the determinant or finding  $\frac{1}{3} \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ .
- (b) The transformation of one-way stretch was not well understood and this part was often left blank. Some recognised the transformation was a stretch rather than a shear (or another transformation) but few were able to give any more detail or the detail was attached to an incorrect transformation. A number of answer spaces were blank.

Answers: (a)  $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{pmatrix}$  or  $\frac{1}{3} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$  (b) one way stretch, parallel to the x axis (or y axis invariant), scale factor 3.

### Question 17

- (a) The inequality  $x > 1$  was recognised more often than  $x + y < 9$ . Some candidates used  $\geq$  when the given inequalities were  $>$ . Some answers bore no resemblance to the lines whose equations were given in the question.
- (b) Many candidates counted the points of intersection of the grid lines, not realising that there were other integer points in between, thus giving common wrong answers of 4 or 3.

Answers: (a)  $x > 1$ ,  $x + y < 9$  (b) 10

### Question 18

- (a) This part of the question on algebraic factorisation was generally well answered.
- (b) This part was often correct. Sometimes the expression was re-written to make the squared term positive, which gave the negative of the correct answer. Candidates should be aware that changing signs can only be done if there is an equation to solve.
- (c) This part, involving factorising a quadratic, caused more problems. This expression was also sometimes re-written to make the squared term positive. Attempts at the quadratic formula were also made.

Answers: (a)  $5p(4 + 5p)$  (b)  $(3 - 2t)(3 + 2t)$  (c)  $(9 - x)(1 + 4x)$

### Question 19

Some good answers, clearly explained, were seen to this question which required knowledge of the internal angles of polygons, but the majority of candidates could improve on their knowledge of polygons, with a number not attempting this question. Incorrect collection of the  $x$  terms caused some problems. Errors in method included equating their angle sum to 360, whether they chose the hexagon or the pentagon. It was not always clear where  $5x = 360$  was derived from; it was often simply using the sum of the given 3 angles equated to 360. Although the use of  $(n - 2) \times 180$  was seen, some tried to use  $\frac{(n - 2) \times 180}{n}$  or  $(n + 2) \times 180$  to find the total sum of the interior angles.

Answer: 72

### Question 20

- (a) This part of the question on functions and their inverses was quite often correct. Sometimes the answer was left in terms of  $y$  or an equation was solved.
- (b) This was one of the more demanding questions on the paper and few candidates were successful. Answers were often left blank. A part mark was sometimes scored for the first step of working or recognition that  $f(-9) = -3$ . This value of  $-3$  was often given for A, with  $\frac{t+3}{2}$  given for B as candidates did not realise that there were two constant terms ( $-3$  and  $\frac{3}{2}$ ) to collect together to find A.

Answers: (a)  $2x - 3$  (b)  $A = -\frac{3}{2}$   $B = \frac{1}{2}$

### Question 21

- (a) Many recognised that the set was  $\{1,2,3,4,5,6,7\}$  and some were then able to give  $n$  as 7.
- (b) Candidates were not always aware that each letter should appear in only one section of the Venn diagram. The letter  $p$  was most often correct with  $r$  being the least often correct, appearing frequently in more than one region and as the letter  $z$ .

Answers: (a) 7 (b) Correct  $p$ ,  $q$  and  $r$

### Question 22

Circle theorems are not well understood. If possible, candidates should not use answers to previous parts as these may be incorrect. It is better to look for a single step answer using the given angles. Angles inside triangles cannot be greater than 180.

- (a) Candidates often recognised that triangle BCT was isosceles but sometimes gave the wrong two base angles. Others thought that angle ACT was 90, giving an answer of 86.
- (b) The answer of 52 could be found directly using the angles in the same segment.
- (c) The answer could be found directly using the alternate segment theorem.
- (d) The answer could be found directly from the alternate segment theorem for angle 52 and angles on a straight line at B.

Answers: (a) 68 (b) 52 (c) 56 (d) 72

### Question 23

Candidates need to develop a better understanding of the topic of speed–time graphs.

- (a) This part was frequently correct. Wrong answers included (-) 20, -0.5, and -2.5 (from  $\frac{0-50}{20}$ ).
- (b) Answers were often given without evidence of working and some answer spaces were left blank. Some candidates tried to use  $D = S \times T$  rather than proportion. A common wrong answer was 30.
- (c) Those candidates who recognised that the distance was given by the area under the graph and used either area of a trapezium or areas of a rectangle and triangle were usually successful. Some candidates only found the area of the triangle as 400 or gave answers of 200 or 800.
- (d) This part was more successful than the previous 2 parts, although a significant number of candidates gave no response. Answers were often given without working. A common wrong answer was from using acceleration =  $1 = \frac{50-u}{20}$ .

Answers: (a) (-) 2 (b) 20 (c) 600 (d) 40

### Question 24

- (a) Finding the coordinates of the midpoint was frequently correct.
- (b)(i) A common error in this question on vectors was in subtracting  $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$  from  $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ . There were a number of answer spaces left blank.

- (ii) Candidates who realised that they needed to add  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$  and then find the length of  $\overrightarrow{AC}$  were generally able to score at least 1 mark. There were sometimes problems with signs when finding  $\overrightarrow{AB}$ . A common error for those who attempted the question was to find the lengths of AB and BC and then add them together. Others tried to use  $\mathbf{a} + \mathbf{b}$  or their answer in (a) for  $\overrightarrow{AB}$ .

Answers: (a) (3, 5) (b)(i) (4, 6) (ii) 29

### Question 25

- (a) Algebraic errors included answers of  $2n - 1$  and  $4n - 1$  or the correct answers reversed. There were a significant number of numerical answers, usually from finding 4 more numerical rows thus giving answers 22 and 24.
- (b) Those candidates who completed the table generally gave at least one correct value.
- (c) Although credit was given to those who were able to follow through their algebraic answer in (a), this part was rarely correct, with many numerical answers. Brackets were an essential part of the answer, although expanded versions were acceptable.
- (d) Very few candidates were able to attempt an algebraic proof – most answers gave numerical examples. To score full marks, all relevant brackets needed to be shown – the common error was to omit the bracket round  $(9n^2 - 6n)$  resulting in the wrong sign for  $6n$ .

Answers: (a)  $3n - 2$   $(3n - 1)$   $3n$  (b) 121 and 120 (c)  $3n(3n - 2)$  (d)  $(3n - 1)^2 - 3n(3n - 2)$  leading correctly to 1

### Question 26

- (a) Those who realised the angle was reflex were usually within range.
- (b) The diagram often had no construction lines shown.  $AD$  was drawn as 10 cm but  $CD$  was not drawn equal to  $CB$ .
- (c)(i) This was the most successful part of the construction, with a number of correct perpendicular bisectors of  $AB$ .
- (ii) Candidates frequently did not recognise that the construction was an angle bisector, with the perpendicular bisector of  $AC$  drawn a number of times.
- (d) Shading in the correct place was rare.

