CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the May/June 2015 series

4037 ADDITIONAL MATHEMATICS

4037/21   Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.
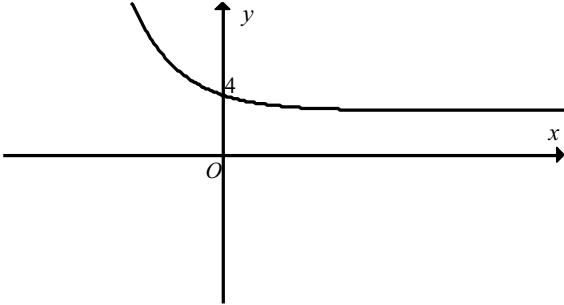
Cambridge will not enter into discussions about these mark schemes.

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Abbreviations

awrt   answers which round to
cao   correct answer only
dep   dependent
FT   follow through after error
isw   ignore subsequent working
oe   or equivalent
rot   rounded or truncated
SC   Special Case
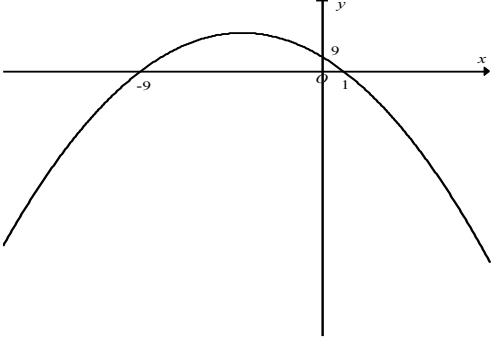soi   seen or implied
www   without wrong working

1	(a)	$\dfrac{\log_3 x}{\log_3 27}$	M1	Can use other interim bases if all correct but M1 when in base 3 only
		$\dfrac{\log_3 x}{3}$ isw	A1	NOT $\log_3 x \div 3$
	(b)	$\log_a 15 - \log_a 3 = \log_a 5$ soi	M1	
		$\log_a 5^3$ or $\log_a a$	M1	
		$\log_a y = \log_a 125a \Rightarrow y = 125a$	A1	
2	(a)	$[\mathrm{f}(x) =]2x - 4$ and $[\mathrm{f}(x) =]-2x + 4$	B1,B1	Condone $y = \ldots$
	(b)		B1 B1 B1	correct shape; y intercept marked or seen nearby; intent to tend to $y = 3$ (i.e. not tending to or cutting x-axis)
3	(a)	$\mathbf{A} = \dfrac{1}{4}\left[\begin{pmatrix} 51 & -8 & 19 \\ 31 & 2 & 65 \end{pmatrix} - \begin{pmatrix} 20 & 0 & -5 \\ 15 & -10 & 25 \end{pmatrix}\right]$	M1	
		$\mathbf{A} = \begin{pmatrix} 8 & -2 & 6 \\ 4 & 3 & 10 \end{pmatrix}$	A1	Integer values
	(b) (i)	The (total) value of the stock in **each** of the 3 shops	B1	Must have "each" oe
	(ii)	The **total** value of the stock in all 3 shops	B1	Must have "total" oe

4	**(i)**	$\dfrac{PT}{8} = \tan\left(\dfrac{3\pi}{8}\right)$ oe	M1	$\dfrac{PT}{\sin\dfrac{3\pi}{8}} = \dfrac{8}{\sin\dfrac{\pi}{8}}$
		$PT = 19.3$	A1	awrt 19.3
	(ii)	$\dfrac{1}{2} \times 8^2 \times \dfrac{3\pi}{4}$ oe (75.4)	M1	or $\dfrac{1}{2} \times 8^2 \times \dfrac{3\pi}{8}$
		$8\tan\left(\dfrac{3\pi}{8}\right) \times 8 - their$ sector oe $(=154.5\text{-}'75.4')$	M1	$8 \times their\ PT - their$ sector
		79.1	A1	awrt 79.1
	(iii)	$8\left(\dfrac{3\pi}{4}\right)$ oe (18.8)	M1	
		$\left[6\pi + 16\tan\left(\dfrac{3\pi}{8}\right)\right] = 57.5$	A1	Accept 57.4 to 57.5
5	**(a)**	Permutation because the order matters oe	B1	
	(b) (i)	$^6C_4 + {}^5C_4 + {}^7C_4$ 55	M1 A1	3 correct terms added
	(ii)	$^2C_1 \times {}^6C_1 \times {}^5C_1 \times {}^7C_1$ 420	M1 A1	4 correct terms multiplied
	(iii)	$^6C_3 \times {}^2C_1$ or $^2C_2 \times {}^5C_1 \times {}^6C_1$ summation 70	M1 M1 A1	for either correct product adding two correct products If 0 scored, then SC1 for 1,1,1,0 and 0,0,2,1 seen
6	**(i)**	$2t^2 - 14t + 12 = 0$ $(t-1)(t-6)$ oe $(t=)\ 1$	M1 A1	Can use formula, etc. If $t = 1$ with no working, then M1A1
	(ii)	$\int\left(2t^2 - 14t + 12\right)\mathrm{d}t$ $(s=)\ \dfrac{2t^3}{3} - \dfrac{14t^2}{2} + 12t$	M1 A2,1,0	 -1 for each error or for $+c$ left in or limits introduced
	(iii)	$(a=)\dfrac{\mathrm{d}v}{\mathrm{d}t}\ \ (4t - 14)$ $[4(3) - 14 =] -2$ cao	M1 A1	

