CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2014 series

4037 ADDITIONAL MATHEMATICS

4037/13 Paper 1, maximum raw mark 80

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1	$a = 3$ $b = 2$ $c = 4$	**B1** **B1** **B1**	
2	$x^2 = 16$ or $y^2 - 4y + 3 = 0$	**M1**	for correct elimination of one variable and attempt to form a quadratic equation in *x* or *y*.
	$x = \pm 4$ $y = 1, 3$ Points (–4, 1) and (4, 3)	**A1** **A1**	
	Line $AB = \sqrt{8^2 + 2^2}$	**M1**	for use of Pythagoras theorem
	$= \sqrt{68}$ or $2\sqrt{17}$	**A1**	allow either form
3 (i)	$\text{n}(A) = 2$ $\text{n}(B) = 3$ $\text{n}(C) = 0$	**B1** **B1** **B1**	**B0** for n(2), {2},{0}, ∅ , {} etc.
(ii)	$A \cup B = \{-1, -2, -3, 3\}$	**B1**	
(iii)	$A \cap B = \{-2\}$	**B1**	
(iv)	ξ, 'the universal set', R, 'real numbers', $\{x : x \in \quad \}$	**B1**	
4 (a)	$\tan x = -\frac{5}{3}$	**M1**	Correct statement or $\tan x = -1.67$
	$x = 121.0°,\ 301.0°$	**A1** **A1ft**	**A1** for either correct solution **ft** from *their* first solution
(b)	$\sin\left(3y + \frac{\pi}{4}\right) = \frac{1}{2}$	**M1**	for dealing correctly with cosec and attempt to solve subsequent equation
	$3y + \frac{\pi}{4} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$	**A1**	for $\frac{\pi}{6}$, $\frac{5\pi}{6}$, or $\frac{13\pi}{6}$, or $\frac{17\pi}{6}$
	$3y = -\frac{\pi}{12}, \frac{7\pi}{12}, \frac{23\pi}{12}, \frac{31\pi}{12}$	**DM1**	for correct order of operations
	$y = \frac{7\pi}{36}, \frac{23\pi}{36}, \frac{31\pi}{36}$ (0.611, 2.01 and 2.71)	**A1, A1**	**A1** for one correct solution **A1** for both the other correct solutions and no others in range.

5 (a) (i)	$\begin{pmatrix} 12 & 2 & 1 \\ 9 & 3 & 0 \\ 8 & 5 & 1 \\ 11 & 2 & 0 \end{pmatrix}\begin{pmatrix} 0.5 \\ 0.4 \\ 0.45 \end{pmatrix} = \begin{pmatrix} 7.25 \\ 5.70 \\ 6.45 \\ 6.30 \end{pmatrix}$	M1	for correct compatible matrices the correct order. Allow 1 error in each matrix. Allow if done in cents
	or $\begin{pmatrix} 0.5 & 0.4 & 0.45 \end{pmatrix}\begin{pmatrix} 12 & 9 & 8 & 11 \\ 2 & 3 & 5 & 2 \\ 1 & 0 & 1 & 0 \end{pmatrix}$	DM1	for a correct method for multiplying their matrices to obtain an appropriate 4 by 1 or 1 by 4 matrix.
	$= \begin{pmatrix} 7.25 & 5.70 & 6.45 & 6.30 \end{pmatrix}$	A2,1,0	**A2** all correct or **A1** 3 correct elements.
(ii)	25.70	B1	Allow 25.7
(b)	$\mathbf{Y} = \mathbf{X}^{-1}$ or $\mathbf{Y} = \mathbf{X}^{-1}\mathbf{I}$	M1	for matrix algebra
	$\mathbf{Y} = \frac{1}{22}\begin{pmatrix} 1 & -4 \\ 5 & 2 \end{pmatrix}$ or $\begin{pmatrix} \frac{1}{22} & -\frac{4}{22} \\ \frac{5}{22} & \frac{2}{22} \end{pmatrix}$	A1	for $\frac{1}{22}\begin{pmatrix} & \\ & \end{pmatrix}$
		A1	for $k\begin{pmatrix} 1 & -4 \\ 5 & 2 \end{pmatrix}$
	Alternative method: $\begin{pmatrix} 2 & 4 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $2a + 4c = 1,\ 2b + 4d = 0$ $-5a + c = 0,\ -5b + d = 1$	M1	for a complete method using simultaneous equations
		A1	$a = \frac{1}{22}$ and $c = \frac{5}{22}$ or $b = -\frac{4}{22}$ and $d = \frac{2}{22}$
	leading to $= \frac{1}{22}\begin{pmatrix} 1 & -4 \\ 5 & 2 \end{pmatrix}$ oe	A1	for correct matrix