Answers: (a)  $264^\circ$  to  $268^\circ$  (b) Acceptable  $ABCD$  (c)(i) Acceptable perpendicular bisector (ii) Acceptable bisector of angle  $ABC$  (d) Correct region (top left hand corner) shaded.

### Question 27

- (a) Many candidates scored at least one mark in this question on matrices. The common error was to give -7 as the top right hand element.
- (b)(i) Common wrong answers were  $2 \times 2$  and  $2 \times 1$  matrices.
- (ii) It was rare to see a correct answer and this part was often blank. Some candidates tried to find  $(8 \ 5) \div \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$  and others wrote  $X$  as  $(x \ x)$  or  $\begin{pmatrix} x \\ y \end{pmatrix}$ . Those forming a pair of simultaneous equations were more successful than those attempting to use the inverse (as the determinant was often given as 7 rather than 6).

Answers: (a)  $\begin{pmatrix} -3 & -1 \\ -2 & -1 \end{pmatrix}$  (b)(i) 1 row, 2 columns (ii)  $(4 \ 3)$

# MATHEMATICS D (CALCULATOR VERSION)

---

Paper 4024/12

Paper 12

## General comments

In order to do well in this paper, candidates need to have covered the whole syllabus; remember necessary formulae and facts; recognise, and carry out correctly, the appropriate mathematical procedures for a given situation; perform calculations accurately and show clearly all necessary working in the appropriate place.

It was noticeable that there were many scripts where candidates had not attempted many questions.

At times, candidates appeared not to read the questions carefully. They should be encouraged to pay careful attention to the wording, the numbers given, the units used and the units required in the answer.

Questions that proved particularly difficult were 10**(b)**, 13**(b)**, 18**(b)(c)**, 20**(b)**, 21**(b)**, 23**(d)** and 26**(c)(ii)**.

It was noticeable that a significant number of candidates need to improve their ability to estimate, approximate and use appropriate degrees of accuracy; and also to understand the integer class of number. Some candidates were very competent at performing standard techniques, and yet seemed to find it difficult to recognise the appropriate mathematical procedure required for a given situation

Candidates need to learn the difference between the algebraic terms “factorise”, “expand”, “solve” and “simplify”. They also need to improve their skills in manipulating fractions and in basic arithmetic. Work, that used correct methods, was spoilt by wrong calculations like  $94 - 58 = 26$ ;  $360/20 = 16$ ;  $9 \times 3 = 18$ , 12 or even 21. Careless mistakes like these were made by candidates of all abilities.

Presentation of the work was usually good. Candidates should bear in mind that it is to their advantage to make sure they provide sufficient working and that this working is set out neatly and legibly. Working makes it possible for marks, where they are available, to be awarded for correct methods and intermediate results.

A few candidates did not heed the instructions on the front page - to write in dark blue or black pen. Except for diagrams and graphs, candidates should not write in pencil. Nor must they overwrite pencil answers in ink, as this makes a double image which can be difficult to read. Some candidates wrote in pencil and even erased their workings, often producing a mass of rubber and paper debris that interfered with the clarity of other answers, particularly with a decimal point and a negative sign.

Candidates should be made aware that only their final answer to each question should be written in the answer space. Alternative offerings and working should not be written there. When an answer is to be changed, it is far better to delete and replace the original one rather than attempt to write over it.

Care must always be taken to ensure that answers obtained in the working are accurately transferred to the answer space.



### Comments on specific questions

#### Question 1

- (a) This part of this question on the order of the four operations showed that many candidates need to get an understanding of place value with decimal numbers. Attempts at evaluating  $8 + 2.6$  yielded 3.4, 34, 11.4. Some candidates seemed unaware of the rules regarding the order of operations and obtained 13, from  $(8 + 2) \times 1.3$ .
- (b) This questions on converting between decimals and fractions was quite well answered, though a significant number of candidates seemed unaware of how to convert a decimal to a fraction. The usual wrong answers were  $\frac{3}{5}$ ;  $\frac{3}{500}$ ;  $\frac{6}{100}$ ;  $\frac{6}{10}$ .

Answer: (a) 10.6      (b)  $\frac{3}{50}$

#### Question 2

- (a) Attempts at this question showed that some candidates need to improve their ability to manipulate fractions and to work with a common denominator. Many did obtain the correct answer.
- (b) Most candidates noted that  $3^0 = 1$  and gave the correct answer. Others left their answer as  $4^1$  or else gave 6, or  $6^1$ .

Answer: (a)  $2\frac{11}{12}$       (b) 4

#### Question 3

- (a) Many candidates overlooked one or two of the unmarked sides in this question on finding the perimeter of a compound shape, or calculated their lengths incorrectly, or else made an error in evaluating  $9 + 8 + 3 + 4 + 6 + 4$ . A common wrong answer was 28, obtained by missing out side 6.
- (b) Some candidates divided the area of the card by the area of one square, not realising that squares of side 4 cm do not fit exactly into the card, and obtained the answer 12. The other common wrong answer was 4, the area of one square.

Answer: (a) 34      (b) 10

#### Question 4

- (a) This question on functions was often answered correctly. Common errors were to obtain  $5 - 3\frac{1}{2}$  from  $5 + 3\left(-\frac{1}{2}\right)$ ; to convert  $\frac{7}{2}$  to a mixed fraction incorrectly; to give  $\frac{3}{2}$ .
- (b) Those candidates competent with algebra and familiar with inverse functions gave a correct answer, although a few gave an answer that was not expressed in terms of  $x$ . The usual wrong answers were  $\frac{5-x}{3}$ ;  $\frac{x+5}{3}$ ;  $\frac{5x}{3}$ .

Answer: (a)  $3\frac{1}{2}$       (b)  $\frac{x-5}{3}$

### Question 5

Most candidates attempted this part of this question on ordering values sensibly, converting the fractions to decimals, or to fractions with a denominator of 20. The usual errors were to place  $-1$  in the centre position; to place  $-\frac{17}{20}$  and  $-\frac{4}{5}$  the wrong way round; to write the first three numbers as  $-\frac{4}{5}$ ,  $-\frac{17}{20}$ ,  $-1$ .

Answer:  $-1$ ,  $-\frac{17}{20}$ ,  $-\frac{4}{5}$ ,  $0$ ,  $\frac{3}{4}$

### Question 6

- (a) In this question on speed, distance and time most candidates could correctly calculate the number of hours taken. A few gave 180 minutes; others divided 3 by 90.
- (b) This question on average speed was often answered correctly. Some candidates did not read the question carefully, and found the average speed from  $B$  to  $C$  instead of from  $A$  to  $C$ . Other answers, such as 40 or 80, showed that there is a need for understanding *average speed* as (*total distance travelled*) / (*total time taken*) and **not** as the *average*, or the *sum*, of the two speeds.

Answer: (a) 3 (b) 35

### Question 7

This question on expanding brackets was answered well by those who knew what the instructions meant.

