



ADDITIONAL MATHEMATICS

4037/21

Paper 2

May/June 2017

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

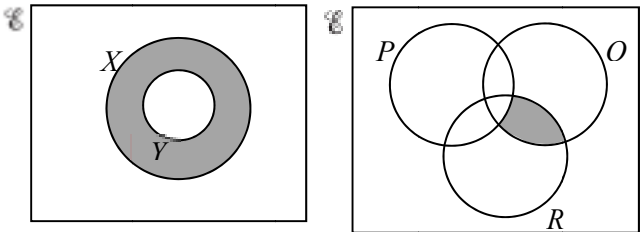
- M Method marks, awarded for a valid method applied to the problem.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	Integrates	M1	must be clear attempt to integrate at least one term
	$[y =]x^4 + x (+c)$	A1	Both terms correct
	$17 = 2^4 + 2 + c$	DM1	Substitution of $x = 2, y = 17$ to find c
	$y = x^4 + x - 1$ cao	A1	must have $y =$
2(a)	$2\sqrt{6} \times 3\sqrt{3} = 6\sqrt{18}$ oe	M1	method must be shown – simplifies and combines product
	$18\sqrt{2}$	A1	If all over common denominator then consider the product for M1A1
	$9\sqrt{2}$ oe soi leading to final answer of $27\sqrt{2}$	B1	
2(b)	$[x =] \frac{6 + \sqrt{3}}{2 - \sqrt{3}}$	M1	Expanding and making x subject – condone slips but must be of equivalent difficulty
	$[x =] \frac{6 + \sqrt{3}}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$ oe and multiplies out numerator and denominator	M1	numerator at least 3 terms; $12 + 2\sqrt{3} + 6\sqrt{3} + 3$
	$15 + 8\sqrt{3}$	A1	
3(i)	$\frac{2x}{x^2 + 1}$ final answer	B2	B1 for $\frac{1}{x^2 + 1} \times (ax + b)$, a or b must be non-zero
3(ii)	$\delta y = \text{their} \left(\frac{2(3)}{(3)^2 + 1} \right) \times h$ or better	M1	Substitutes $x = 3$ into <i>their</i> $\frac{dy}{dx}$ and multiplies by h
	$\frac{6}{10}h$ oe	A1	
4(a)(i)	36	B1	
4(a)(ii)	7	B1	
4(b)	$[y =] 5 \sin 4x + 7$	B4	B1 for each of 5, 4 and 7 and B1 for sine Accept $a = 5, b = 4, c = 7$ for B3

Question	Answer	Marks	Guidance
5(i)	$16 + 32ax + 24a^2x^2 + 8a^3x^3 + a^4x^4$	B2	B1 for at most 2 terms incorrect or missing or for correct but unsimplified form SC1 for $16 + 32ax + 24ax^2 + 8ax^3 + ax^4$ or all terms correct listed
5(ii)	$24a^2 = 8a^3$ and solves to given answer	B1	or verifies that $a = 3$ leads to coeff of 216 for both terms must be from correct terms in (i)
5(iii)	$x = -0.01$ or $ax = -0.03$ soi	M1	
	$16 + 32(3)(-0.01) + 24(9)(-0.01)^2$ leading to $16 - 0.96 + 0.0216$ or $15.06\dots$ isw	A1	Must show clear substitution into their expansion for A1 and reach a value which rounds to 15.1
6(i)	$(\mathbf{M} =) \begin{pmatrix} 90 & 10 & 30 \\ 0 & 45 & 0 \\ 25 & 0 & 15 \\ 10 & 0 & 100 \end{pmatrix}$	B1	columns and/or rows may be interchanged but must be consistent
6(ii)	$(\mathbf{LM} =) (1 \ 1 \ 1 \ 1) \begin{pmatrix} 90 & 10 & 30 \\ 0 & 45 & 0 \\ 25 & 0 & 15 \\ 10 & 0 & 100 \end{pmatrix} = (125 \ 55 \ 145)$	B1	Answer must be of correct order and must be consistent with a correct M
6(iii)	The total numbers of each type of ticket sold by all 4 cinemas oe	B1	
6(iv)	$(\mathbf{N} =) \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$	B1	Calculation not required
	The total income of all (4) cinemas or other valid comment e.g. total income from all ticket sales	B1	Total cost/value of tickets etc.
7(a)		B2	B1 for each
7(b)(i)	$n(M \cap D) = 0$ or $M \cap D = \emptyset$	B1	No additional brackets e.g. $M \cap D = \{\emptyset\}$ is B0

