UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

MATHEMATICS (SYLLABUS D)   **4024/02**

Paper 2

May/June 2006

2 hours 30 minutes

Additional Materials: Answer Booklet/Paper
                          Electronic calculator
                          Geometrical instruments
                          Graph paper (1 sheet)
                          Mathematical tables (optional)

READ THESE INSTRUCTIONS FIRST

Write your answers and working on the separate Answer Booklet/Paper provided.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Section A
Answer **all** questions.

Section B
Answer any **four** questions.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 100.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

UNIVERSITY *of* CAMBRIDGE
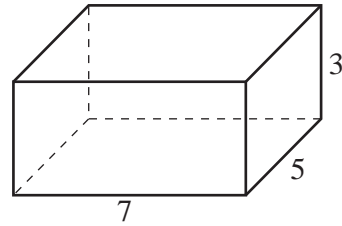International Examinations

[Turn over

Section A [52 marks]

Answer **all** questions in this section.

1   (a) Solve the equation $3x^2 - 4x - 5 = 0$, giving your answers correct to two decimal places.   [4]

(b) Remove the brackets and simplify $(3a - 4b)^2$.   [2]

(c) Factorise completely $12 + 8t - 3y - 2ty$.   [2]

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2   (a) A solid cuboid measures 7 cm by 5 cm by 3 cm.

(i)   Calculate the total surface area of the cuboid.   [2]

(ii) A cube has the same volume as the cuboid.
Calculate the length of an edge of this cube.   [2]

(b) [The volume of a cone is $\frac{1}{3}$ × base area × height.]

[The area of the curved surface of a cone of radius r and slant height l is $\pi r l$.]
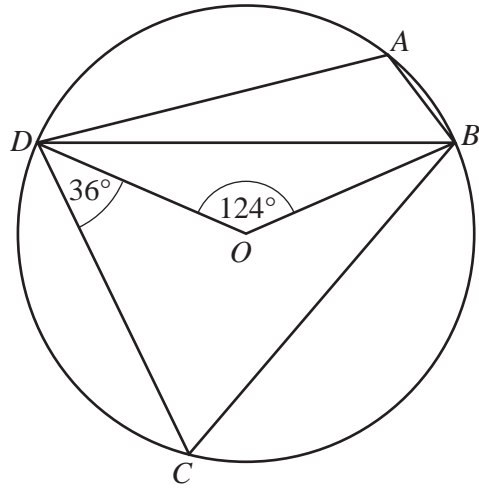
A solid cone has a base radius of 8 cm and a height of 15 cm.

Calculate
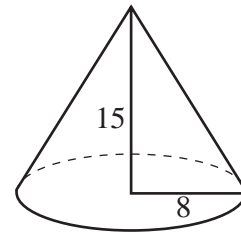
(i)   its volume,   [2]

(ii) its slant height,   [1]

(iii) its curved surface area,   [2]
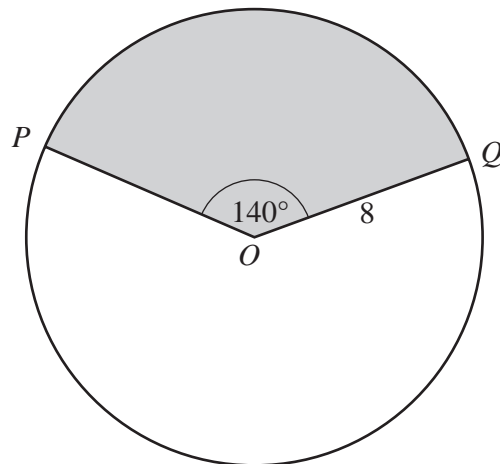
(iv) its **total** surface area.   [1]

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3 **(a)** In the diagram, the points *A*, *B*, *C* and *D* lie on a circle, centre *O*.

$D\hat{O}B = 124°$ and $C\hat{D}O = 36°$.

Calculate

(i) $D\hat{C}B$, [1]

(ii) $D\hat{A}B$, [1]

(iii) $O\hat{D}B$, [1]

(iv) $C\hat{B}O$. [1]

(b) The diagram shows a circle, centre *O*, with the sector *POQ* shaded.