- (a) Common wrong answers were  $8k - 1$ ;  $7k + 1$ ;  $7k - 1$ ;  $k = -1/8$ ;  $4k + 1$ ;  $4k - 5$ .
- (b) Some candidates expanded the brackets and simplified the expression obtained correctly. However, sometimes this result was then factorised, not always correctly, and these factors were written in the Answer Space. Sometimes  $2x$  was written instead of  $2x^2$ , or  $2x^2 + 5x$  became  $7x$  or  $7x^3$ . Other common wrong answers were  $2x^2 + 5x + 12$ ;  $2x^2 + x + 12$ ;  $7x^2 - 12$ ;  $7x - 12$ .

Answer: (a)  $8k + 1$  (b)  $2x^2 + 5x - 12$

### Question 8

- (a) This part showed that many candidates need to improve their understanding of bearings. The usual wrong answers were  $105^\circ$ ;  $285^\circ$ ;  $195^\circ$  from  $180 + (90 - 75)$ .
- (b) This part showed that many candidates need to improve their understanding of the 24-hour clock and skills in working with time. Answers were very varied, usually with 6, 8 or 9 hours and 7, 13, 33 or 47 minutes. Some candidates used 100 minutes in 1 hour. Others gave 16 hours 7 minutes from 21 40 – 5 33.

Answer: (a)  $255^\circ$  (b) 7 hours 53 minutes

### Question 9

- (a) Most did not realise that the mode had to be greater than the 5 given in the table. The usual answer was 0.
- (b) Many did not realise that a median of 1 means that there are as many as  $(5 + 4 + 2)$  values above 1 as there are below it. Common wrong answers were 1; 2, from the middle value of 0, 1, 2, 3, 4; 5, the middle value of  $x$ , 1, 5, 4, 2. Some started with  $(12 + 1)/2$ .

Answer: (a) 6 (b) 11

### Question 10

- (a) Most candidates seemed to have some idea of what was required in this question on prime factorisation, but errors in division frequently led to the wrong answer. Some gave  $2 \times 2 \times 5 \times 9$ . Others gave a list of some *divisors* of 180.
- (b) Many candidates did not attempt this part, or made a very spurious attempt. Some started from the beginning and obtained for example, 23 from  $3\sqrt{20}$ . Candidates were expected to use part (a) and write  $\sqrt{180}$  as  $6\sqrt{5}$  from  $2 \times 3 \times \sqrt{5}$ .

Answer: (a)  $2^2 \times 3^2 \times 5$  (b) 11

### Question 11

- (a) This question on indices was often answered correctly, with 8, from  $\frac{a}{4} = 2$ , being the usual wrong answer.
- (b) This question on fractional indices was less often correct. Common errors were to give  $b = \frac{1}{4}$ , from  $8b = 2$ ; or the answer 3; or  $-3$ .

Answer: (a) 6 (b)  $\frac{1}{3}$

### Question 12

This question was, on the whole, well attempted by those who noted “directly” and “square of  $x$ ”. The common error was to use direct proportion, obtaining the answer 24. Others used inverse variation; or the square root of  $x$ ; or made an error in evaluating  $\frac{32}{16}$ ; or having obtained  $y = 2x^2$ , went on to evaluate  $2 \times 3$ .

Answer: 18

### Question 13

Many candidates seem to lack understanding of the topic of upper bounds and the principles involved.

- (a) This upper bound was sometimes correct. Some gave 9.4. Others seemed to interpret “nearest tenth” as “nearest 10” and gave 14.4.
- (b) This question involving using the upper and lower bounds was rarely correct. Few realised that to get the greatest possible difference it is necessary to subtract the lower bound for Tom (7.5) from the upper bound for Sam (9.45). The common wrong answers were 1.4, from  $9.4 - 8$  or  $9.45 - 8.05$  or  $9.9 - 8.5$ ; 0.95 from  $9.45 - 8.5$ .

Answer: (a) 9.45 (b) 1.95

### Question 14

- (a) Most candidates attempted to complete the Venn diagram and the value of  $p$  was usually correct. Finding the value of  $q$  was more difficult. Some calculated  $n(H) + n(B) + n(S) - 60$ .

Many candidates made a good attempt at parts (b) and (c).

Some candidates need to be made aware of the meaning of  $n(\ )$  and ‘ (complement) when applied to Sets.

Others confused the number of candidates in each subset with the number of numbers written in the Venn diagram.

- (b) Common wrong answers were  $\{15, 5, 9, 6\}$ ; 4.

- (c) Common wrong answers were { 15, 3, 16}; 3; 38 (from S')

Answer: (a)  $p = 6, q = 4$  (b) 33 (c) 34

### Question 15

- (a) This question on factorisation was answered correctly by many. Common wrong answers were  $2p(8p + 2p)$ ;  $(4p + 2p)(4p - 2p)$ ;  $20p^3$ .
- (b) Most candidates could factorise a quadratic and knew how to tackle this question and usually obtained the correct factors.

Answer: (a)  $4p(4 + p)$  (b)  $(x + 2a)(y + 3a)$

### Question 16

Some candidates made a very superficial attempt at this question on probability and seemed to have difficulty in understanding the situation described.

- (a) There were many correct answers, mostly written as 0, when candidates realised that it was not possible to get a total of 3 with two cards. The answer  $\frac{1}{3}$  occurred frequently. Weaker candidates offered "Card B".
- (b) Attempts varied. Some gave only three entries, corresponding to AB (5), BC (7) and CA (6), overlooking the fact that order is important; some provided 9 entries, the extra ones being AA (4) BB (6) and CC (8); others gave fractions.
- (c) Better candidates gave the correct answer. Some gave "card B and card C". Some gave a value, commonly 2, that is greater than 1. Others did not attempt this part.

Answer: (a) 0 (b)

A	A	B	B	C	C
B	C	A	C	A	B
5	6	5	7	6	7

(c)  $\frac{\text{number of 7s}}{\text{total no. of outcomes}}$

### Question 17

The majority of candidates seemed to be reasonably familiar with standard form, though they often made errors in finding the correct index.

- (a) Some answered this part correctly. Others demonstrated a misunderstanding of the concept of significant figures by: counting all digits to the right of the decimal point as significant; replacing the last 2/3/4 digits with zeros; removing the zero place holder between the 4 and 5. Common wrong answers were 0.041; 0.040; 0.406; 0.0405; 0.040600; 40.589.
- (b) Many candidates found the addition of two numbers expressed in standard form difficult to calculate. Some reached 0.00068 and then found difficulty in writing this in standard form. Others multiplied the two numbers. Common wrong answers were based on 1.4; 4.8; 8.6 with varied powers of 10; or  $6.8 \times 10^4$ .
- (c) Some candidates listed the square numbers then still did not choose the closest. The numbers 81 and 25 were chosen frequently. Some used  $\sqrt{40}$  for  $\sqrt{35}$ . Others used long multiplication to evaluate numbers like  $5.9^2$  and  $9.8^2$ . Candidates were expected to approximate  $\sqrt{97}$  with the value 10, and  $\sqrt{35}$  with the value 6.  
Some candidates thought that taking a square root is the same as dividing by 2. Another invalid method was to write  $\sqrt{97} - \sqrt{35} = \sqrt{62}$ , occasionally then giving 8.

