# UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE Joint Examination for the School Certificate and General Certificate of Education Ordinary Level <br> <br> MATHEMATICS (SYLLABUS D) <br> <br> MATHEMATICS (SYLLABUS D) <br> 4024/2 <br> Dollar version 

## PAPER 2

OCTOBER/NOVEMBER SESSION 2001
2 hours 30 minutes

Additional materials:
Answer paper
Electronic calculator
Geometrical instruments
Graph paper (3 sheets)
Mathematical tables (optional)

TIME 2 hours 30 minutes

## INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the separate answer paper/answer booklet.
Write your answers and working on the separate answer paper provided.
Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.

## Section A

Answer all questions.

## Section B

Answer any four questions.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100 .
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

## Section A [52 marks]

Answer all the questions in this section.


In the diagram, $A B$ is a vertical wall.

A beam, $C D$, of length 11 metres, rests with one end, $D$, on horizontal ground.
It is held in place by two cables, $B C$ and $B D$.

Given that $A D=8$ metres, $B D=15$ metres and angle $B D C=55^{\circ}$, calculate
(a) the length $A B$,
(b) the length of the cable $B C$,
(c) the angle between the beam $C D$ and the ground.

2 Part of a pattern of numbers is shown in the table below.

| 1 | 2 | 3 | 4 | 5 | $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 8 | 11 | 14 | $p$ | $x$ |
| 4 | 9 | 16 | 25 | $q$ | $y$ |
| 1 | 8 | 27 | 64 | $r$ | $z$ |
| 10 | 25 | 54 | 103 | $s$ | $t$ |

(a) Study the patterns and write down the value of $p$, the value of $q$, the value of $r$ and the value of $s$.
(b) Find expressions, in terms of $n$, for each of $x, y, z$ and $t$.


In the diagram, $A, B, C, D$ and $E$ lie on a circle and $E B=E C$.
The lines $E D, A C$ and $F B$ are parallel.
$E \hat{B} C=75^{\circ}$ and $A \hat{C} E=20^{\circ}$.
Calculate
(a) $C \hat{E} D$,
(b) $C \hat{D} E$,
(c) $E \hat{C} D$,
(d) $A \hat{C} B$,
(e) $A \hat{B} F$.

4 (a) Given that $v=u+\frac{F t}{m}$,
express $t$ in terms of $F, m, u$ and $v$.
(b) An elastic string hangs from a nail $N$.

When a mass of $m$ grams is attached to its lower end, the elastic is stretched so that its total length is $x \mathrm{~cm}$, as shown in the diagram.

The table below shows the results of two experiments.

| Length $(x \mathrm{~cm})$ | 43 | 49 |
| :--- | :--- | :--- |
| Mass $(m$ grams $)$ | 50 | 80 |



It is known that $x$ and $m$ are connected by the equation $x=c+d m$, where $c$ and $d$ are constants.
(i) Use this information to write down two equations in $c$ and $d$.
(ii) Solve your equations to find the value of $c$ and the value of $d$.
(iii) Find the mass at the end of the string when its length is 40 cm .
(iv) What does the value of $c$ represent?


Diagram I


Diagram II

A water tank, shown in Diagram I, is a circular cylinder of radius 24 cm and height 125 cm . It is open at one end and full of water.
(a) Calculate
(i) the volume, in litres, of water in the tank,
(ii) the total area, in square metres, of the outside of the open tank.
(b) Diagram II shows a rectangular trough of length 150 cm and width 20 cm .

The trough was completely filled with $48000 \mathrm{~cm}^{3}$ of water from the tank.
Calculate the depth of the trough.
(c) After the trough had been filled, water started to leak from the tank.

In 2 hours 30 minutes it was found that $20000 \mathrm{~cm}^{3}$ ran out of the tank.
Calculate the rate at which the level of water in the tank was falling.
Express your answer in centimetres per hour.

## 6 Answer the whole of this question on a sheet of graph paper.

Triangle $A$ has vertices $(6,-2),(8,-2)$ and $(6,-5)$.
(a) Using a scale of 1 cm to represent 1 unit on each axis, draw axes for values of $x$ and $y$ in the ranges $-6 \leqslant x \leqslant 12$ and $-6 \leqslant y \leqslant 6$.

Draw and label triangle $A$.
(b) The translation T is represented by the column vector $\binom{4}{3}$.

The translation T maps triangle $A$ onto triangle $B$, so that $\mathrm{T}(A)=B$.
Draw and label triangle $B$.
(c) The transformation R is a rotation through $90^{\circ}$ clockwise, centre $(3,4)$.

The transformation R maps triangle $A$ onto triangle $C$, so that $\mathrm{R}(A)=C$.
Draw and label triangle $C$.
(d) Given that $\operatorname{TR}(A)=D$, draw and label triangle $D$.
(e) Triangle $E$ has vertices $(2,-2),(4,-2)$ and $(-4,-5)$.

The single transformation H maps triangle $A$ onto triangle $E$.
Describe fully the transformation H .

## Section B [48 marks]

Answer four questions in this section.
Each question in this section carries 12 marks.
7 (a) Express as a single fraction in its simplest form

$$
\begin{equation*}
\frac{200}{x}-\frac{200}{x+4} . \tag{2}
\end{equation*}
$$

(b) When driven in town, a car runs $x$ kilometres on each litre of petrol.
(i) Find, in terms of $x$, the number of litres of petrol used when the car is driven 200 km in town.
(ii) When driven out of town, the car runs $(x+4)$ kilometres on each litre of petrol.

