

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

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MARK SCHEME for the October/November 2014 series

4037 ADDITIONAL MATHEMATICS

4037/12

Paper 1, maximum raw mark 80

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

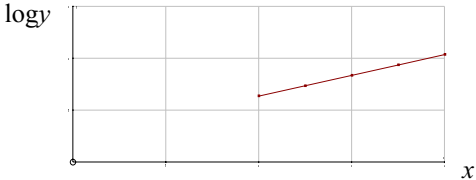
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<p>4 (i)</p> <p>(ii)</p>	$5y^2 - 7y + 2 = 0$ $(5y - 2)(y - 1) = 0$ $y = \frac{2}{5}, x = \frac{\ln 0.4}{\ln 5}$ $x = -0.569$ $y = 1, x = 0$	<p>B1, B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p>	<p>B1 for 5, B1 for -7</p> <p>for solution of quadratic equation from (i)</p> <p>for use of logarithms to solve equation of the type $5^x = k$</p> <p>must be evaluated to 3sf or better</p>
<p>5 (i)</p> <p>(ii)</p>	$\frac{dy}{dx} = 3x^2 - \frac{1}{x}$ <p>When $x = 1, y = 1$ and $\frac{dy}{dx} = 2$</p> <p>Tangent: $y - 1 = 2(x - 1)$</p> $(y = 2x - 1)$ <p>Mid-point (5, 9)</p> $9 = 2(5) - 1$ <p>Alternative Method: Tangent equation $y = 2x - 1$ Equation of line joining (-2, 16) and (12, 2) $y = -x + 14$ Solve simultaneously $x = 5, y = 9$</p> <p>Mid-point (5, 9)</p>	<p>M1</p> <p>B1</p> <p>DM1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p>	<p>for attempt to differentiate</p> <p>for $y = 1$</p> <p>for attempt to find equation of tangent</p> <p>allow equation unsimplified</p> <p>for midpoint from given coordinates</p> <p>for checking the mid-point lies on tangent</p> <p>for a complete method to find the coordinates of the point of intersection</p> <p>for midpoint from given coordinates</p>
<p>6 (i)</p> <p>(ii)</p>	$(2 + px)^6 = 64 + 192px + 240p^2x^2 \dots$ $240p^2 = 60$ $p = \frac{1}{2}$ $(3 - x)(64 + 192px + 240p^2x^2 \dots)$ <p>Coefficient of x^2 is $180 - 192p$ $= 84$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1 ft</p> <p>M1</p> <p>A1</p>	<p>for $240p^2$ or $240p^2x^2$ or ${}^6C_2 \times 2^4 \times (px)^2$ or ${}^6C_2 \times 2^4 \times p^2$ or ${}^6C_2 \times 2^4 \times p^2x^2$</p> <p>for equating <i>their</i> term in x^2 to 60 and attempt to solve</p> <p>ft for $192p, 96$ or $192 \times \text{their } p$</p> <p>for $180 - 192p$</p>