Answer: (a) 0.0406 (b)  $6.8 \times 10^{-4}$  (c) 4

### Question 18

Correct answers to all parts of this question on angles and triangles were rare. Candidates frequently made wrong assumptions.

There were many blank responses for parts **(b)** and **(c)**.

- (a)** Those who realised that triangles  $AQP$  and  $QCP$  are situated on the same base line,  $AQC$ , and had a common height from this line to the vertex  $P$ , were able to give the correct answer from  $6 \times \frac{2}{4}$ . Some correctly calculated the common height as 3 cm. Some fortuitously obtained the same answer by using  $QP = 3$  cm. Others obtained 1.5 by wrongly assuming that triangles  $AQP$  and  $CQP$  are similar.
- (b)** Those who realised that triangles  $ABC$  and  $QPC$  are similar usually obtained the correct answer. Other attempts tried, unsuccessfully, to use  $\frac{1}{2} \times \text{base} \times \text{height}$ .
- (c)** Few obtained the correct answer. Many attempts tried to use incorrect strategies with similar triangles or  $\frac{1}{2} \times \text{base} \times \text{height}$  and incorrect lengths.

Answer: **(a)** 3    **(b)**  $13\frac{1}{2}$     **(c)**  $4\frac{1}{2}$

### Question 19

- (a)** Most attempted this question on matrices sensibly, though there were often arithmetic errors in the scalar multiplication of  $\mathbf{M}$  and in its subtraction from the given matrix. The usual error was to multiply  $\mathbf{M}$  by  $-2$  and then subtract, effectively adding  $2\mathbf{M}$ .
- (b)** Most candidates were well trained in finding inverses. Common errors were to evaluate the determinant as 2; to find the wrong adjoint; to omit the determinant; to multiply, instead of divide, by the determinant. A few found the inverse of their answer to **(a)**.

Answer: **(a)**  $\begin{pmatrix} -6 & 0 \\ -2 & -6 \end{pmatrix}$     **(b)**  $\begin{pmatrix} 3 & -1 \\ 4 & 4 \\ 1 & 1 \\ 4 & 4 \end{pmatrix}$

### Question 20

- (a) (i)** Most made a reasonable attempt at this part of the question on inequalities, though some gave an answer in the form of coordinates. A common wrong answer was 1, from  $5 - 4$ .
- (ii)** This part was less well answered. Some candidates seemed to confuse the “ $k$ ” as being the constant “ $c$ ” in a straight line equation and used the point  $(1, 5)$  as lying on the line  $y = \frac{1}{2}x + c$  to obtain the value 4.5. The usual wrong answers were  $\frac{1}{2}$ ;  $(4, 2)$ .
- (b)** There were some completely correct answers. Some candidates confused  $x$  with  $y$  and gave  $a = 2$ ,  $b = 1$ ; others did not seem to understand this part and gave either multiple values, or coordinates of points, for the unknown quantities.

Answer: **(a)(i)** 4    **(ii)** 2    **(b)**  $a = 1, b = 2, c = 6$

### Question 21

- (a) Most candidates made a good attempt at this part of this question on matrix transformations, and found a matrix with the correct order, though sometimes with one or more incorrect elements. Others added elements to obtain a 2 by 2 or a 2 by 1 matrix.
- (b) Few candidates recognised that the transformation was a one-way stretch, and fewer could give a full description although the scale factor was often correct. Of those who attempted this part, most thought the transformation was a shear, or an enlargement. A few drew a letter T on the grid, with no description of a transformation.

Answer: (a)  $\begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$  (b) one way stretch, x-axis invariant, stretch factor  $\frac{1}{2}$

### Question 22

- (a) This question on coordinates was often answered correctly. Common wrong answers were (18, 2); (5.5, 1.5); (2, 1).
- (b) The correct answer was given most often, by those who attempted this part, with “trapezium”, or “rectangle”, being the usual wrong answers.
- (c) Many did not attempt this part of the question, which involved finding the area of the shape. Of those who recognised that the quadrilateral was a parallelogram, only a small proportion found the base (= 3 units), the height (= 9 units), and the area correctly. It was quite common to see attempts at using the formula for calculating the distance between two points, especially  $BB'$ .

Answer: (a) (11, 3) (b) parallelogram (c) 27

### Question 23

In general, attempts at this question on angles within circles showed that it is vital to look carefully at the diagram. Many candidates seemed to assume at least one of the following: that the quadrilateral  $ABOE$  was cyclic; that the arc  $BC$  subtends the angle  $\hat{p}$  at the circumference; that  $EA$  is parallel to  $DC$ ; that  $EA$  is parallel to  $OB$ ; that  $ED$  is parallel to  $BC$ . Some otherwise correct attempts were ruined by careless errors such as  $180 - 56 = 134$ .

Many candidates obtained answers that they should have realised were clearly the wrong size just by looking at the diagram. For example, giving an obtuse angle for something that was clearly acute. Although the diagram is not drawn to scale, it is not intended to mislead.

- (a) The common wrong answers were 118; 100.
- (b) The common wrong answer was 100.
- (c) Most candidates recognised the “angle in a semicircle” and answered this part correctly. Common wrong answers were 28; 121.
- (d) The common wrong answers were 31; 28; 100.

Answer: (a)  $124^\circ$  (b)  $118^\circ$  (c)  $31^\circ$  (d)  $38^\circ$

### Question 24

- (a) In this question on polygons, those who used the “sum of exterior angles is  $360^\circ$ ”, leading to  $n = \frac{360}{180 - 160}$  were more successful than those who attempted to use the result  $(n - 2) \times 180 = 160n$ . This result was often mis-quoted as  $(n - 2) \times 180 = 160$ , or mis-used as  $n - 360 = 160n$  from  $n - 2 \times 180 = 160n$ .

- (b) Some candidates did not connect part (b) to the regular polygon with an interior angle of  $160^\circ$ , as shown in the question layout. The statement on the first line applies to all parts of the question. It is essential to use  $\hat{A}BC = \hat{B}CD = 160^\circ$ . A few tried to use a polygon with three sides  $AB$ ,  $BC$ ,  $CD$ .
- (i) Usually answered correctly by those who used  $\hat{A}BC = 160^\circ$  and noted that triangle  $ABC$  is isosceles. Most candidates, however, did not do this.
- (ii) Attempts at this part, and the answers obtained, varied a great deal.

Answer: (a) 18    (b)(i)  $10^\circ$     (ii)  $20^\circ$

### Question 25

Attempts at this question showed that some candidates need to get a better understanding of the properties of a speed-time graph, and to appreciate that the “D-S-T triangle” is not a valid method when there is an acceleration.

- (a) Those who realised the the gradient of the line was required usually obtained a correct answer. Common wrong answers were  $\frac{4u}{10}$ ;  $\frac{3u-u}{t}$ .
- (b)(i) More able candidates realised that the area under the graph was required, and used this fact correctly. Common wrong answers were 20; 4, from  $s = \frac{d}{t} = \frac{40}{10}$ ;  $\frac{4}{3}$ , from  $3u \times 10 = 40$ .
- (ii) Often omitted. Many attempts drew a straight line to the point (10, 40), though not all of these started at (0, 0). Very few graphs were curved in the manner required.

