Cambridge O Level

Cambridge International Examinations
Cambridge Ordinary Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7631681188

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2015**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

DC (KN/SW) 107047/1

CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

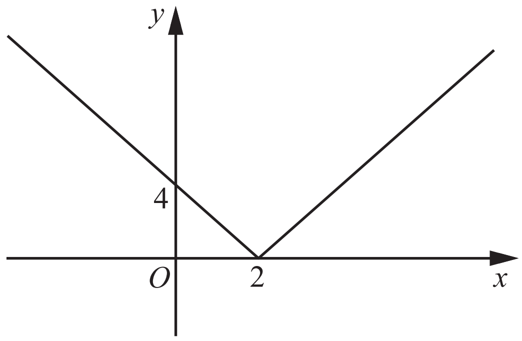
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Write $\log_{27}x$ as a logarithm to base 3. [2]

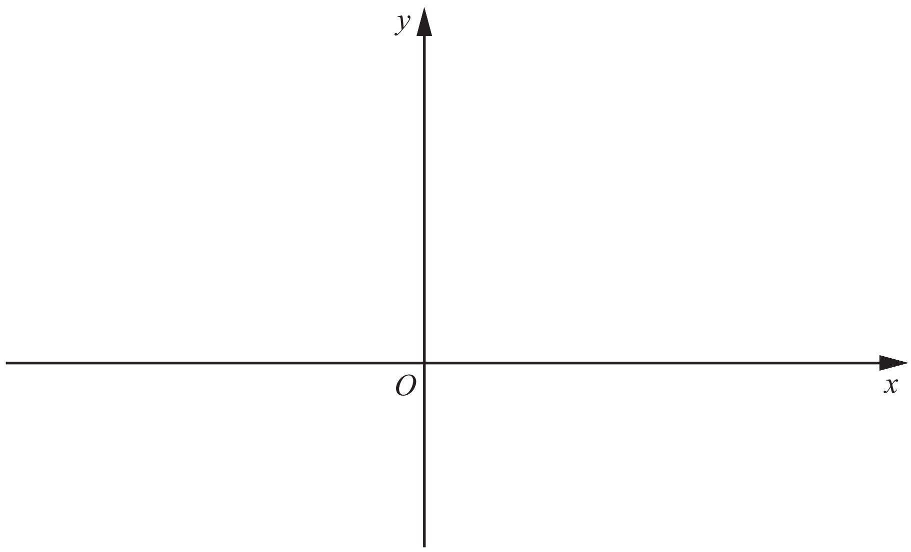
(b) Given that $\log_a y = 3(\log_a 15 - \log_a 3) + 1$, express y in terms of a. [3]

2 **(a)**



The diagram shows the graph of $y = |f(x)|$ passing through $(0, 4)$ and touching the x-axis at $(2, 0)$. Given that the graph of $y = f(x)$ is a straight line, write down the two possible expressions for $f(x)$. [2]

(b) On the axes below, sketch the graph of $y = e^{-x} + 3$, stating the coordinates of any point of intersection with the coordinate axes. [3]



3 **(a)** Find the matrix **A** if $4\mathbf{A} + 5\begin{pmatrix} 4 & 0 & -1 \\ 3 & -2 & 5 \end{pmatrix} = \begin{pmatrix} 52 & -8 & 19 \\ 31 & 2 & 65 \end{pmatrix}$. [2]

(b) $\mathbf{P} = \begin{pmatrix} 30 & 25 & 65 \\ 70 & 15 & 80 \\ 50 & 40 & 30 \\ 40 & 20 & 75 \end{pmatrix}$ $\qquad \mathbf{Q} = \begin{pmatrix} 650 & 500 & 450 & 225 \end{pmatrix}$

The matrix **P** represents the number of 4 different televisions that are on sale in each of 3 shops. The matrix **Q** represents the value of each television in dollars.

(i) State, without evaluation, what is represented by the matrix **QP**. [1]

(ii) Given that the matrix $\mathbf{R} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, state, without evaluation, what is represented by the matrix **QPR**. [1]

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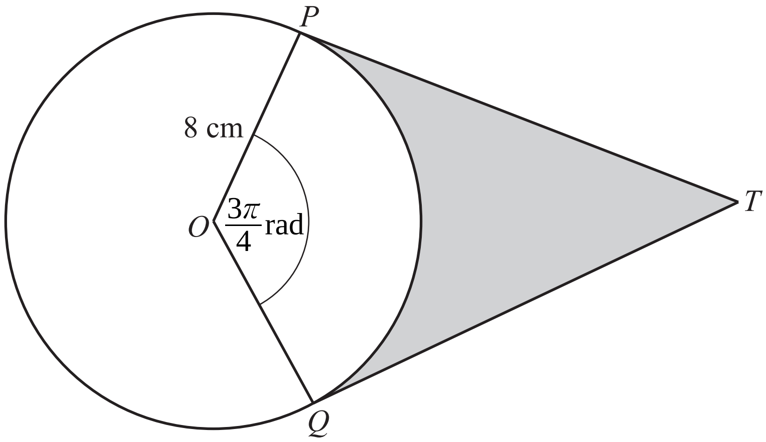


The diagram shows a circle, centre O, radius 8 cm. The points P and Q lie on the circle. The lines PT and QT are tangents to the circle and angle $POQ = \frac{3\pi}{4}$ radians.