It uses 5 litres less petrol to go 200 km out of town than to go 200 km in town.
Use this information to write down an equation involving $x$, and show that it simplifies to

$$
\begin{equation*}
x^{2}+4 x-160=0 . \tag{3}
\end{equation*}
$$

(c) Solve the equation $x^{2}+4 x-160=0$, giving both answers correct to two decimal places.
(d) Calculate the total volume of petrol used when the car is driven 40 km in town and then 120 km out of town.

## 8 Answer the whole of this question on a sheet of graph paper.

The length of time taken by 80 drivers to complete a journey is given in the table below.

| Time <br> $(t$ minutes $)$ | $60<t \leqslant 80$ | $80<t \leqslant 90$ | $90<t \leqslant 95$ | $95<t \leqslant 100$ | $100<t \leqslant 110$ | $110<t \leqslant 130$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number <br> of drivers | 4 | 10 | 14 | 20 | 24 | 8 |

(a) Using a scale of 2 cm to represent 10 minutes, draw a horizontal axis for times between 60 minutes and 130 minutes.
Choose a suitable scale for the vertical axis and draw a histogram to represent the information in the table.
(b) In which interval does the median of the distribution lie?
(c) Calculate an estimate of the mean time taken to complete the journey.
(d) One driver is chosen at random.

Expressing your answer as a fraction in its lowest terms, calculate the probability that she took 90 minutes or less for the journey.
(e) Two drivers are chosen at random.

Expressing each answer as a fraction in its lowest terms, calculate the probability that
(i) both took more than 110 minutes for the journey,
(ii) one took 80 minutes or less for the journey and the other took more than 110 minutes.

## 9 Answer the whole of this question on a sheet of graph paper.

The table gives some values of $x$ and the corresponding values of $y$, correct to one decimal place, where

$$
y=4 x+\frac{25}{x^{2}} .
$$

| $x$ | 1 | 1.25 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $p$ | 21.0 | 17.1 | 14.3 | 14.0 | 14.8 | 16.0 | 17.6 |

(a) Find the value of $p$.
(b) Using a scale of 4 cm to represent 1 unit, draw a horizontal $x$-axis for $0 \leqslant x \leqslant 4$.

Using a scale of 4 cm to represent 10 units, draw a vertical $y$-axis for $0 \leqslant y \leqslant 30$.
On your axes, plot the points given in the table and join them with a smooth curve.
(c) Use your graph to find
(i) a solution of $4 x+\frac{25}{x^{2}}=20$,
(ii) the least value of $4 x+\frac{25}{x^{2}}$.
(d) By drawing a tangent, find the gradient of the curve at the point where $x=1.5$.
(e) On the axes used in part (b), draw the graph of the straight line

$$
\begin{equation*}
y=25-3 x \text { for values from } x=0 \text { to } x=4 \text {. } \tag{2}
\end{equation*}
$$

(f) (i) Write down the $x$ coordinates of the points at which the two graphs intersect.
(ii) Find the equation, in the form $a x^{3}+b x^{2}+c x+d=0$, which is satisfied by the values of $x$ found in part (f)(i).

## 10 [The value of $\pi$ is $\mathbf{3 . 1 4 2}$ correct to three decimal places.]



Diagram I


Diagram II

Diagram I shows a regular pentagon, $A B C D E$, inscribed in a circle of radius 1.5 cm and centre $O$.
(a) Explain why $A \hat{O} B=72^{\circ}$.
(b) Calculate the area of the pentagon $A B C D E$.
(c) Diagram II shows a design for a new coin.

The vertices of the regular pentagon $A B C D E$ are joined by circular arcs whose centres are the opposite vertices.
For example, the arc $A B$ has centre $D$ and radius $D A$.
(i) Explain why $A \hat{D} B=36^{\circ}$.
(ii) Show that the length of $D A$ is approximately 2.85 cm .
(iii) Calculate the area of triangle $D A B$.
(iv) Calculate the area of the segment shaded in Diagram II.
(v) Calculate the area of the face $A B C D E$ of the coin.


In the diagram, $O A B$ is a triangle.
$C$ is the point on $A B$ such that $A C: C B=2: 1$.
The side $O B$ is produced to the point $D$ such that $O B: B D=3: 2$.
It is given that $\overrightarrow{O A}=\mathbf{a}$ and $\overrightarrow{O B}=\mathbf{b}$.
(a) Express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,
(i) $\overrightarrow{A B}$,
(ii) $\overrightarrow{A C}$,
(iii) $\overrightarrow{O C}$,
(iv) $\overrightarrow{O D}$.
(b) Show that $\overrightarrow{C D}=\mathbf{b}-\frac{1}{3}$ a.
(c) It is given that $E$ is the point on $O A$ such that $\overrightarrow{O E}=\frac{5}{9}$ a.

Express, as simply as possible, in terms of $\mathbf{a}$ and $\mathbf{b}$, the vector $\overrightarrow{E D}$.
(d) (i) Show that $\overrightarrow{E D}=k \overrightarrow{C D}$, where $k$ is a constant.
(ii) Write down two facts about $E D$ and $C D$.
(e) Calculate $\frac{\text { the area of triangle } A E C}{\text { the area of triangle } O E C}$.

## BLANK PAGE