Answer: (a)  $\frac{u}{5}$     (b)(i) 2    (b)(ii) continuous curve, concave upwards, from (0, 0) to (10, 40)

### Question 26

Answers to this question showed that many candidates need to be able to distinguish between the *position* of a term in a sequence and the *value* of the term.

For example, in the sequence 1, 3, 5, 7, 9, the 4th term has the value 7.

- (a) Of those who attempted this question, most gave an answer that involved 223, or  $\frac{2008}{9}$ . Few gave 2011, the *value* of the 223rd term.
- (b) There were many correct answers from those who started with  $(9 \times 10 + 4) - (9 \times 6 + 4)$ , and did not make a careless slip. The usual wrong answer was 4, from  $10 - 6$ .
- (c)(i) There were many correct answers from those who used  $(9x + 4) - (9y + 4)$ . Some candidates gave  $9x - 9y + 8$ , from  $9x + 4 - 9y + 4$ ; or  $x - y$ . Many did not attempt this part.
- (ii) Very few candidates gave an acceptable explanation.

Answer: (a) 2011    (b) 36    (c)(i)  $9x - 9y$     (ii) 123 is not a multiple of 9

### Question 27

A substantial number of candidates did not attempt this question on loci and constructions.

- (a) Usually correct, with (approx.)  $53^\circ$  the usual wrong answer. A few candidates interpreted  $\hat{A}BC$  as being the sum of the lengths  $AB + BC$ .

- (b) Those who read the question carefully produced an acceptable quadrilateral. Others drew a parallelogram, or else constructed  $BD$ , instead of  $AD$ , as being equal to  $AB$ .
- (c) There were some good loci constructed. Candidates should be made aware that the instruction “**inside** the quadrilateral  $ABCD$ ” implies that the loci is expected to go across the whole quadrilateral and not just a small part of it. The common errors were to construct a perpendicular bisector of  $AB$  or  $BC$  or to draw the diagonal  $BD$ .
- (d) Competent candidates identified  $P$  correctly, and obtained it to an acceptable degree of accuracy.

Answer: (a)  $127^\circ$  (b) correct  $ABCD$  (c)(i) circular arc, centre  $C$  (c)(ii) bisector of angle  $ABC$   
(d) approx. 2.24 cm



# MATHEMATICS D (CALCULATOR VERSION)

---

Paper 4024/21

Paper 21

## Key Messages

To succeed in this paper, candidates need to have completed full syllabus coverage, remember necessary formulae, show all necessary working clearly and use a suitable level of accuracy.

## General Comments

Many candidates continue to be unaware of the level of accuracy required in numerical questions. It is essential that at least 4 figure accuracy is maintained in calculations if the final answer is to be given to 3 significant figures. Premature approximation usually results in the loss of accuracy marks and this may be substantial when several questions are involved. Generally the scripts were well presented with candidates providing working to support their answers.

It is important that care is taken when transferring answers from the working to the answer line as mistakes can lead to the loss of marks.

Candidates are advised to check that their calculators are set in degree mode before evaluating trigonometrical expressions.

The questions which candidates had most difficulty with were 2(c)(ii), 2(c)(iii), 5(d)(i), 5(d)(ii), 6(b)(ii), 7(c), 8(e)(ii), 9(d), 10(c) and 11(a)(ii)

## Comments on Specific Questions

### *Section A*

#### **Question 1**

Throughout this question on trigonometry, candidates using the most direct methods for right-angled triangles were the most likely to obtain accurate answers. Correct trigonometrical reasoning was sometimes spoiled by poor calculator work.

- (a) The expected answer of 4.28 comes directly from  $4.5\cos 18$ . This was often achieved. Some candidates lost a mark due to the accuracy of their answer. They need to remember that answers are expected to be given to 3 significant figures, or at least to have written a more accurate answer in the working space prior to the final answer. The most common error was to use 'sin' instead of 'cos'.
- (b)(i) The accurate answer of  $36.0^\circ$  comes directly from  $\sin^{-1}(6/10.2)$ . This was often achieved, but those candidates who found BC first and then used their value of BC to calculate the angle were likely to lose 3-figure accuracy. More complex methods were also seen, such as the use of the cosine rule and the sine rule. Candidates should avoid these approaches when the situation clearly relates to a right-angled triangle.

- (ii) The expected strategy here was two applications of Pythagoras. The efficient way of doing this avoids finding BC. If BC is evaluated, it must be sufficiently accurate to maintain 3-figure accuracy for the complete calculation. This mark was easily lost with DC=11.7 leading to AD=5.7 frequently seen. Some candidates tried to find AD directly by treating triangle ABD as a right-angled triangle. Other errors included  $10.2 / 14.3 = 6 / AD$  and  $AD = 14.3 - 10.2$ . For some, the only mark scored was for subtracting 6 from their value for CD.

Answer: (a) 4.28 (b)(i)  $36.0^\circ$  (b)(ii) 5.68 or 5.69

### Question 2

- (a)(i) Several candidates scored full marks in this question on simplifying algebraic expressions. Some removed the brackets accurately but were then unable to simplify the resulting terms correctly. Others wrote  $1-5p$  as  $4p$ , whilst some non-existent brackets were seen by some candidates, leading to quadratic expressions.
- (ii) This question on inequalities was often well done. A common incorrect answer was  $x > -1$ . Some candidates made basic errors at the outset leading to  $2x > 5+3$ , for example.
- (b)(i) A mixed response was seen to this algebra question. Correct substitutions were often seen, but followed by wrong answers such as  $3A$ ,  $3A/A$  and  $2A$ .
- (ii) This question was worked to a successful conclusion by a good number of candidates. Those who did not score full marks usually had a correct start by reaching the stage  $xy = A + 2x$
- (c)(i)(ii) It was apparent here that a number of candidates did not seem to understand what was required. There were some attempts to work backwards from the answers. These were not convincing. These parts were omitted by a number of candidates. There were only a few derivations of the given equations that were convincing.
- (iii) Candidates who omitted or struggled with parts (i) and (ii) often gave up and did not attempt this part. These were relatively straightforward simultaneous equations to solve, and many candidates were successful. A number who successfully reached  $4x = 24$  were unable to obtain  $x = 6$ . Many candidates made this question more difficult than it needed to be by rearranging the equations and then making transposition errors.

Answers: (a)(i)  $10p + 1$  (ii)  $x < -1$  (b)(i) 3 (ii)  $\frac{A}{y-2}$  (c)(iii)  $x = 6$   $y = 31$

### Question 3

- (a)(i) Most candidates answered this question on percentages correctly. A common wrong answer was 170.
- (ii) A mixed response to this question on percentage profit with a fair number of correct solutions seen. Again, as in trigonometry, some candidates found it difficult to maintain the required 3 significant figure accuracy throughout the question. Common errors were to omit the subtraction of \$30 or to use a denominator of 170 or 200.
- (b) This question proved demanding. There were many confused attempts using the given amounts and percentages in incorrect combinations. Some used 115% rather than 85%, whilst others subtracted 160 from 647.50 instead of adding. There were a few successful attempts to tackle this problem algebraically by constructing an appropriate equation from the given data. This approach is to be recommended.