Given that $P\hat{O}Q = 140°$ and the radius of the circle is 8 cm, calculate

(i) the area of the shaded region, [2]

(ii) the **total** perimeter of the **unshaded** region. [3]

4 **(a)** These are the prices for a ride in an amusement park.

| Adult | $3.60 |
| Child | $2.25 |

    (i) A family of two adults and three children went on the ride.
        They paid with a $20 note.

        Calculate the change they received. [1]

  (ii) Express $2.25 as a percentage of $3.60. [1]

(b) Diagram I represents part of the framework of the ride.

    The points *A*, *B*, *C*, *D*, *E* and *F* are on the framework.
    The points *H*, *C*, *G*, *E* and *F* lie on a horizontal line.
    The lines *BH* and *DG* are vertical.

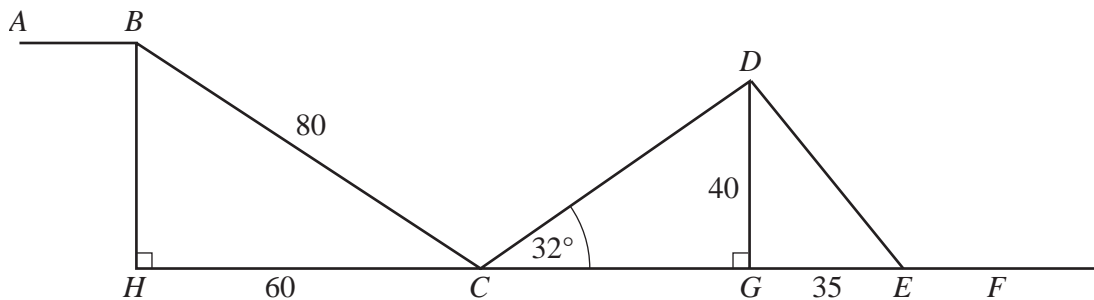 $BC = 80$ m, $HC = 60$ m, $DG = 40$ m, $GE = 35$ m and $D\hat{C}G = 32°$.

Diagram I

    Calculate

    (i) $H\hat{C}B$, [2]

  (ii) *CD*, [3]

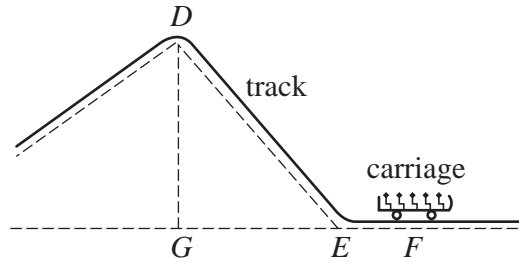 **(iii)** the angle of depression of *E* from *D*. [2]
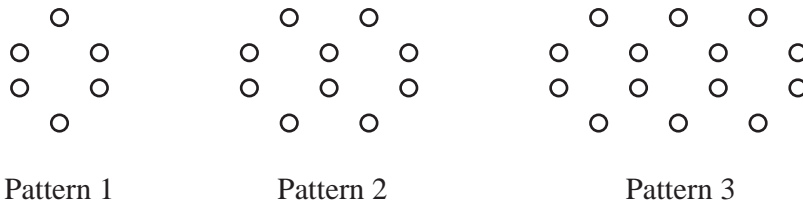
(c)

Diagram II

Diagram II shows part of the ride.
The carriage that carried the family was 4.6 m long.
It was travelling at a constant speed of 15 m/s as it passed the point F.

(i) Calculate, correct to the nearest hundredth of a second, the time taken for the carriage to pass the point F. [2]

(ii) Express 15 m/s in kilometres per hour. [1]

_____

5

Pattern 1   Pattern 2   Pattern 3

Counters are used to make patterns as shown above.
Pattern 1 contains 6 counters.
The numbers of counters needed to make each pattern form a sequence.

(a) Write down the first four terms of this sequence. [1]

(b) The number of counters needed to make Pattern n is $An + 2$.
Find the value of A. [1]

(c) Mary has 500 counters.
She uses **as many of** these counters as she can to make **one** pattern.