<p>7 (i)</p> <p>(ii)</p>	$\mathbf{A}^{-1} = \frac{1}{5ab} \begin{pmatrix} b & -2b \\ a & 3a \end{pmatrix}$ $\mathbf{X} = \mathbf{BA}^{-1}$ $= \begin{pmatrix} -a & b \\ 2a & 2b \end{pmatrix} \begin{pmatrix} \frac{1}{5a} & -\frac{2}{5a} \\ \frac{1}{5b} & \frac{3}{5b} \end{pmatrix}$ $= \begin{pmatrix} 0 & 1 \\ \frac{4}{5} & \frac{2}{5} \end{pmatrix}$	<p>B1, B1</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>A1</p>	<p>B1 for $\frac{1}{5ab}$, B1 for $\begin{pmatrix} b & -2b \\ a & 3a \end{pmatrix}$</p> <p>for post-multiplication by inverse matrix</p> <p>for correct attempt at matrix multiplication, needs at least one term correct for their \mathbf{BA}^{-1} (allow unsimplified)</p> <p>for each correct pair of elements, must be simplified</p>
<p>8 (i)</p> <p>(ii)</p> <p>(iii)</p>	<p>$\overline{AB} = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$, at P, $x = -2 + \frac{1}{4}(12)$ so at P, $x = 1$ $y = 3 + \frac{1}{4}(16)$, $y = 7$</p> <p>Gradient of $AB = \frac{16}{12}$, so perp gradient = $-\frac{3}{4}$</p> <p>Perp line: $y - 7 = -\frac{3}{4}(x - 1)$ $(3x + 4y = 31)$</p> <p>$Q\left(0, \frac{31}{4}\right)$</p> <p>Area $AQB = 12.5$</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1 ft</p> <p>M1</p> <p>A1</p>	<p>for convincing argument for $x = 1$</p> <p>for $y = 7$</p> <p>for finding gradient of perpendicular</p> <p>for equation of perpendicular through their P</p> <p>Allow unsimplified</p> <p>ft on their perpendicular line, may be implied</p> <p>for any valid method of finding the area of the correct triangle, allow use of <i>their</i> Q; must be in the form $(0, q)$.</p>

9	(i)	$\log y = \log a + x \log b$ <table border="1" style="margin: 10px 0;"> <tr> <td>x</td> <td>2</td> <td>2.5</td> <td>3</td> <td>3.5</td> <td>4</td> </tr> <tr> <td>$\lg y$</td> <td>1.27</td> <td>1.47</td> <td>1.67</td> <td>1.87</td> <td>2.07</td> </tr> </table> <table border="1" style="margin: 10px 0;"> <tr> <td></td> <td>2</td> <td>2.5</td> <td>3</td> <td>3.5</td> <td>4</td> </tr> <tr> <td>$\ln y$</td> <td>2.93</td> <td>3.39</td> <td>3.84</td> <td>4.31</td> <td>4.76</td> </tr> </table> 	x	2	2.5	3	3.5	4	$\lg y$	1.27	1.47	1.67	1.87	2.07		2	2.5	3	3.5	4	$\ln y$	2.93	3.39	3.84	4.31	4.76	<p>B1 for the statement, may be seen implied in later work,</p> <p>M1 for attempt to draw graph of x against $\log y$</p> <p>A2,1,0 –1 each error in points plotted</p>
	x	2	2.5	3	3.5	4																					
$\lg y$	1.27	1.47	1.67	1.87	2.07																						
	2	2.5	3	3.5	4																						
$\ln y$	2.93	3.39	3.84	4.31	4.76																						
(ii)	<p>Gradient = $\log b$ $\lg b = 0.4$ or $\ln b = 0.92$</p> <p>$b = 2.5$ (allow 2.4 to 2.6)</p> <p>Intercept = $\log a$ $\lg a = 0.47$ or $\ln a = 1.10$</p> <p>$a = 3$ (allow 2.8 to 3.2)</p> <p>Alternative method: Simultaneous equations may be used provided points that are on the plotted straight line are used.</p> <p>$a = 3$ (allow 2.8 to 3.2) $b = 2.5$ (allow 2.4 to 2.6)</p>	<p>DM1 for attempt to find gradient and equate it to $\log b$, dependent on M1 in (i)</p> <p>A1</p> <p>DM1 for attempt to equate y-intercept to $\log a$ or use <i>their</i> equation with <i>their</i> gradient and a point on the line, dependent on M1 in (i)</p> <p>A1</p> <p>DM1 for a pair of equations using points on the line, dependent on M1 in (i)</p> <p>DM1 for solution of these equations, dependent on M1 in (i)</p> <p>A1 A1 for each</p>																									