Answer: (a)(i) 30 (ii) 29.0 (b) 950

#### Question 4

- (a) (i) Several candidates obtained the correct answer of 20 in this question on angles, but some gave 40 or 140.
- (ii) This question was answered quite well. The meaning of the word 'bisect' was not generally understood.
- (iii) Most candidates sensed that the figure was a rectangle. Very few were able to show how the previous answers of  $20^\circ$  and  $70^\circ$  were crucial in justifying why one of the angles was  $90^\circ$ .
- (b) (i) Many candidates used the data well to show that the corresponding sides were proportional. This was spotted possibly more frequently than the equal vertically opposite angles, so that complete solutions were relatively rare.
- (ii) A good number of correct calculations were seen.

Answer: (a)(i)  $20^\circ$  (ii)  $70^\circ$  (iii) Rectangle (b)(ii) 1.8

#### Question 5

- (a) Candidates found this question on areas of parts of circles difficult with only a small number obtaining the correct answer. The two arcs of A were not generally seen as being equivalent to one semi-circle.
- (b) Similarly here for the two arcs of B. Even solutions adding 10 to part (a) were rarely seen.
- (c) (i) Candidates found this mark relatively easy to obtain. Some did not draw all the lines of symmetry.
- (ii) This part was answered well by the majority. Some did not understand the question and gave answers such as clockwise or  $90^\circ$ .
- (d) (i) Many did not realise that the area of the shaded shape was a quarter of the area of the square. Some attempted to calculate the areas of the two parts and then add them together. In this case one of the areas found was often incorrect.
- (ii) There were few successful solutions to this part. Although the mensuration of a circle is probably well known, not many candidates were able to use it effectively in the given context.

Answer: (a) 15.7 (b) 25.7 (c)(ii) 4 (d)(i) 25 (ii) 14.3

#### Question 6

- (a) The required technique for finding the mean of grouped data is well known here, but care is needed in its application. A good number of correct solutions were seen. Some candidates consistently used wrong values from within the intervals and then correctly divided by 80. Others divided by 6.
- (b) (i) This part on probability was well answered.
- (ii) This question on probability was not so well done. The use of  $\frac{24}{80}$  was a common error. Many candidates did not appreciate that a product of probabilities was required here. Some added two probabilities.
- (iii) A pleasing number of correct, well-drawn histograms were seen. Weaker candidates struggled with the more difficult frequency densities, but usually managed to add at least one correct column.

Answer: (a) 98.2 (b)(i)  $\frac{28}{80}$  (ii)  $\frac{992}{6320}$

## Section B

### Question 7

- (a) Candidates produced a variety of valid ways to analyse this problem, and achieved much success. There were some who performed the appropriate calculations correctly but were unable to interpret the results accurately.
- (b)(i) Several candidates were able to evaluate the volume of the cylinder correctly. Errors included the use of the diameter instead of the radius, and the use of an incorrect formula such as  $2\pi r^2h$  or  $\frac{1}{3}\pi r^2h$ .
- (ii) This part required more care in the selection and evaluation of the appropriate formulae. Some calculated the volume instead of the total external surface area.
- (iii) There were few successful solutions to this part. Common errors included omitting the answer to part (ii) in the calculation, or forgetting to adjust to square metres at the end.  $30 \times 30\,000$  was a popular incorrect answer.
- (c) There were relatively few correct solutions to this question on the volumes of similar shapes. The usual error was to forget to take the cube root of the ratio  $1000 : 512$

Answer: (a) 130 g tin (b)(i) 423 to 424 (ii) 319 (iii) 1050 (c) 7.2

### Question 8

- (a) The answer to this part of this question on the graphs of functions was usually correct.
- (b) The plotting of the points was usually accurate with many good graphs seen. The most common error was the misplotting of the point  $(-0.5, 0.4)$  which was frequently plotted at  $(-0.5, -0.4)$  occasionally candidates used a ruler to join the points.
- (c) This question was not answered particularly well. Sometimes not all the solutions to the equation were given.
- (d) Some reasonable attempts to draw the correct tangent were seen. As usual, the calculation of the gradient needed care.
- (e)(i) This part was often omitted. Only a few correct answers were seen with many candidates giving  $a = 1$  as their solution.
- (ii) Again, this was often omitted. There was rarely any attempt to draw  $y = x + a$ . Only a few scored full marks. There were a few solutions by factorisation, but for full marks, the method of intersecting graphs was required.

Answer: (a) 4.1 (c) correct readings from graph (d) 1 to 2 (e)(i) -1 (ii) -1, 1, 2

### Question 9

- (a) Many candidates recognised that the sine rule was required here and scored well. Some candidates thought that the sides of the triangle were proportional to the opposite angles. A few started with the correct sine rule, but lost the 'sin' by the time the expression was evaluated. A common error resulted in  $AB = 65\sin 60 / \sin 48$ .
- (b) There were many correct answers to this part, but there were several who just calculated  $\frac{1}{2} \times 84 \times 65$ .
- (c) Many candidates recognised that the application of the cosine rule was required here and evaluated it correctly. The most common wrong answer resulted from those who attempted to use Pythagoras in the triangle ACD.

- (d) This question involving the angle of elevation was rarely answered correctly. Marks were available for finding a correct AP in triangle ABC, and for a clear attempt to find the tangent of an angle in a vertical plane.

Answer: (a) 59.2 (b) 2360 (c) 129 (d) 31.9°

#### Question 10

- (a) Most candidates were able to earn the mark by the correct use of Time = Distance / Speed.
- (b) This part was well answered. Unsuccessful attempts usually had  $(80 - x)$  or  $(80 + x)$  as the denominator.
- (c) There was a mixed response to this question. The required derivation of the given equation was successfully negotiated by a fair number of candidates. Some lost marks because they had the subtraction the wrong way round at the outset. There were others who had the subtraction correct but could not accurately rearrange into the given equation.
- (d) The formula for solving a quadratic equation is well known and continues to be a source of good marks for many candidates. The roots were expressed correctly to one decimal place in most cases. The inability to cope with the negative coefficient of  $x$  and / or the negative constant proved costly for some.
- (e) Candidates usually had the correct expression to evaluate. The main error at this stage was in expressing the answer in the form required. 2.15 hours was usually given as 2 hours 15 minutes.