Given that this is Pattern m, find

(i) the value of m, [1]

(ii) how many counters are **not** used. [1]

_____

**6** **(a)** The results of a survey of 31 students are shown in the Venn diagram.

$\mathscr{E}$ = {students questioned in the survey}
M = {students who study Mathematics}
P = {students who study Physics}
S = {students who study Spanish}
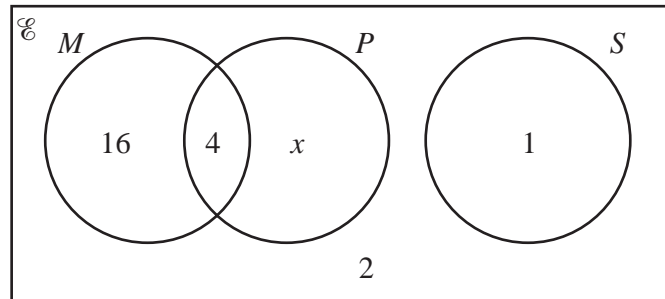
(i) Write down the value of

    (a) x,   [1]

    (b) $n(M \cap P)$,   [1]

    (c) $n(M \cup S)$,   [1]

    (d) $n(P')$.   [1]

(ii) Write down a description, in words, of the set that has 16 members.   [1]

(b) In the diagram, triangle AQR is similar to triangle ABC.

$AQ = 8$ cm, $QB = 6$ cm and $AR = 10$ cm.
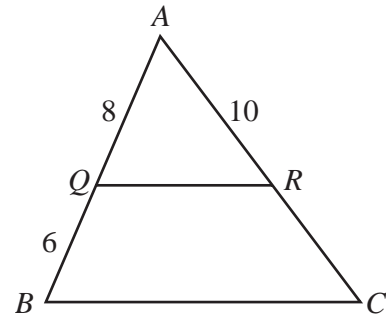
(i) Calculate the length of RC.   [2]

(ii) Given that the area of triangle AQR is 32 cm^2, calculate the area of triangle ABC.   [2]

Section B [48 marks]

Answer **four** questions in this section.

Each question in this section carries 12 marks.

7   James and Dan are partners in a small company.
From each year's profit, James is paid a bonus of $15 000 and the remainder is shared between James and Dan in the ratio $2:3$.

   (a)   In 1996 the profit was $20 000.

   Show that Dan's share was $3000.   [1]

   (b)   In 1997 the profit was $21 800.

   Calculate

   (i)   the percentage increase in the profit in 1997 compared to 1996,   [2]

   (ii)   the total amount, including his bonus, that James received in 1997.   [2]

   (c)   In 1998 Dan received $7500.

   Calculate the profit in 1998.   [3]

   (d)   In 1999, the profit was $$x$, where $x > 15 000$.

   (i)   Write down an expression, in terms of x, for the amount Dan received.   [1]

   (ii)   Given that Dan received half the profit, write down an equation in x and hence find the amount that Dan received.   [3]

                                           [Turn over

8   **Answer the whole of this question on a sheet of graph paper.**

The table below gives some values of x and the corresponding values of y, correct to one decimal place, where

$$y = \frac{x^2}{8} + \frac{18}{x} - 5.$$

x	1	1.5	2	2.5	3	4	5	6	7	8
y	13.1	7.3	4.5	3.0	2.1	1.5	1.7	p	3.7	5.3

(a) Find the value of p.   [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $0 \leqslant x \leqslant 8$.

Using a scale of 1 cm to 1 unit, draw a vertical y-axis for $0 \leqslant y \leqslant 14$.

On your axes, plot the points given in the table and join them with a smooth curve.   [3]

(c) Use your graph to find

    (i) the value of x when $y = 8$,   [1]

    (ii) the least value of $\dfrac{x^2}{8} + \dfrac{18}{x}$ for values of x in the range $0 \leqslant x \leqslant 8$.   [1]

(d) By drawing a tangent, find the gradient of the curve at the point where $x = 2.5$.   [2]

(e) On the axes used in part **(b)**, draw the graph of $y = 12 - x$.   [2]