6 (i)	$\cos 0.9 = \frac{6}{OC}$ or $\frac{OC}{\sin 0.9} = \frac{12}{\sin(\pi - 1.8)}$ $OC = \frac{6}{\cos 0.9} = 9.652...$	**M1**	for correct use of cosine, cosine rule or any other valid method
	or $OC = \frac{12 \sin 0.9}{\sin(\pi - 1.8)} = 9.652...$	**A1**	for manipulating correctly to $OC = 9.652(35\ldots)$ Must have 4th figure (or more) for rounding
(ii)	Perimeter = $(0.9 \times 12) + 9.652 + (12 - 9.652)$	**B1** **M1**	for arc length for attempt to add the correct lengths
	$= 22.8$	**A1**	
(iii)	Area $= \left(\frac{1}{2} \times 12^2 \times 0.9\right) - \left(\frac{1}{2} \times 9.652^2 \sin(\pi - 1.8)\right)$	**B1**	for area of sector, allow unsimplified
		B1	for area of isosceles triangle $\frac{1}{2}(9.65(2...))^2 \sin(\pi - 1.8)$ or $\frac{1}{2}(12 \times 6 \tan 0.9)$ or $\frac{1}{2}(12 \times 9.65(2...) \times \sin 0.9)$, allow unsimplified.
	64.8 – 45.36 = 19.4 to 19.5	**B1**	for answer in range 19.4 to 19.5
	Alternative Method: $\frac{1}{2}(12 - 9.652) \times 9.652 \times \sin 1.8$	**B1**	for area of triangle *ACB*, unsimplified
	$\frac{1}{2} 12^2 (0.9 - \sin 0.9)$	**B1**	for area of segment, unsimplified
	11.04 + 8.40 Area =19.4 to 19.5	**B1**	answer in range 19.4 to 19.5
7	$1 + 2\log_5 x = \log_5(18x - 9)$	**B1, B1**	**B1** for dealing with '1', **B1** for dealing with '2'
	$\log_5 5 + \log_5 x^2 = \log_5(18x - 9)$	**M1**	for a correct use of addition or subtraction of logarithms
	$5x^2 = 18x - 9$ $(5x - 3)(x - 3) = 0$	**DM1**	for elimination of logarithms to form a 3 term quadratic and for solution of quadratic
	$x = \frac{3}{5}$, 3	**A1**	for both x values

8 (i)	$f'(x)=\left(x\times\frac{3x^2}{x^3}\right)+\left(\ln x^3\right)$	**M1**	for differentiation of a product
		B1	for differentiation of $\ln x^3$
	$=3+3\ln x,\ \ =3(1+\ln x)$	**A1**	for simplification to gain <u>given answer</u>
	or $f(x)=3x\ln x$	**B1**	for use of $\ln x^3=3\ln x$
	$f'(x)=\left(3x\times\frac{1}{x}\right)+3\ln x$,	**M1**	for differentiation of a product
	$=3(1+\ln x)$	**A1**	for simplification to gain <u>given answer</u>
(ii)	$\int 3(1+\ln x)\mathrm{d}x=x\ln x^3$ or $3x\ln x$	**M1**	for realising that differentiation is the reverse of integration and using **(i)**
	$\int 1+\ln x\mathrm{d}x=\frac{1}{3}x\ln x^3$ or $x\ln x$	**A1**	
(iii)	$x\ln x-\int 1\mathrm{d}x$ or $\left[\frac{1}{3}x\ln x^3\right]-\int 1\mathrm{d}x$	**DM1**	for using answer to **(ii)** and subtracting $\int 1\mathrm{d}x$ dependent on M mark in **(ii)**
	$[x\ln x-x]_1^2$ or $\left[\frac{1}{3}x\ln x^3-x\right]_1^2$	**DM1**	for correct application of limits
	$=2\ln 2-2+1$ $=-1+\ln 4$	**A1**	from correct working
9 (a)	$5^p=625$, so $p=4$	**B1**	
	${}^4C_1 5^{p-1}(-q)=-1500$ $4\times125(-q)=-1500$	**M1**	*their p* substituted in ${}^pC_1 5^{p-1}(-q)$ or in ${}^pC_1 5^{p-1}(-qx)$ unsimplified
	$q=3$	**A1**	
	${}^4C_2 5^{p-2}q^2=r$	**M1**	*their p* and *q* substituted in ${}^pC_2 5^{p-2}(-q)^2$ or ${}^pC_2 5^{p-2}(-qx)^2$ unsimplified
	$r=1350$	**A1**	
(b)	${}^{12}C_3(2x)^9\left(\frac{1}{4x^3}\right)^3$	**M1**	for identifying correct term
		DM1	for attempt to evaluate correct expression
	Term is 1760	**A1**	must be evaluated

