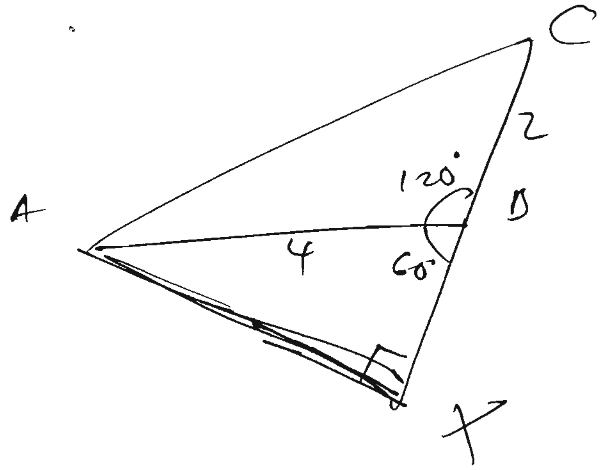
① i) $\frac{AX}{4} = \sin 60^\circ$

$\frac{AX}{4} = \frac{\sqrt{3}}{2}$

Ans $AX = 2\sqrt{3}$ cm



ii) $\frac{BX}{4} = \cos 60^\circ$

$BX = 2$

$AX = 4$

$\tan A\hat{C}B = \frac{AX}{CX}$

$\tan A\hat{C}B = \frac{2\sqrt{3}}{4}$

$\tan A\hat{C}B = \frac{\sqrt{3}}{2}$

$A\hat{C}B = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (shown)

(2) $9^x(27)^y = 1$

$(3^{2x})(3^{3y}) = 3^0$

$3^{2x+3y} = 3^0$

$2x+3y = 0$ —— (1)

$8^y \div (\sqrt{2})^x = 16\sqrt{2}$

$(2^{3y}) \div 2^{\frac{1}{2}x} = (2^4)(2^{\frac{1}{2}})$

$2^{3y-\frac{1}{2}x} = 2^{4.5}$

$3y - \frac{1}{2}x = 4.5$

$\frac{1}{2}x = \cancel{4.5}\ 3y - 4.5$

$x = \cancel{6y}\ 6y - 9$ —— (2)

Sub (2) into (1),

$2(6y-9)+3y = 0$

$12y - 18 + 3y = 0$

$15y = 18$

$y = \frac{6}{5}$

$y = 1.2$

From (2) $x = 6(1.2) - 9$

$= -1.8$

Ans: $x = -1.8$, $y = 1.2$

(3) $\text{Det } A = (7)(6) - (-8)(1)$

$= 50$

Ans $A^{-1} = \frac{1}{50}\begin{pmatrix} 6 & 8 \\ -1 & 7 \end{pmatrix}$

$8p - 7q + 11 = 0$

$7q - 8p = 11$

$6p + q + 7 = 0$

$q + 6p = -7$

$\begin{pmatrix} 7 & -8 \\ 1 & 6 \end{pmatrix}\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} 11 \\ -7 \end{pmatrix}$

$\begin{pmatrix} q \\ p \end{pmatrix} = \frac{1}{50}\begin{pmatrix} 6 & 8 \\ -1 & 7 \end{pmatrix}\begin{pmatrix} 11 \\ -7 \end{pmatrix}$ 66-56