(f) The x coordinates of the points where the two graphs intersect are solutions of the equation
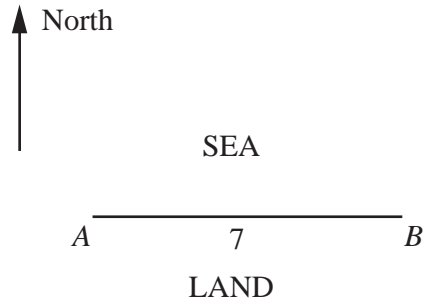
$$x^3 + Ax^2 + Bx + 144 = 0.$$

Find the value of A and the value of B.   [2]

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9 In the diagram, *A* and *B* are two points on a straight coastline.
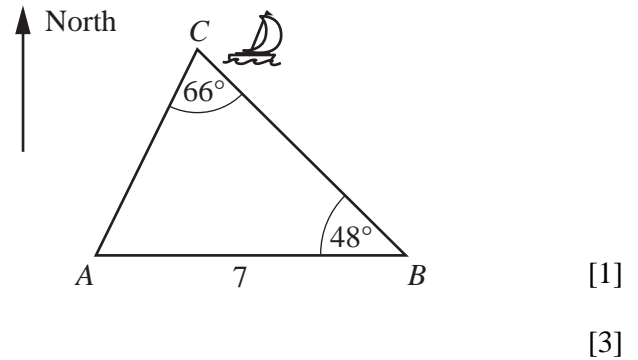
B is due east of *A* and *AB* = 7 km.

The position of a boat at different times was noted.

North

SEA

$\overline{}$
A   7   *B*

LAND

(a) At 8 a.m., the boat was at *C*, where
$A\hat{C}B = 66°$ and $A\hat{B}C = 48°$.

North

C

66°

48°

A   7   *B*

Calculate

(i) the bearing of *B* from *C*,   [1]

(ii) the distance *AC*.   [3]

(b) At 9 a.m., the boat was at *D*, where
$AD = 6.3$ km and $D\hat{A}B = 41°$.

North

D

6.3

41°

A   7   *B*

Calculate

(i) the area of triangle *ADB*,   [2]

(ii) the shortest distance from the boat to the coastline.   [2]
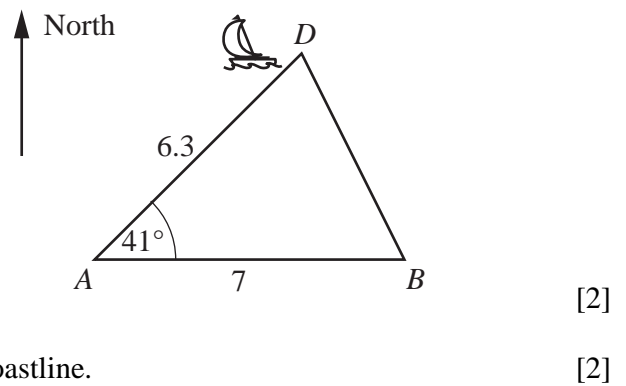
(c) At 11 a.m., the boat was at *E*, where
AE = 9 km and *BE* = 5 km.