Question	Answer	Marks	Guidance
7(b)(ii)		B3	<p>B1 correct intersection of circles with 12 and 25 correct</p> <p>B1 33, 2, 11 correctly placed</p> <p>B1FT 17; must be on the Venn diagram and identified as the required answer</p> <p>FT on 100– (sum of <i>their</i> 5 correctly positioned values)</p>
8(a)	${}^{30}P_2 = 870$	B1	
8(b)(i)	${}^2C_1 \times {}^{14}C_{10}$ oe (2×1001)	M1	Condone $\binom{14}{4}$ for $\binom{14}{10}$
	2002	A1	implies M1
8(b)(ii)	$({}^2C_1 \times {}^5C_4 \times {}^9C_6) + ({}^2C_1 \times {}^5C_5 \times {}^9C_5)$ oe $(840 + 252)$ ${}^2C_1 \times {}^{14}C_{10}$ – or $({}^2C_1 \times {}^5C_1 \times {}^9C_9 + {}^2C_1 \times {}^5C_2 \times {}^9C_8 + {}^2C_1 \times {}^5C_3 \times {}^9C_7)$ $\{2002 - (10 + 80 + 720)\}$	M3	<p>M3 for fully correct method soi</p> <p>M2 for all necessary products but not summed with no extra products seen soi</p> <p>M1 for one correct three term product soi</p>
	1092	A1	implies M3
9(i)	Substitution of $y = 2(1 - x)$	M1	Must be attempt at full substitution. Condone one sign error in substitution. Condone omission of = 0 or incorrect rhs
	$-3x^2 + 2x + 1 = 0$ oe $(3x^2 - 2x - 1 = 0)$	A1	Terms collected
	Solving <i>their</i> quadratic found from eliminating one variable $(3x + 1)(1 - x)$ or $(3x + 1)(x - 1)$	M1	can be implied by a correct pair of x values
	$\left(-\frac{1}{3}, \frac{8}{3}\right)$ oe and $(1, 0)$ oe isw nfw	A2	A1 for each or A1 for a correct pair of x -coordinates or a correct pair of y -coordinates

Question	Answer	Marks	Guidance										
9(ii)	$[m =] \frac{1}{2} \text{ cao}$	B1											
	$\left(\frac{1}{3}, \frac{4}{3}\right)$	B1	FT										
	$y - \text{their} \frac{4}{3} = \text{their} \frac{1}{2} \left(x - \text{their} \frac{1}{3}\right)$	M1	or $y = \text{their} \frac{1}{2}x + c$ and substitutes their midpoint and reaches $c = \dots$										
	$6y - 3x = 7$	A1	allow any equivalent form with integer coeffs/constant										
10(i)	<table border="1"> <tbody> <tr> <td>t</td> <td>1</td> <td>1.5</td> <td>2</td> <td>2.5</td> </tr> <tr> <td>$\ln P$</td> <td>1.48</td> <td>2.12</td> <td>2.76</td> <td>3.4(0)</td> </tr> </tbody> </table>	t	1	1.5	2	2.5	$\ln P$	1.48	2.12	2.76	3.4(0)	M1	allow $\ln P$ values to 1 dp rounded or truncated (1.5, 2.1, 2.8, 3.4)
	t	1	1.5	2	2.5								
$\ln P$	1.48	2.12	2.76	3.4(0)									
single ruled line drawn within tolerance at least for t between 1 and 2.5	A1	All points within 1 square of line / must not pass through origin											
10(ii)	$e^{\text{their}3}$	M1											
	18 to 22.2	A1											
10(iii)	$(0, c)$ with $0.1 \leq c \leq 0.3$ (0.2)	B1	allow $y = c$ condone $c = \dots$										
	m in the range $1.25 \leq m \leq 1.34$ (1.28)	B1											
10(iv)	$\ln P = (\text{their}1.28)t + \text{their}0.2$	M1	or $\ln P = (\ln b)t + \ln a$										
	$P = e^{(\text{their}1.28)t + \text{their}0.2}$	M1	or $\ln b = m = \text{their}1.28$ and $\ln a = c = \text{their}0.2$										
	$P = e^{\text{their}0.2} e^{(\text{their}1.28)t}$	A1	or $1.10 \leq a \leq 1.35$ $3.49 \leq b \leq 3.82$										
10(v)	$1000 * e^{\text{their}0.2} \times e^{\text{their}1.28t}$ or $1000 * \text{their} a \times \text{their} b^t$	M1	A correct relationship e.g. $1.3t * \ln(1000) - 0.2$ where * is = or an inequality sign										
	5.3	A1	5.2 to 5.5 must be to 1dp										

Question	Answer	Marks	Guidance
11(i)	$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$ oe	B2	B1 for either $\cot x = \frac{\cos x}{\sin x}$ or $\tan x = \frac{\sin x}{\cos x}$ used B1 for correctly placing over a common denominator or for splitting into 3 correct terms not just for stating or working from both sides
	Valid use of Pythagorean identity e.g. $\cos^2 x + \sin^2 x = 1$	B1	
	Simplification to $\sec x$ (correct solution only)	B1	not if working from both sides
11(ii)	$\cos x = \frac{1}{2}$ soi	M1	
	60, 300	A1	Correct pair
	$\cos x = -\frac{1}{2}$ soi	M1	
	120, 240	A1	Correct pair
12(i)	$\left[v = \frac{d(3t - \cos 5t + 1)}{dt} = \right] 3 + 5 \sin 5t$	B2	B1 for either with no other terms or for both with 1 extra
	<i>their</i> $(3 + 5 \sin 5t) = 0$	M1	Must be from an attempt to differentiate
	awrt 0.76	A1	0.7570187525
	awrt 1.13	A1	1.12793684
	substitutes <i>their</i> t values into s (4.07..., 3.58...)	DM1	must be two values
	0.48 to 0.49 [m]	A1	Final A1 may imply earlier A1 s
12(ii)	$25 \cos 5t$	M1	Differentiating <i>their</i> v correctly providing at least 2 terms with one trig function
	-25	A1	Ignore +25 following -25