(i) Find the length of PT. [2]

(ii) Find the area of the shaded region. [3]

(iii) Find the perimeter of the shaded region. [2]

5 **(a)** A lock can be opened using only the number 4351. State whether this is a permutation or a combination of digits, giving a reason for your answer. [1]

(b) There are twenty numbered balls in a bag. Two of the balls are numbered 0, six are numbered 1, five are numbered 2 and seven are numbered 3, as shown in the table below.

Number on ball	0	1	2	3
Frequency	2	6	5	7

Four of these balls are chosen at random, without replacement. Calculate the number of ways this can be done so that

(i) the four balls all have the same number, [2]

(ii) the four balls all have different numbers, [2]

(iii) the four balls have numbers that total 3. [3]

6 A particle P is projected from the origin O so that it moves in a straight line. At time t seconds after projection, the velocity of the particle, v ms^{-1}, is given by $v = 2t^2 - 14t + 12$.

(i) Find the time at which P first comes to instantaneous rest. [2]

(ii) Find an expression for the displacement of P from O at time t seconds. [3]

(iii) Find the acceleration of P when $t = 3$. [2]

7 **(a)** The four points O, A, B and C are such that

$$\overrightarrow{OA} = 5\mathbf{a}, \qquad \overrightarrow{OB} = 15\mathbf{b}, \qquad \overrightarrow{OC} = 24\mathbf{b} - 3\mathbf{a}.$$

Show that B lies on the line AC. [3]

(b) Relative to an origin O, the position vector of the point P is $\mathbf{i} - 4\mathbf{j}$ and the position vector of the point Q is $3\mathbf{i} + 7\mathbf{j}$. Find

(i) $|\overrightarrow{PQ}|$, [2]

(ii) the unit vector in the direction $\overrightarrow{PQ}$, [1]

(iii) the position vector of M, the mid-point of PQ. [2]

8 **(a)** **(i)** Find $\int e^{4x+3}\,dx$. [2]

(ii) Hence evaluate $\int_{2.5}^{3} e^{4x+3}\,dx$. [2]

(b) **(i)** Find $\int \cos\left(\frac{x}{3}\right)dx$. [2]

(ii) Hence evaluate $\int_{0}^{\frac{\pi}{6}} \cos\left(\frac{x}{3}\right)dx$. [2]

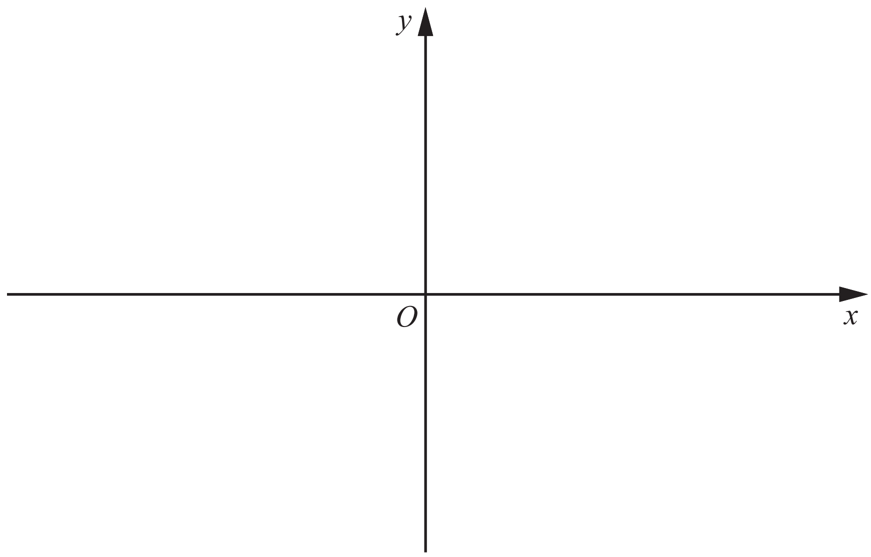
(c) Find $\int (x^{-1} + x)^2 \mathrm{d}x$. [4]