North

E

9

5

A   7   *B*

Calculate the bearing of *E* from *A*.   [4]
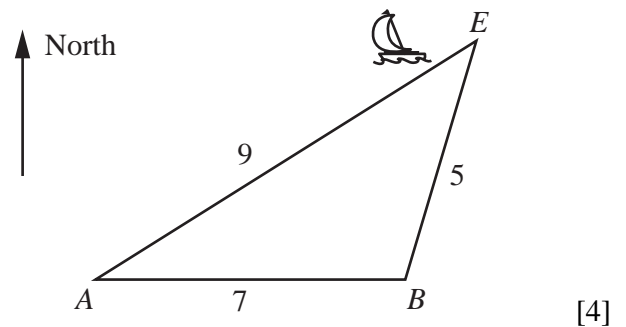
10 (a) The lengths of 120 leaves were measured.
The cumulative frequency graph shows the distribution of their lengths.
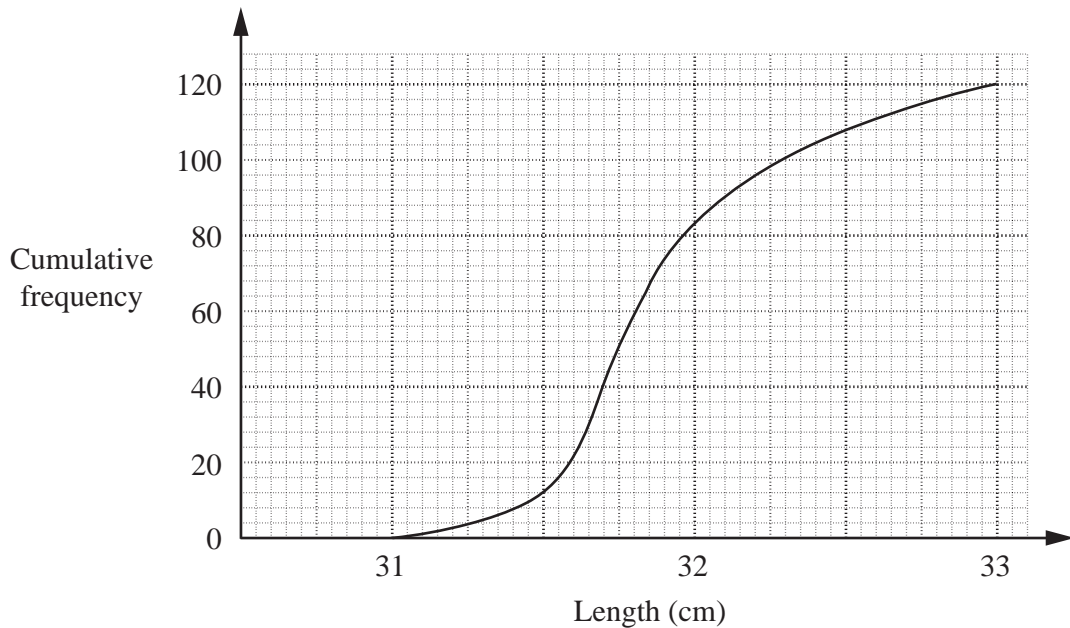
Use this graph to estimate

(i) the median, [1]

(ii) the interquartile range, [2]

(iii) the number of leaves whose length is more than 31.5 cm. [1]

(b) Each member of a group of 16 children solved a puzzle.
The times they took are summarised in the table below.

Time (t minutes)	$5 < t \leqslant 10$	$10 < t \leqslant 12$	$12 < t \leqslant 14$	$14 < t \leqslant 16$	$16 < t \leqslant 20$
Frequency	2	4	6	3	1

(i) Write down an estimate of the number of children who took less than 13 minutes. [1]

(ii) Calculate an estimate of the mean time taken to solve the puzzle. [3]

(iii) Two children are chosen at random.

Calculate, as a fraction in its simplest form, the probability that one of these children took more than 10 minutes and the other took 10 minutes or less. [2]

(iv) A histogram is drawn to illustrate this information.
The height of the rectangle representing the number of children in the interval $10 < t \leqslant 12$ is 8 cm.
Calculate the height of the rectangle representing the number of children in the interval $5 < t \leqslant 10$. [2]

11 **(a)** $\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 3 & -2 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} -2p & 3p \\ -3p & p \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

   (i) Evaluate $4\mathbf{C} - 2\mathbf{A}$. [2]

   (ii) Given that $\mathbf{B} = \mathbf{A}^{-1}$, find the value of p. [2]

   (iii) Find the 2×2 matrix $\mathbf{X}$, where $\mathbf{AX} = \mathbf{C}$. [2]

   (iv) The matrix $\mathbf{C}$ represents the **single** transformation T.

   Describe, fully, the transformation T. [2]

(b) $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -9 \end{pmatrix}$ $\overrightarrow{PR} = \begin{pmatrix} h \\ -6 \end{pmatrix}$ $\overrightarrow{QU} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$ $\overrightarrow{PS} = \begin{pmatrix} 17 \\ k \end{pmatrix}$

   (i) Given that R lies on PQ, find the value of h. [1]

   (ii) Express $\overrightarrow{PU}$ as a column vector. [1]

   (iii) Given that U is the midpoint of QS, find the value of k. [2]

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