<p>10 (a) (i) (ii) (iii)</p> <p>(b) (i)</p> <p>(ii)</p>	<p>360 60 36</p> <p>${}^8C_5 \times {}^{12}C_5$</p> <p>$56 \times 792 = 44352$</p> <p>4 places are accounted for Gender no longer 'important'</p> <p>Need ${}^{16}C_6 = 8008$</p> <p>Alternative Method $({}^6C_6 \times {}^{10}C_0) + ({}^6C_5 \times {}^{10}C_1) \dots ({}^6C_0 \times {}^{10}C_6)$ $1 + 60 + 675 + 2400 + 3150 + 1512 + 210 = 8008$</p>	<p>B1 B1 B1</p> <p>B1, B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p>	<p>B1 for each, allow unevaluated with no extra terms</p> <p>B1 Final answer must be evaluated and from multiplication</p> <p>M1 for realising that 4 places are accounted or that gender is no longer important</p> <p>A1 for 8008</p> <p>M1 for at least 5 of the 7 cases, allow unsimplified</p>
<p>11 (a)</p> <p>(b)</p>	<p>$2 \cos 3x - \frac{\cos 3x}{\sin 3x} = 0$</p> <p>$\cos 3x \left(2 - \frac{1}{\sin 3x} \right) = 0$</p> <p>Leading to $\cos 3x = 0, 3x = 90^\circ, 270^\circ$</p> <p>$x = 30^\circ, 90^\circ$</p> <p>and $\sin 3x = \frac{1}{2}, 3x = 30^\circ, 150^\circ$</p> <p>$x = 10^\circ, 50^\circ$</p> <p>$\cos \left(y + \frac{\pi}{2} \right) = -\frac{1}{2}$</p> <p>$y + \frac{\pi}{2} = \frac{2\pi}{3}, \frac{4\pi}{3}$</p> <p>so $y = \frac{\pi}{6}, \frac{5\pi}{6}$ (0.524, 2.62)</p>	<p>M1</p> <p>DM1</p> <p>A1</p> <p>DM1</p> <p>A1</p> <p>M1</p> <p>DM1</p> <p>A1, A1</p>	<p>for use of $\cot 3x = \frac{\cos 3x}{\sin 3x}$, may be implied</p> <p>for attempt to solve $\cos 3x = 0$ correctly from correct factorisation to obtain x</p> <p>A1 for both, no excess solutions in the range</p> <p>for attempt to solve $\sin 3x = \frac{1}{2}$ correctly to obtain x</p> <p>A1 for both, condone excess solutions</p> <p>for dealing with $\sec \left(y + \frac{\pi}{2} \right)$ correctly</p> <p>for correct order of operations, must not mix degrees and radians</p>

12 (i)	$\overline{AQ} = \lambda \mathbf{b} - \mathbf{a}$	B1	
(ii)	$\overline{BP} = \mu \mathbf{a} - \mathbf{b}$	B1	
(iii)	$\overline{OR} = \mathbf{a} + \frac{1}{3}(\lambda \mathbf{b} - \mathbf{a})$ or $\lambda \mathbf{b} - \frac{2}{3}(\lambda \mathbf{b} - \mathbf{a})$ $= \frac{2}{3}\mathbf{a} + \frac{1}{3}\lambda \mathbf{b}$	M1 A1	for $\mathbf{a} + \frac{1}{3}$ their (i) Allow unsimplified
(iv)	$\overline{OR} = \mathbf{b} + \frac{7}{8}(\mu \mathbf{a} - \mathbf{b})$ or $\mu \mathbf{a} - \frac{1}{8}(\mu \mathbf{a} - \mathbf{b})$ $= \frac{1}{8}\mathbf{b} + \frac{7}{8}\mu \mathbf{a}$	M1 A1	for $\mathbf{b} + \frac{7}{8}$ their (ii) Allow unsimplified
(v)	$\frac{2}{3}\mathbf{a} + \frac{1}{3}\lambda \mathbf{b} = \frac{1}{8}\mathbf{b} + \frac{7}{8}\mu \mathbf{a}$ $\frac{2}{3} = \frac{7}{8}\mu, \mu = \frac{16}{21}$ Allow 0.762 $\frac{1}{3}\lambda = \frac{1}{8}, \lambda = \frac{3}{8}$ Allow 0.375	M1 A1 A1	for equating (iii) and (iv) and then equating like vectors