9 **(a)** Find the set of values of x for which $4x^2 + 19x - 5 \leqslant 0$. [3]

(b) **(i)** Express $x^2 + 8x - 9$ in the form $(x + a)^2 + b$, where a and b are integers. [2]

(ii) Use your answer to part **(i)** to find the greatest value of $9 - 8x - x^2$ and the value of x at which this occurs. [2]

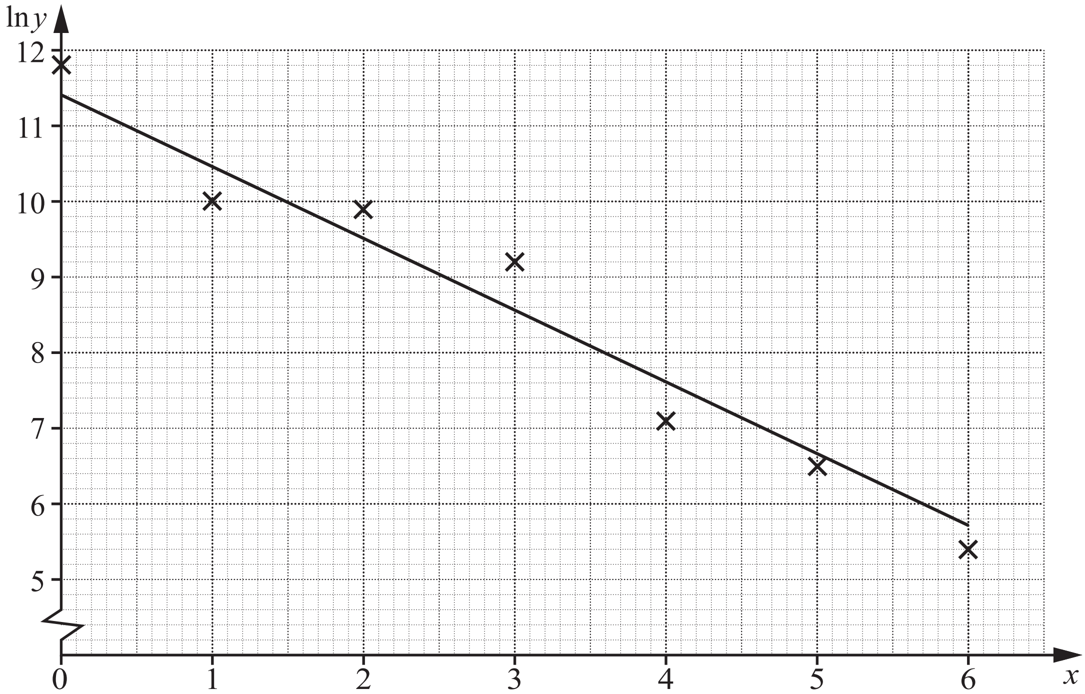
(iii) Sketch the graph of $y = 9 - 8x - x^2$, indicating the coordinates of any points of intersection with the coordinate axes. [2]



10 The relationship between experimental values of two variables, x and y, is given by $y = Ab^x$, where A and b are constants.

(i) By transforming the relationship $y = Ab^x$, show that plotting $\ln y$ against x should produce a straight line graph. [2]

(ii) The diagram below shows the results of plotting $\ln y$ against x for 7 different pairs of values of variables, x and y. A line of best fit has been drawn.



By taking readings from the diagram, find the value of A and of b, giving each value correct to 1 significant figure. [4]

(iii) Estimate the value of y when $x = 2.5$. [2]

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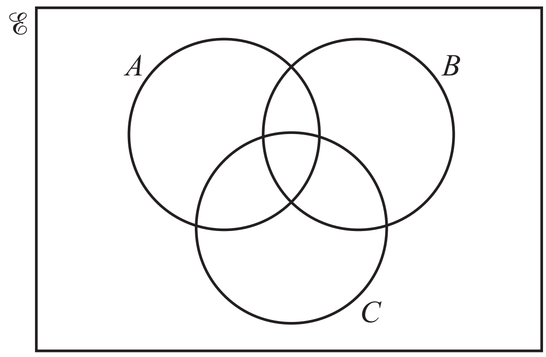

The Venn diagram above shows the sets A, B and C. It is given that

$n(A \cup B \cup C) = 48$,
$n(A) = 30$, $n(B) = 25$, $n(C) = 15$,
$n(A \cap B) = 7$, $n(B \cap C) = 6$, $n(A' \cap B \cap C') = 16$.

(i) Find the value of x, where $x = n(A \cap B \cap C)$. [3]

(ii) Find the value of y, where $y = n(A \cap B' \cap C)$. [3]

(iii) Hence show that $A' \cap B' \cap C = \varnothing$. [1]

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