7	(a)	$\overrightarrow{AB} = 15\mathbf{b} - 5\mathbf{a} = 5(3\mathbf{b} - \mathbf{a})$ or	B1	Any correct simplified vector
		$\overrightarrow{BC} = 24\mathbf{b} - 3\mathbf{a} - 15\mathbf{b} = 3(3\mathbf{b} - \mathbf{a})$ or	B1	Any second simplified vector
		$\overrightarrow{AC} = 24\mathbf{b} - 3\mathbf{a} - 5\mathbf{a} = 8(3\mathbf{b} - \mathbf{a})$		
		Comment: e.g. the vectors are scalar multiples of each other AND they have a common point (A, B or C as appropriate)	B1dep	Dep on both B marks being awarded.
	(b) (i)	$2\mathbf{i} + 11\mathbf{j}$ soi	B1	
		$\Rightarrow \sqrt{2^2 + 11^2}$		
		$\sqrt{125}$ or $5\sqrt{5}$ or 11.2 (3 s.f.) or better)	B1fT	ft *their* $2\mathbf{i} + 11\mathbf{j}$ (not $\overrightarrow{OP}$ or $\overrightarrow{OQ}$)
	(ii)	$\dfrac{1}{5\sqrt{5}}(2\mathbf{i} + 11\mathbf{j})$ isw	B1fT	ft *their* answers from **(i)**
	(iii)	$\dfrac{\mathbf{i} - 4\mathbf{j} + 3\mathbf{i} + 7\mathbf{j}}{2}$ or $\mathbf{i} - 4\mathbf{j} + \dfrac{2\mathbf{i} + 11\mathbf{j}}{2}$ or $3\mathbf{i} + 7\mathbf{j} - \dfrac{2\mathbf{i} + 11\mathbf{j}}{2}$	M1	
		$2\mathbf{i} + 1.5\mathbf{j}$	A1	
8	(a) (i)	$k\mathrm{e}^{4x+3} (+c)$ oe	M1	any constant, non-zero k
		$k = \dfrac{1}{4}$ oe	A1	
	(ii)	$\dfrac{1}{4}\left(\mathrm{e}^{4(3)+3} - \mathrm{e}^{4(2.5)+3}\right)$ or better	DM1	ft *their* integral attempt
		$706\,650.99\ldots = 707\,000$ to 3 sf or better	A1	Accept $\dfrac{1}{4}\left(\mathrm{e}^{15} - \mathrm{e}^{13}\right)$
	(b) (i)	$k\sin\left(\dfrac{x}{3}\right) (+c)$	M1	any constant, non-zero k
		$k = 3$	A1	
	(ii)	$3\sin\left(\dfrac{\pi}{6} \times \dfrac{1}{3}\right) - 3\sin(0)$	DM1	Dep on *their* integral attempt in sin; condone omission of lower limit
		$0.520\,944\ldots = 0.521$ to 3 sf or better	A1	Accept $3\sin\left(\dfrac{\pi}{18}\right)$
	(c)	$\displaystyle\int\left(x^{-2} + 2 + x^2\right)\mathrm{d}x = \dfrac{x^{-1}}{-1} + 2x + \dfrac{x^3}{3}$	B1 M1 A1	Expands – accept unsimplified integration of *their* 3 term expansion Fully correct
		$+ c$	B1	$+c$

9	(a)	$(4x-1)(x+5)\ [\leqslant 0]$	M1	Solves quadratic
		critical values $\dfrac{1}{4}$ and -5 soi	A1	
		$-5 \leqslant x \leqslant \dfrac{1}{4}$	A1	Accept: $\left[-5, \dfrac{1}{4}\right]$; $-5 \leq x$ AND $x \leq 0.25$
	(b) (i)	$(x+4)^2 - 25$ or $a = 4$ and $b = -25$	B1, B1	
	(ii)	(Greatest value =) 25 $x = -4$	B1ft B1ft	Must be clear
	(iii)		B1 B1	Correct shape with maximum in second quadrant and crossing positive and negative axes correctly All 3 intercepts correctly shown on graph
10	(i)	$\ln y = \ln(Ab^x) \implies \ln y = \ln A + \ln b^x$ $\implies \ln y = \ln A + x\ln b$	M1 A1	
	(ii)	$\ln A = 11.4 \implies A = e^{their\,11.4}$ $A = 90\,000$ cao $\ln b = -1$ $b = 0.4$ cao	M1 A1 M1 A1	condone misread of scale for M1 (11.2 only) Allow awrt -1
	(iii)	$x = 2.5 \implies \ln y = 9$ $y = e^9$ or 8000 to 1 sf	M1 A1	Allow awrt 8100
11	(i)	$7 - x, x, 6 - x$ oe	B1	
		their attempt at $7 - x + x + 6 - x + 16 = 25$ oe	M1	
		$x = 4$	A1	Condone $x = 4$ for all 3 marks
	(ii)	$23 - y, y, 9 - y$ oe	B1	or $n(A \cup C) = 48 - 16 = 32$
		$48 = 30 + 25 + 15 - 7 - 6 - (their\ 4 + y) + their\ 4$ oe soi	M1	or $32 = 30 + 15 - (their\ 4 + y)$ or $48 = (23 - y) + 3 + 16 + y + 4$ $\qquad\qquad\qquad + 2 + (9 - y)$
		$y = 9$	A1	Condone $y = 9$ for all 3 marks
	(iii)	$n(C) = 15$ and $y + n(B \cap C) = 9 + 6 = 15$ [and so $A' \cap B' \cap C = \varnothing$].	B1	or equivalent deduction