$= \frac{1}{50}\begin{pmatrix} 10 \\ -60 \end{pmatrix}$

$= \begin{pmatrix} 0.2 \\ -1.2 \end{pmatrix}$

Ans : $p = -1.2$, $q = 0.2$

4 i) $\frac{d}{dx}(x^3 \ln x)$

$= (3x^2)(\ln x) + \left(\frac{1}{x}\right)(x^3)$

$= 3x^2 \ln x + x^2$

ii) $\int x^2 \ln x \, dx$

$= \frac{1}{3}\int 3x^2 \ln x \, dx$

$= \frac{1}{3}\int (3x^2 \ln x + x^2 - x^2) \, dx$

$= \frac{1}{3}\left[x^3 \ln x - \frac{x^3}{3} \right] + c$

$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + c$

5ii) let $\dfrac{8x-46}{(x-5)(x+1)} = \dfrac{A}{(x-5)} + \dfrac{B}{(x+1)}$

$8x-46 = A(x+1) + B(x-5)$

let $x = -1$

$-54 = 0 + B(-6)$

$B = 9$

Let $x = 5$

$40-46 = 6A + 0$

$A = -1$

Ans: $\dfrac{8x-46}{(x-5)(x+1)} = \dfrac{9}{x+1} - \dfrac{1}{x-5}$

$y = \dfrac{8x-4}{(x-5)(x+1)}$

$y = 9(x+1)^{-1} - (x-5)^{-1}$

$\dfrac{dy}{dx} = 9(-1)(x+1)^{-2}(1) - (-1)(x-5)^{-2}(1)$

$\dfrac{dy}{dx} = \dfrac{-9}{(x+1)^2} + \dfrac{1}{(x-5)^2}$

When $x = 2$

$\dfrac{dy}{dx} = \dfrac{-9}{9} + \dfrac{1}{9}$

$\dfrac{dy}{dx} = \dfrac{-8}{9}$

Ans: Gradient of curve at point where $x=2$ is $\dfrac{-8}{9}$.

6 i) $v = 6t - \frac{1}{2}t^2$

$\cancel{\frac{dv}{dt} = 6 - t}$

~~when~~ $\cancel{\frac{dv}{t}}$ when $v = 0$

$6t - \frac{1}{2}t^2 = 0$

$t^2 - 12t = 0$

$t(t-12) = 0$

$t = 0$ or $t = 12$

Ans $\therefore t = 12$

Time taken = 12 seconds

$\cancel{s = \int v\,dt}$

iii) Distance AB $= \int_0^{12} v\,dt$

$= \int_0^{12} (6t - \frac{1}{2}t^2)\,dt$

$= \left[3t^2 - \frac{1}{6}t^3\right]_0^{12}$

$= 432 - 288$

$= 144m.$

7.

$$y = \frac{\sin x}{2 - \cos x}$$

$$\frac{dy}{dx} = \frac{(\cos x)(2 - \cos x) - (\sin x)(\sin x)}{(2 - \cos x)^2}$$

$$= \frac{2\cos x - (\sin^2 x + \cos^2 x)}{(2 - \cos x)^2}$$

$$= \frac{2\cos x - 1}{(2 - \cos x)^2}$$

Parallel to x-axis, gradient $= 0$

$$\frac{dy}{dx} = 0$$

$$\frac{(2\cos x - 1)}{(2 - \cos x)^2} = 0$$

$$2\cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3} \#$$

Ans : x-coordinate is $\frac{\pi}{3}$.

Si; L.H.S

$= \sin 3x + \sin x$

$= 2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)$

$= 2\sin 2x \cos x$

$= 2(2\sin x\cos x)(\cos x)$

$= \cancel{4}\ 4\sin x\cos^2 x$

$=$ R.H.S (proven).

$3\sin 3x + \sin x = 2\cos^2 x$

$4\sin x\cos^2 x = 2\cos^2 x$

$4\sin x\cos^2 x - 2\cos^2 x = 0$

$2\sin x\cos^2 x - \cos^2 x = 0$

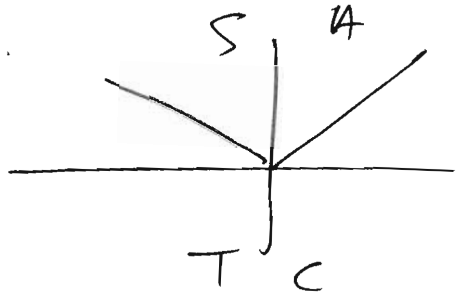
$\cos^2 x(2\sin x - 1) = 0$

$\cos^2 x = 0$ or $\sin x = \frac{1}{2}$

$\cos x = 0$ ~~(NA)~~

$x = \frac{\pi}{2}$

Basic $\angle = \frac{\pi}{6}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Ans 2

$\hookrightarrow x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

(9) Let Ann be x yrs old.
" Betty " y " "

Twice the square of Betty's age $= 2y^2$
Square of Ann's age $= x^2$

$6 \times$ Difference of age $= 6(x-y)$

$$\boxed{x^2 - 2y^2 = 6(x-y)} \quad \text{— (1)}$$

Sum of their ages $= x+y$
$5 \times$ difference of age $= 5(x-y)$

$$x+y = 5(x-y)$$
$$6y = 4x$$
$$y = \frac{2}{3}x \quad \text{— (2)}$$

(1) into (2)

$$x^2 - 2\left(\frac{2}{3}x\right)^2 = 6\left(x - \frac{2}{3}x\right)$$
$$x^2 - 2\left(\frac{4}{9}x^2\right) = 6x - 4x$$
$$\frac{1}{9}x^2 = 2x$$
$$x^2 - 18x = 0$$
$$x(x-18) = 0$$
$$x = 0 \text{ (N/A)} \quad \text{or} \quad x = 18$$
$$y = \frac{2}{3}(18) = 12$$

Ans: Ann is 18yrs old and Betty is 12yrs old.