10 (a)	$\frac{5^x}{5^{2(3y-2)}}=1$ or $\frac{3^x}{3^{3(y-1)}}=3^4$ oe	**M1**	for obtaining one correct e... in powers of 5, 3, 25, 27 or 8...
	$x=6y-4$	**A1**	for $x=6y-4$ oe linear equation
	$x=3y+1$	**A1**	for $x=3y+1$ oe linear equation
		M1	for attempt to solve linear simultaneous equations which have been obtained correctly
	Leads to $x=6,\ y=\frac{5}{3}$	**A1**	for both.
(b)	Using the cosine rule: $\left(1+2\sqrt{3}\right)^2=\left(2+\sqrt{3}\right)^2+2^2-4\left(2+\sqrt{3}\right)\cos A$	**M1**	for correct substitution in cosine rule, may use in form of $\cos A=\ldots$
	$\cos A=\frac{\left(13+4\sqrt{3}\right)-\left(7+4\sqrt{3}\right)-4}{-4\left(2+\sqrt{3}\right)}$ oe	**DM1**	for attempt to make cos*A* subject and simplify
	$\cos A=\frac{-1}{2\left(2+\sqrt{3}\right)}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$	**DM1**	for rationalisation.
	$\cos A=-1+\frac{\sqrt{3}}{2}$	**A1**	

11 (i)	$\frac{dy}{dx}=(x+5)2(x-1)+(x-1)^2$	**M1** **A1**	for differentiation of a product, allow unsimplified correct
	$\frac{dy}{dx}=(x-1)(3x+9)$		
	When $\frac{dy}{dx}=0$	**DM1**	for equating to zero and solution of quadratic
	$x=1$	**A1**	
	$x=-3$	**A1**	
	Alternative method:		
	$y=x^3+3x^2-9x+5$	**M1**	for expansion of brackets and differentiation of each term of a 4 term cubic
	$\frac{dy}{dx}=3x^2+6x-9$	**A1**	
	When $\frac{dy}{dx}=0$	**DM1**	for equating to zero and solution of 3 term quadratic
	$x=1$	**A1**	from correct quadratic equation
	$x=-3$	**A1**	from correct quadratic equation
(ii)	$\int x^3+3x^2-9x+5\,dx$	**M1**	for correct attempt to obtain and integrate a 4 term cubic
	$=\frac{x^4}{4}+x^3-\frac{9x^2}{2}+5x\ (+c)$	**A2,1,0**	**A2** for 4 correct terms or **A1** for 3 correct terms
(iii)	$\left[\frac{x^4}{4}+x^3-\frac{9x^2}{2}+5x\right]_{-5}^{1}$	**M1**	for correct substitution of limits 1 and –5 for *their* **(ii)**
	$=\left(\frac{1}{4}+1-\frac{9}{2}+5\right)-\left(\frac{625}{4}-125-\frac{225}{2}-25\right)$		
	$=108$	**A1**	
(iv)	When $x=-3, y=32$	**M1**	for realising that the *y*-coordinate of the maximum point is needed.
	$k>32$	**A1**	