Answer: (a)  $\frac{320}{x}$  (b)  $\frac{320}{x - 80}$  (d) 148.8 -68.8 (e) 2 h 9 mins

#### Question 11

- (a)(i)(a) A good number of correct vectors were seen.
- (b) Again, this part was answered well.
- (c) A mixed response to this question. Weaker candidates found this part demanding.
- (ii) Conclusions were required here and there were very few completely correct answers. Candidates usually gave only one conclusion, either 'equal' or 'parallel'.
- (b)(i) This question was answered quite well. A common mistake was to see the triangle reflected in the  $y - axis$ .
- (ii) A good number of correct triangles were drawn.
- (iii) Some completely correct descriptions were seen, often regardless of whether candidates had drawn the correct triangles previously. Many descriptions gave an incorrect centre. Sometimes a second transformation was given.

Answer: (a)(i)(a)  $\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{r}$  (b)  $\mathbf{r} + \mathbf{p} - \mathbf{q}$  (c)  $\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{r}$  (ii) Equal and Parallel (b)(i) Triangle with vertices  $(-2,0)$ ,  $(0,6)$ ,  $(0,7)$  (ii) Triangle with vertices  $(-2,0)$ ,  $(0,0)$ ,  $(0,-1)$  (iii) Rotation, 90° anticlockwise centre  $(0,3)$

# MATHEMATICS D (CALCULATOR VERSION)

---

Paper 4024/22

Paper 22

## Key messages

To succeed in this paper, candidates need to have completed full syllabus coverage and remember necessary formulae. All working must be clearly shown, and in numerical work, a suitable level of accuracy must be used in any given calculation, and maintained throughout a series of linked calculations.

## General Comments

Scripts covering the whole mark range were seen. Many were well presented, using the working and answer spaces effectively. Clarity and neatness in presentation remain a problem for some candidates. Working done in pencil is not always clear, and working done in pencil that is then over written in ink is not always easy to read.

Candidates generally kept within the rubric when answering **section B** questions.

Much of the algebra and trigonometry in **Questions 1, 2 and 9**, gave candidates at all levels opportunities to gain marks.

Plotting and drawing in the graphical question was generally successful.

There was a good degree of success in the vector and transformation question.

The construction of algebraic expressions, as required in the inequality in **Question 2(d)**, and in the quadratic equation in **Question 10 (b)**, was less successful than the manipulation of algebraic expressions already given as in **Question 2**. Confidence in this aspect of mathematics could with advantage be applied elsewhere, such as in reverse percentage questions, where rules have been forgotten or have become confused. Algebra would provide a means of thinking through a problem from first principles.

**Question 5**, and part **(a)(i)** of **Question 7**, required the use of remembered mensuration formulae. Candidates need to be secure with these in order to tackle such questions successfully.

The responses to the three dimensional aspects of **Question 5(c)** and **Question 9(d)** showed that candidates would benefit from practical work in these topics.

Where results found in one part of a question can be used in later parts of a question, such as in **Questions 1, 5 and 7**, candidates need to maintain a suitable degree of accuracy in order to reach an accurate final answer.

### Comments on specific questions

#### Section A

#### Question 1

- (a) The angle asked for in the question could be found directly using the tangent ratio for a right-angled triangle. Candidates using this approach were generally successful. Those who calculated  $AC$  first either did not continue any further, or lost final accuracy by the time they had found  $\hat{ACB}$  by a different route. The answer was given as 56.9 by a number of candidates.
- (b)(i) It was expected that Pythagoras's Theorem would be used here to find  $BD$  first, leading to  $BD = 10$  for  $AD$ . This was the most successful method. The length of  $AD$  was not always converted accurately from metres into metres and centimetres. It was common to see two final answers, one in metres and the other in centimetres. Those candidates who rounded  $BD$  to an approximate value at this stage did not reach the correct final answer. Other methods included the calculation of additional angles in order to use the cosine rule for  $AD$ . Long methods such as this were often methodically incorrect and rarely accurate.
- (ii) Again, the most successful candidates used the direct method available for a right-angled triangle.

Answers: (a)  $57.0^\circ$  (b) (i) 5 (m) 6 (cm) (ii)  $66.7^\circ$ .

#### Question 2

- (a) Some candidates spotted the difference of two squares and factorised accurately. Unsuccessful attempts included such as  $(4x - 1)(4x + 1)$ ,  $x(4x - 1)$  and  $(2x)^2 - 1$ .
- (b)(i) The idea of substitution seemed to be generally understood. There were some correct substitutions with the resulting fraction left with  $Q$  not cancelled. There were some incorrect cancellations leading to answers such as 2 or  $3Q$ , and some misunderstandings such as  $2Q + Q = 2Q^2$ .
- (ii) Successful solutions usually began with  $RP = 2Q + R$  and often candidates reached  $PR - R = 2Q$ . Other forms of rearrangement were not usually successful, with some candidates introducing squares and square roots.
- (c) The method of equalising coefficients was well understood and often totally successful. Care is always needed whenever subtraction is required. The method of substitution, once the substitution had been made, seemed to present more obstacles than the method of equalising coefficients. The fractions were rarely removed successfully from the resulting equation.
- (d)(i) Responses here showed that candidates need more confidence in constructing algebraic expressions from situations described in words. Some answers given were purely numerical.
- (ii) Again, more confidence is needed in constructing an inequality that matches the situation described in the question. Sometimes correct algebraic expressions were used with a less than rather than a greater than inequality sign.
- (iii) Some solutions offered here bore no relation to the work shown in part (ii).

Answers: (a)  $(2x - 1)(2x + 1)$  (b) (i) 3 (ii)  $\frac{2Q}{P-1}$  (c)  $(x =) 7$   $(y =) -1$  (d) (i)  $3.2x + 16$  (ii)  $x > 73.125$   
(iii) 74.

### Question 3

- (a) (i) Candidates seemed to understand the idea of discount. Some gave the discounted price, \$(1080 – 43.2) as the answer. Candidates did not always make clear in their working whether they were using dollars or cents. Some candidates found 4% of 3000.
- (ii) This part was generally well answered. Using the denominator 45 was a common error. Some candidates gave the answer 9.
- (iii) Again, this was generally understood and successfully solved. The calculation was sometimes left incomplete, with only  $\frac{1302.75}{3000 \times 0.45}$  evaluated. A common error in this part was to divide by 1302.75. Some candidates were still trying to apply the 4% from part (a)(i).
- (b) Clearly, some candidates remembered the correct procedure to solve this problem. In a few cases, a correct algebraic equation was constructed and solved. Candidates could well be encouraged to develop this latter approach. Some candidates stopped after finding \$4.80. The most common misconception here was to apply the  $12\frac{1}{2}\%$  increase to \$5.40. 87.5% and 112.5% of \$5.40 were also seen.

Answers: (a) (i) 43.20 (ii) 25 (iii) 3.5 (b) 0.6.

### Question 4

- (a) (i) The angle sum of a triangle was often well used here.
- (ii) Most candidates spotted the vertically opposite angles.
- (iii) Candidates showed confidence in applying the relevant property of parallel lines.
- (b) (i) The basic elements required here were generally well known, candidates gained at least one of the marks for a correct pair of equal angles. Solutions that did not gain full marks usually omitted to state a valid reason for the equal angles chosen. A common misconception was the assumption that  $AB$  and  $ED$  were parallel. There were attempts to show similarity using ratios.
- (ii) Care was needed here to use the factor  $\frac{2}{5}$  correctly, and to remember to add 5 to 2.2 when reached. The common error here was to use ratios involving  $2 \times 5$ .