(10) a) $D < 0$

$$(5)^2 - (4)(a)(2) < 0$$

$$25 - 8a < 0$$

$$8a > 25$$

$$a > \frac{25}{8}$$

$$a > 3\tfrac{1}{8}$$

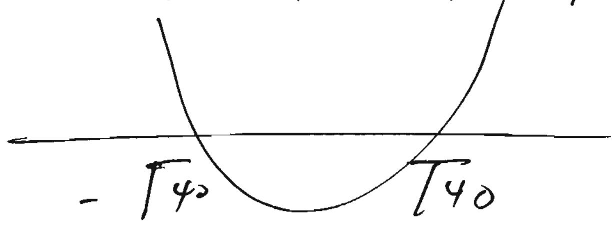
Ans Smallest integer value $= 4$

b) $D < 0$

$$(b)^2 - 4(-5)(-2) < 0$$

$$b^2 - 40 < 0$$

$$(b - \sqrt{40})(b + \sqrt{40}) < 0$$



$$-\sqrt{40} < b < \sqrt{40}$$

$$-\sqrt{40} = -6.32$$

Ans Small integer value of $b = -6$ #

(11) $\left(x+\frac{k}{x}\right)^7$

$$= x^7+\binom{7}{1}(x)^6\left(\frac{k}{x}\right)^1+\binom{7}{2}(x)^5\left(\frac{k}{x}\right)^2+\binom{7}{3}(x^4)\left(\frac{k}{x}\right)^3$$

$$= x^7+(7k)x^5+(21k^2)\cdot x^3+(35k^3)x+\dots$$

$$21k^2 = 35k^3$$

$$35k^3-21k^2=0$$

$$5k^3-3k^2=0$$

$$k^2(5k-3)=0$$

$$k=0\ (NA) \quad \text{or} \quad k=\frac{3}{5}\ \#$$

$$\left(x+\frac{k}{x}\right)^7 = \left(x^7+\frac{21}{5}x^5+\dots\right)$$

$$(1-5x^2)\left(x+\frac{3}{5x}\right)^7$$

$$= (1-5x^2)\left(x^7+\frac{21}{5}x^5+\dots\right)$$

$$= x^7-21x^7+\dots$$

$$= -20x^7+\dots$$

Co-efficient of $x^7 = -20$ #

(12)

$Y = mX + c$

$Y = mX + 1.3$

At $(11, 0.8)$

$0.8 = 11m + 1.3$

$-\frac{1}{2} = 11m$

$m = -\frac{1}{22}$

$Y = -\frac{1}{22}X + 1.3$

$\lg y = -\frac{1}{22}(x) + 1.3$

$y = 10^{-\frac{1}{22}x + 1.3}$

$y = (10^{1.3})(10^{-\frac{1}{22}})^x$

$k = 10^{1.3}$

$k = 19.9$

$k = 20$ (2sf)

$b = 0.9006$

$b = $ (2sf) 0.90

(ii) when $x = 8$,

$\lg y = -\frac{1}{22}(8) + 1.3$

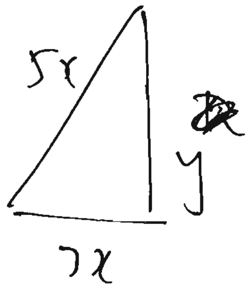
$y = \cancel{46.7} \cancel{(3sf)}$

$= \cancel{8.64}\ 8.637$

≈ 8.64 (3sf)

(13)

$(5x)^2 = y^2 + (3x)^2$



$y^2 = 16x^2$

$y = 4x$

$P = 360$

$5x + 5x + h + 6x + h = 360$

$2h = 360 - \cancel{76}x$

$h = 180 - 8x$

$A = (180 - 8x)(6x) + \frac{1}{2}(4x^2)(6x)$

$= 1080x - 48x^2 + 12x^2$

$A = 1080x - 36x^2$ (shown)

(ii) $\frac{dA}{dx} = 1080 - 72x$

when $\frac{dA}{dx} = 0$

$1080 - 72x = 0$

$x = \frac{1080}{72}$

$x = 15$

$A = 1080(15) - 36(15)^2$

Stationary value of $A = 8100$

$\frac{d^2A}{dx^2} = -72$ (max)

This stationary value is a maximum.