Answers: (a) (i) 102 (ii) 102 (iii) 78 (b) (ii) 7.2

### Question 5

- (a) The relevant formula for arc length was reasonably well known, but completely correct solutions in this part were rare. The successful solutions were those that had a clear grasp of the four separate parts of the given perimeter. The arc  $AD$  was frequently subtracted from the arc  $BC$ . The individual arcs were sometimes seen with 25's and 20's included. Fractions of  $\frac{1}{2}$  and  $\frac{1}{3}$  instead of  $\frac{150}{360}$  were also seen, and some candidates used the formula for the area of a sector. Some candidates thought they were using radians.
- (b) Subtraction was relevant here so candidates were more successful in this part, although the addition of areas was sometimes seen. Some candidates found the difference of the radii before applying the area formula. Again, inappropriate radian forms of the area formula were sometimes used.



- (c) The relationship between sector and cone was generally missed. Common speculative answers given included 20 and 10. Some candidates used an area rather than an arc length.

Answers: (a) 220 (b) 2130 (c) 8.33.

### Question 6

- (a) Generally, there was a good response to this question on data handling, with candidates exhibiting clear ideas of the structure of the appropriate formula. There was some use of class widths that led to answers such as  $\frac{1500}{150}$ . Division by 6 was sometimes seen.
- (b)(i) Candidates used the given table to good advantage here. Some candidates forgot to include the 25 from the next class interval. There were some whole number probabilities offered.
- (ii) The correct procedure and probabilities were used by some candidates. Using  $\frac{40}{150}$  led to the answer  $\frac{16}{75}$  which was seen on occasion. For those candidates who appreciated that another probability was required here, the usual choice seemed to be  $\frac{30}{149 \text{ or } 150}$ . It was often not appreciated that there were two possibilities to consider. For these candidates,  $\frac{8}{75}$  was a common wrong answer.
- (c) A good number of well drawn histograms were seen. Some candidates did not appreciate the significance of frequency density, giving heights that assumed an interval width of 10 throughout.

Answers: (a) 158 (b) (i)  $\frac{60}{150}$  (ii)  $\frac{4800}{22350}$ .

### Section B

#### Question 7

- (a)(i) Usually a mark was gained for using a relevant area formula in this question on mensuration. A common incorrect answer was 168 from  $8 \times 21$ . There was some confusion between curved surface area and volume in the choice of formulae here. The open cylinder was not always successfully negotiated.
- (ii) Responses to this part did not always use the information given in the question correctly. For example, 150 could be included in the calculation without being multiplied by 30 000, or  $150 \times 30\,000$  could be evaluated, ignoring the answer from part (a)(i).  $\frac{30000}{150}$  was also evaluated. The conversion to square metres was not always successful.
- (b)(i) In this part, the formulae for a sphere were given at the start of the question. Many candidates forgot to divide by 2 when applying them to the hemisphere.
- (ii) Usually four times the previous answer was carried over to this part, but dealing correctly with the five circular areas involved proved challenging for many candidates.
- (iii) The formula for the volume of a sphere, stated at the start of part (b), was not always successfully used to deal with the four hemispheres involved. As well as using four whole spheres, a common error seemed to be to omit any reference to the cylinder.

Answers: (a)(i) 874 (ii) 3070 (b)(i) 77.0 (ii) 500 (iii) 2410.

### Question 8

- (a) Candidates were usually able to complete the table accurately in this question on functions.
- (b) There were a good number of well drawn curves using the axes given in the question.
- (c) The essence of this part of the question was generally understood. Candidates usually managed one or two solutions, if not always all three.
- (d) A good number of correct tangents were seen. Care is always needed in calculating the gradient of a tangent, especially when it is negative.
- (e) Candidates were not always able to deduce from the question that drawing the line  $y = x$  was an essential part of solving the given equation. Those candidates who did realise this completed the solution accurately.
- (f) Again, the implications of this part were not always appreciated. Candidates need more confidence in dealing with ranges of this sort.

Answers: (a) - 2.1 (c) - 0.6, 1, 1.6 (d) - 2.75 (e) 1.7 (f)  $1 < k < 2$

### Question 9

- (a) As expected, most candidates used the sine rule here. Some lost the “sin” when transposing their formula. Or perhaps they had cancelled it. There were a number who started with ratios that did not include “sin”.
- (b) There were many correct evaluations of the cosine rule here. The angle required in the formula,  $122^\circ$ , had to be deduced from the diagram. Not all candidates were successful in this.
- (c) The formula required here is clearly well known, and candidates generally used it accurately. Some candidates forgot to include  $\sin 122^\circ$ .
- (d) This problem had two components. Candidates needed to find  $AP = 30\sin 58^\circ$  from the original 2-D diagram. They then needed to use it accurately in a separate right-angled triangle in a vertical plane to calculate the height of the tower. Part marks were awarded for each of these components, but they were rarely seen together forming a complete solution.

Answers: (a) 42.3 (b) 83.9 (c) 814 (d) 17.2.

### Question 10

- (a) Candidates generally gained part marks here. The usual reason for not gaining full marks was that  $PS = PQ$  was used without justification.
- (b)(i) Again, finding the relevant algebraic expression was challenging.  $10 - x$  appeared quite often.
  - (ii) There were a number of fully explained solutions based on the area of triangle  $PBQ$  or on the length of  $PQ$ . In some cases, where  $40 - x$  had been found in the previous part, candidates appeared to be working back from the answer.
- (c)(i) This was straightforward for some candidates. Others did not get anywhere by starting with  $x^2 - 40x + 250 = 1100$ .
  - (ii) The correct formula appeared to be well known. Care is always needed when evaluating the roots.
- (d) This was no problem for some candidates. Many candidates omitted this part.

Answers: (b)(i)  $40 - x$  (c) (ii) 7.8 32.2.

**Question 11**

- (a)** Most candidates choosing this question understood the nature of vectors. Some candidates used expressions with upper case letters or expressions containing mixtures of vectors and numbers that were not acceptable as vectors.
- (i)(a)** This part was generally well done.
- (b)** Again, the general principles required in this part were understood. Care was needed in combining the relevant vectors correctly.
- (c)** This part was generally well done.
- (ii)** That the given points all lay on the same straight line was obvious to most candidates. This observation had to be backed up from correct work in the preceding parts. Getting the ratio of appropriate lengths correct proved tricky.
- (b)(i)** There were many correct triangles seen in answer to this part.
- (ii)** The correct position of triangle C proved harder to find.
- (iii)** Candidates usually found some marks in this part, and there were a good number of complete answers. Sometimes the transformation was thought to be a shear or an enlargement.

Answers: **(a)(i)(a)** -  $p + q$  **(b)**  $\frac{1}{3}(4q - p)$  **(c)**  $2q - p$